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Prospective Elementary and Middle School Teachers' Knowledge of Linear Functions: A Quantitative Approach

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Built upon existing studies, this study is designed to take a closer look at prospective teachers' mathematics knowledge with a proposed knowledge structure, i.e., four constructs in representation flexibility and one construct of procedural skills. Statistical analyses of data supported the proposed knowledge structure. The results show that prospective teachers perform better on procedural skills than representation flexibility. A careful examination of their performance on various constructs of representation flexibility reveals further details on prospective teachers' knowledge strength and weakness. Across programs, middle prospective teachers performed better than elementary prospective teachers on almost all constructs of the test.

Key words: Prospective teachers, teacher education, representation, algebra.

Rationale and Objectives

As teaching has been recognized as a vital part of enhancing students' academic achievement (NCTM, 1991; NCTAF, 1996), educational research in the past decade has dramatically increased emphasis on preparing and developing teachers who will take the lead in providing high quality classroom instruction (e.g., Sikula, 1996). Specifically, research findings have led to a general consensus that teachers' understanding the content knowledge they teach is essential to effective teaching (e.g., Ball & Bass, 2003; Brophy, 1991; Leinhardt & Smith, 1985; Stein, Baxter, & Leinhardt, 1990). It is widely known that American pre-service and in-service teachers lack sound mathematical understanding and skills (e.g., An, Kulm, & Wu, 2004; Ball, 1990; Ma, 1999). This paper does not intend to offer more evidence of teachers' lack of mathematical understanding, nor will it propose possible remedies that are often readily available in the educational system. Instead, our study is designed to take a closer look at prospective teachers' knowledge of mathematics content. Built upon other existing studies, we took a structural approach and provide diagnostic information about prospective teachers' understanding of mathematics. In particular, many studies have focused on students' use of multiple representations and the understanding of the connections between them (e.g., Eisenberg & Dreyfus, 1996; Moschkovichs, Schoenfled & Arcavi, 1993). This study took a similar perspective in examining prospective teachers' ability to transfer between and within multiple representations, which we termed as *representation flexibility*. Unlike other studies on teachers' content knowledge using a qualitative approach, we took a quantitative method approach to detect possible strengths and weaknesses in prospective teachers' understanding of mathematical knowledge. By focusing on the topic of linear function, this study aimed to answer the following questions:

1. How well do prospective elementary and middle teachers understand the topic of linear functions as measured with a knowledge structure of representation flexibility and procedural skills?
2. Does prospective teachers' content knowledge of linear functions vary for those in elementary and middle school teacher education programs?

Literature Review

Algebraic, graphical and tabular forms are three prominent representations of functions (Moschkovich et al., 1993). The ability to translate between various representational systems, such as visual representations (tabular and graphical) and symbolic representations, is a strong indicator for conceptual understanding (Lesh, Post & Behr, 1987). Therefore, besides procedural skills, this study defined the ability to transfer between and within various representations as measures of conceptual understanding of linear functions. Teachers conceptual understanding was segmented into four constructs based on representation flexibility. They are (a) flexibility within algebraic representations (Brenner et al., 1997; Stump, 2001), (b) flexibility within visual representations (e.g., O'Callaghan, 1998), (c) flexibility between visual and algebraic representations (Knuth, 2000; Moschkovich et al.), and (d) flexibility with real-life situations (e.g., Leinhardt, Zaslavsky, & Stein, 1990) (see Table 1).

Table 1

Framework of Representation Flexibility

Flexibility within algebraic representations	Flexibility within visual representations	Flexibility between visual and algebraic representations	Flexibility with real-life situations
• Transformations among various algebraic representations • Different expressions for slopes	• Local property transformation • Global property transformation	• Cartesian connection • Entity-oriented connections	• Situation → Equation • Equation → Situation

Flexibility within Algebraic Representations (FWAR)

There are several forms that can be used to express linear functions. These include the general form $ax + by + c = 0$, slope-intercept form $y = \mathrm{m}x + \mathrm{b}$ or $y = -(\mathrm{a/b})\, x - \mathrm{c/b}$, the point slope form $(y - y_1) = \mathrm{m}\,(x - x_1)$ and the two-point form $(y - y_1)\,(x_2 - x_1) = (x - x_1)\,(y_2 - y_1)$. Some forms are easier to use than others based on specific problem situations. Students need to be able to understand the similarities and differences of different forms.

Flexibility within Visual Representations (FWVR)

FWVR is subcategorized into (a) local property transformations, which are ordered pairs and graph transformations, such as plotting a number of points (ordered pairs) and finding their graphical properties (ordered pairs); (b) global property transformations, which is more focused on trend direction, such as examining the properties of increasing and decreasing intervals rather than considering individual ordered pairs.

Flexibility between Visual and Algebraic Representations (FBVAR)

FBVAR refers to the ability to transfer flexibly between visual and algebraic representations, such as, to recognize relevant properties of algebraic and visual representations, and to make connections among them when treating functions as an entity. "From the process perspective, a function is perceived of as linking x and y values: For each value of x, the function has a corresponding y value" (Moschkovich et al., 1993, p. 71). The crucial information contained in the process perspective is in the statement of a *Cartesian connection* which is "a point is on the graph of the line L if and only if its coordinates satisfy the equation of L" (p. 73). In other words, when a graph goes through a point, then coordinates of that particular point satisfy the equation of that graph. In comparison, the object perspective refers to when "a function or relation and any of its representations are thought of as entities—for example, algebraically as members of parameterized classes, or in the plane as graphs that, in colloquial language, are thought of as being 'picked up whole' and rotated or translated" (p. 71). Based on Moschkovich et al.'s framework, two aspects of Flexibility between Algebraic and Visual representations are organized: (a) Cartesian connection and (b) Entity-oriented connections, which involve recognizing relevant properties of algebraic and visual representation, and making connections among them when treating functions as an entity. For example, the size of the slope m in the equation of $y = mx + b$ relates to the steepness of the graph, and whether the slope m is negative or positive decides the direction of the graph.

Flexibility with Real-life Situations (FWRS)

FWRS consists of two characteristics: (a) the ability to transfer from functions to a word problem situation, and (b) the ability to transfer from word problem situations to various forms of functions.

Procedural Skills (PS). Since procedural skills are one component of teachers' content knowledge, it is also included as part of this theoretical framework in addition to the four constructs of representation flexibility. Procedural skills consist of manipulating symbols and performing algebraic procedures.

Method

Participants

This study included prospective teachers who were in their last stages of study in teacher education programs. Specifically, a total of 104 prospective teachers from a large, southern university enrolled in elementary and middle school teacher education programs in the Fall of 2006 participated in the study. They had already taken all of the required mathematics courses and were completing methods courses in teaching mathematics. A majority of the participants were seniors and the rest were a few juniors.

Instrumentation

Three instruments were employed to collect the data: a general survey, a test, and selected participant interviews.

Survey

A survey was designed to obtain three aspects of participants' background information: (a) mathematics courses they completed, (b) mathematics education courses being taken, (c) their overall GPA, and (d) their mathematics GPA.

Test

A test was designed to assess prospective teachers' content knowledge (CK) based on the above framework on representation flexibility and procedural skills. It consisted of a total 15 test items with four constructs on representation flexibility and one construct on procedural skills (3 items for each construct). Some of the test items were adopted from Brenner et al. (1997), CBMS (2001), Knuth (2000), Leinhardt et al. (1990), and O'Callaghan (1998). The rest of the items were created for the study. The test was revised based on a pilot study and reviewed by other experts in mathematics education.

Data Analysis

Both quantitative methodologies were employed to analyze the data. Descriptive statistics were used on the data obtained from all five sections of the test. The mean and standard deviation for each item across different participants were reported, as well as for each participant across different items.

A confirmatory factor analysis applying structural equation modeling (SEM) techniques was conducted to establish the structure of different constructs of prospective teachers' content knowledge in linear functions, as well as to verify the validity and reliability of the test. A Split-half reliability test using SPSS was used to check its reliability. Moreover, Multiple Analysis of Variance (MANOVA) and discriminant analysis were conducted to analyze prospective teachers' performance on their content knowledge of linear functions across elementary and middle school teacher education programs. A predetermined design of contrast was used for the MANOVA analysis.

Results

Overview of Prospective Teachers' Content Knowledge (CK) in Linear Functions

Overall, results showed that prospective teachers perform better on items testing their procedural skills than those on representation flexibility, which indicates their insufficient conceptual knowledge of linear functions. Furthermore, a careful examination of their performance on four different constructs of representation flexibility revealed that prospective teachers had especially low scores on items on flexibility between visual and algebraic representations (FBVRAR) and flexibility with real-life situations (FWRS). A simple regression analysis suggested that participant's Grade Point Averages in Mathematics (GPAM), but not overall GPA, is a good predictor of their CK as measured in this study.

Measurement Models: Prospective Teacher's CK Measures and Structure

A confirmatory factor analysis using structural equation modeling (SEM) techniques was performed to confirm the components of the test. Both a Measurement Model of the test (see Figure 1) and a Second Order Model (see Figure 2) were constructed. The chi-square statistic, Comparative Fix Index (CFI), and Root Mean Square Error of Approximation (RMSEA) are reported in Table 1. Both the Measurement Model and Second Order Model fit the data well according to the chi-square statistic with p-value .239 and .175 respectively. Further, CFI larger than .90 and RMSEA smaller than .05 also indicate a good fit for the models. In addition, as shown in Table 2, the standardized path loadings of the five constructs on overall latent construct of CK in the second order measurement model suggested that prospective teachers' performance on items on flexibility within visual representations was the most important component in predicting their CK.

Table 2

Goodness of Fit Indices for Testing Prospective Teachers' CK

Model	Chi-square	df	P-Value	CFI	RMSEA (C.I.)
Measurement Model	90.726	82	.239	.957	.032 (.000-.065)
Second Order Model	99.214	87	.175	.940	.037 (.000-.067)

Table 3

Standardized Path Loading of Latent Constructs on CK

	Standardized Path Loading	S.E.
FWAR	.570	.422
FWVR	.859	.087
FBVRAR	.095	.068
FWRS	.493	.368
PS	.558	.141

Analysis of Variance: Elementary and Middle School Prospective Teachers' CK

MANOVA and discriminant analysis were performed to compare performance by levels and constructs of content knowledge (CK). As shown in Table 3, elementary and middle school prospective teachers' scores were significantly different on CK. The Wilks' Lambda is statistically significant, justifying the discriminant function of the five CK constructs in classifying prospective teachers from different programs based on their performances. Moreover, the univariate ANOVA information shows that elementary and middle school prospective teachers differed significantly on items assessing flexibility within algebraic representations, flexibility within visual representations, flexibility between visual and algebraic representations, and procedural skills ($p < .01$), but not on items assessing flexibility with real-life situations ($p = .769$). Compared to middle school prospective teachers, their elementary counterparts had poorer performance not only on almost all constructs of representation flexibility but also on procedural skills. No differences were found on items assessing flexibility with real-life situations (FWRS). This may be due to the challenge it presents to both elementary and middle school prospective teachers.

Table 4
Multivariate Analysis of Prospective Teachers' CK between Different Levels of Teacher Education

	Wilks' Lambda	Sig.	Box's M	Sig	Log Determinants Group 0	Log Determinants Group 1
Level of Teacher Education	.015	.000	59.665	.000	2.122	-1.516

Significance of the Study

Previous studies on teachers' content knowledge generally relied on a qualitative description of interviews and surveys. This study was designed to take a closer look at prospective teachers' content knowledge through a structural and quantitative method approach. The results of the study allow us to develop a better understanding of prospective teachers' strengths and weaknesses in the case of linear functions. The results of this study show that flexibility with visual representations is the most important factor in predicting CK. This suggests that it would be beneficial for teacher educators to provide prospective elementary and middle school teachers more opportunities to solve problems with graphical representations of linear functions. Both elementary and middle school prospective teachers had difficulties with items assessing flexibility with real-life situations. This is of particular concern that should also be addressed. Therefore, it is incumbent upon teacher educators to expose both elementary and middle school prospective teachers to more problems related to real-life situations. Results show the lower performance of elementary prospective teachers on items assessing flexibility within algebraic representations, flexibility within visual representations, flexibility between visual and algebraic representations and procedural skills. This indicates that additional help is needed for elementary prospective teachers from their teacher educator as algebra is becoming an increasing important topic in elementary school curriculum. Reforming the preparation of mathematics teachers based on these ideas should make prospective teachers better prepared to teach the future elementary and middle school students they will serve.

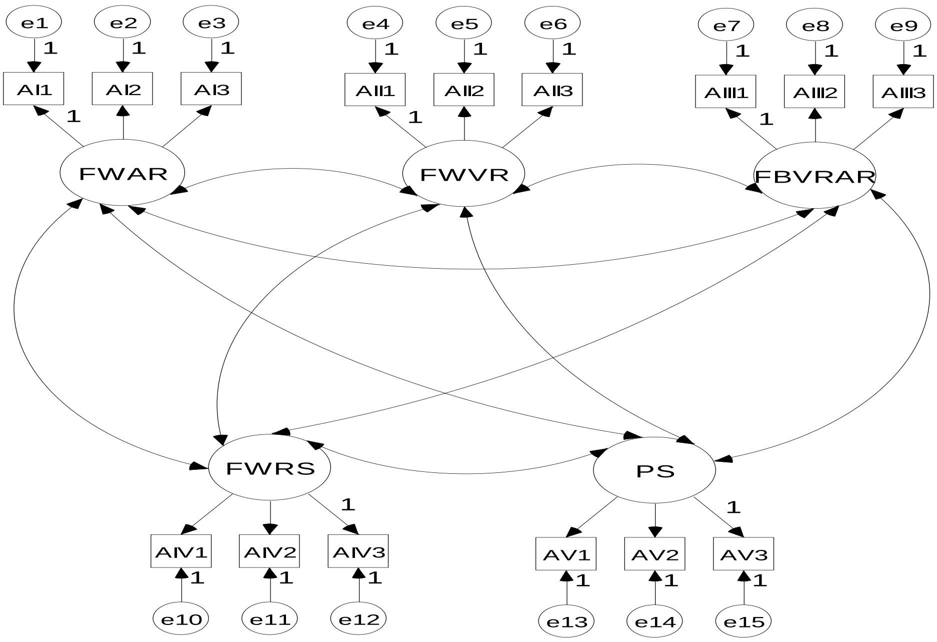

Figure 1. **Measurement model for prospective teachers' CK.**

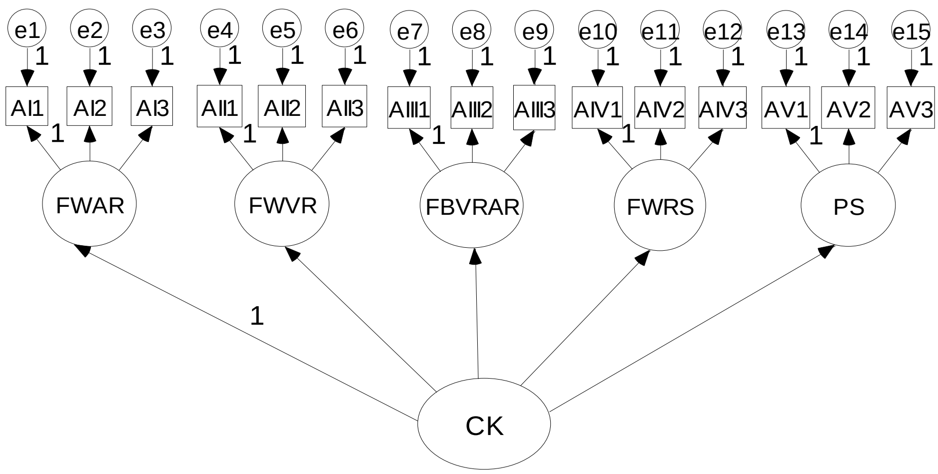

Figure 2. **Second order model for prospective teachers' CK.**

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