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The Structure of Mathematics Lessons in China

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This paper employs transcript coding research of lesson structure. This coding system was modified by the Japan researcher Shimizu based on the earlier coding edition of TIMSS. It divides lesson structure into 13 sub-codes, which are: 1.reviewing the previous lesson, 2.checking homework, 3.presenting the topic, 4.formulating the problem for the day, 5.presenting the problems for the day, 6.working on sub-problem, 7.working on the problem individually or in groups, 8.presentation by students, 9.discussing solution methods, 10.practicing, 11.highlighting and summarizing the main point, 12.assigning homework and 13.announcement of the next topic. According to coding analysis, the basic parts of Chinese mathematics lesson structure are: 1. "PP→DS", 2. "PP→WP", 3. "PP→PS", 4. "PP→WS→DS→WS→DS".

Key Words: LPS, video, mathematics lesson structure, coding analysis.

Problem Formulation

The study of mathematics lessons is of great importance, and related research attracts much attention from the international mathematics education sector. International lessons video studies, such as Third International Mathematics and Science Study Video (TIMSS) and The Learner's Perspective Study (LPS), are studies of this kind.

While the TIMSS Video Studies focused mostly on identifying the model lessons in order to describe the systems of teaching in each country (Givvin et al., 2005; Hiebert et al., 2003; Stigler & Hiebert, 1999), the LPS project is largely engaged in understanding teachers' instructional strategies by reporting the variety of forms and functions in which particular lesson events are carried out in classrooms by competent teachers (Clarke *et al*., 2006). In characterising national norms of teaching practice, the TIMSS video study accepted certain limitations. Only one camera was used, the primary focus of

data collection and analysis was the teacher, and only one lesson was videotaped for each classroom sampled. The Learners' Perspective Study intends to supplement the TIMSS Videotape Classroom Study data by in-depth documentation of the student perspective over several lessons in the same classroom. The available technology is utilized to combine videotape data with participants' reconstructions of classroom events. It will also use video-stimulated recall in interviews conducted immediately after the lesson to obtain participants' reconstructions of the lessons and the meanings that particular events held for them personally.

In 2003, Shimizu argues whether it is reasonable to choose a single lesson as an analysis unit in the study of the TIMSS video. He points out that it is not enough to describe and summarize teaching and learning characteristics of Japanese mathematics lessons according to only a single lesson. In Japan, the teacher usually plans a learning unit which contains several lessons. In this case, structures of a single lesson and a teaching unit will have significant differences. Moreover, these structures may play different roles in different lessons. It is also unreasonable to summarize a teaching mode based only on a single lesson from each country when making comparisons of mathematics lessons from different countries. Thus, to obtain some national norms, a sequence of lessons or a whole teaching unit should be chosen for study. Shimizu suggested capturing the lesson structure according to the separation of components within a lesson's structure. These specific components, referred to as a "lesson event", have attracted much attention in the LPS project.

China mainland has not taken part in the TIMSS study. China Hong Kong has a high ranking in the TIMSS study (4th of 38 countries in 1999; 3rd of 45 countries in 2003; Singapore occupies 1st place all the time). Thus, what are the characteristics of Chinese mathematics lessons and what is the lesson structure? These are the questions which our study attempts to answer.

Design of Research

In this paper, videos of twenty lessons presented by two teachers from Shanghai were chosen. Based on the lessons transcript analysis, we focused on the characteristics of "continuous lessons structure" and on the two teachers, expecting to discuss the essential characteristics of Chinese mathematics lessons. "Chinese mathematics lessons" in this paper are lessons on algebraic equations presented by two teachers from Shanghai. Although they are representative, they cannot represent all Chinese mathematics lessons.

The study of Chinese junior mathematics lessons structure has been poor until now. Thus, for a better comparison research of Chinese and international mathematics lessons in the LPS project, we must learn more about the characteristics of Chinese mathematics lessons. The analysis of lessons structure is significant.

In this paper, the following questions will be addressed: Does any lesson activity or structure appear more than once in one lesson taught by one teacher? Does any lesson structure or activity appear more than once in twenty lessons taught by two teachers? If yes, does this structure appear in other teacher lessons? If no, what is the difference?

Method

Data collection

This is how we obtain the data which we can analyze. Coding the video of SH1 and SH3's ten lessons about linear equations in two unknowns was done as follows:

(1) Watch video of SH1 and SH3's ten lessons.

(2) Coding behavior of teacher and students according to the transcript.

(3) Record the coding results of each lesson and make a coding statistics table (see table 1).

Specifications of Data

For these two teachers, the content as well as the process of these ten lessons are almost the same. Difference in education plans only appears in the 9^{th} and 10^{th} lessons. For SH1 the, 9^{th} lesson is about the graphical method of solving simultaneous linear equations in two unknowns, and the 10^{th} lesson is an exercise lesson on simultaneous linear equations in two unknowns. For SH3, the 9^{th} lesson is an exercise lesson on simultaneous linear equations in two unknowns, and the 10^{th} lesson is about the graphical method of solving simultaneous linear equations in two unknowns. The two teachers' plans on the content of the graphical method of solving simultaneous linear equations in two unknowns are almost the same. During the exercise lesson, the content of SH1 covers the whole chapter and is more than that of SH3, which only covers the method of solving simultaneous linear equations in two unknowns (to judge which method is more appropriate). For simplicity, we exchange

the order of the 9th and 10th lessons of SH3.

Data

Table 1

Statistics on the Coding of SH1 and SH3's Ten Lessons

	SH1	SH3
1	Linear equations in two unknowns and solution. RP，PT，FP，PP，WS，DS，WS，DS，PP，DS，P，HS，AH	PT，FP，PP，DS，WP，HS，PT，FP，PP，WP，DS，P，HS，AH
2	Rectangular coordinate plane and coordinates (a). RP，P，HS，PT，FP，PP，DS，HS，AH	PT，FP，PP，DS，P，DS，P，PS，P，HS，AH
3	Rectangular coordinate plane and coordinates (b). RP，PP，DS，PP，DS，P，DS，PP，DS，PP，DS，WP，PS，HS，AH	RP，PP，DS ，PP，WP，PS，HS，P，PS，PP，DS，PP，WP，PS，PP，DS，PP，DS，PP，WP，DS，P，HS，AH
4	Graphics of linear equations in two unknowns. RP，PP，DS，P，DS，PP，DS，PP，DS，PP，DS，HS，HS，AH	RP，PP，DS，WP，DS，PP，DS，P，HS，DS，P，HS，AH
5	Simultaneous linear equations in two unknowns. RP，P，PP，WP，WS，PS，WS，PS，HS，PP，DS，WS，WP，DS，PP，DS，PP，DS，P，HS，AH	PT，FP，PP，DS，HS，PP，DS，FP，P，PS，DS，P，DS，P，PP，WP，DS，HS，HS，AH
6	Method of substitution. RP，PP，DS，PP，DS，PP，DS，HS，P，HS，AH	RP，PP，WP，PS，HS，PP，WP，DS，PP，WP，PS，HS，P，DS，HS，AH
7	Method of elimination (a). RP，PP，DS，HS，PP，DS，PP，DS，P，DS，P，DS，HS，AH	FP，PP，DS，PP，WP，PS，PP，WP，DS，P，PP，WP，DS，HS，P，DS，HS，AH
8	Method of elimination (b).	

	RP，PP，DS，HS，P，DS，P，DS，HS，AH	RP，CH，WP，PS，DS，PP，DS，P，HS，AH
	Graphical method.	
9	RP，PP，DS，HS，PP，DS，P，DS，HS，HS，AH	RP，P，DS，PP，WP，PS，PP，PS，HS，P，DS，HS，AH
	Exercise lesson	
10	PP，DS，PP，DS，HS，PP，DS，PP，DS，HS，PP，PS ，DS，AH	RP，PP，WP，PS，PP，PS，DS，PP，PS ，DS，P，DS，HS，AH

Comment: As the order of the 9th lesson and 10th lesson are converse for SH1 and SH3, we exchanged the order of the 9th and 10th lessons of SH3 for simplicity.

Result and Analysis of the Study

The statistics of coding of mathematics problems proposed or converted by SH1 and SH3, are as follows in Table 2:

Table 2

Statistics on the Coding times of Presenting the Problems of SH1 and SH3

SH1	2	1	4	4	4	3	3	1	2	4
SH3	2	1	7	2	3	3	4	1	2	3

Among the ten lessons of SH1 and SH3, every lesson employs the coding of "presenting the topic". During the lessons of SH1, this coding is employed at least once a lesson, at most four times a lesson, and the average frequency is 2.8 times per lesson, while during the lessons of SH3, it is employed at least once a lesson, at most seven times a lesson, and 2.9 times per lesson for an average. This illustrates that in Chinese mathematics lessons, there are specific mathematics problems in every lesson, and solving mathematics problems runs throughout the whole process.

During the lessons of SH1 and SH3, there are mainly four modes for solving mathematics problems.

Model 1: "PP→DS"

Mode of “PP→DS” appears frequently in the lessons of SH1 and SH3. Following are the statistics about the times and the percentage of “PP→DS” mode in the ten lessons of SH1 and SH3.

Table 3

Statistics on Appearance Times of "PP->DS" Mode During Ten Lessons of SH1 and SH3

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
SH1	1	1	4	4	3	3	3	1	2	3
SH3	1	1	4	2	2	0	1	1	0	0

Table 4

Statistics on Appearance Percentage of "PP->DS" Mode During Ten Lessons of SH1 and SH3

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
SH1	50%	100%	100%	100%	75%	100%	100%	100%	100%	75%
SH3	50%	100%	57%	100%	67%	0%	25%	100%	0%	0%

It can be observed from Table 3 and Table 4 that: “PP→DS” mode is extensively utilized in lessons of SH1 and SH3. For SH1, every lesson utilizes “PP→DS”, and for SH3, the majority of lessons (7/10) utilize “PP→DS”. Thus SH1 uses it more than SH3. Among the ten lessons of SH1, the 2nd, 3rd, 4th, 6th, 7th, 8th and 9th lessons utilize “PP→DS” mode completely (100%). The majority of the 5th and 10th lessons (75%) utilize this mode. Part of the 1st lesson (50%) utilizes this mode. Among the ten lessons of SH3, the 2nd, 4th and 8th lessons utilize “PP→DS” mode completely (100%). The majority of the 5th lesson (67 %) utilizes this mode. Part of the 3rd and 1st lessons (57% and 50%) utilizes this mode. Minority of the 7th lesson (25%) utilizes this mode. The 6th, 9th and 10th lessons do not utilize this mode at all (0%).

According to the above analysis, it is observed that “PP→DS” mode is extensively utilized in Chinese mathematics lessons. Since the capacity of Chinese mathematics lessons is great, teachers usually employ the mode of

"first propose problems and then discuss the solution". This is an efficient way to achieve a great capacity. During the process of discussing solutions, teachers can adjust the scheduling based on students' reaction, helping students learn more within a limited time. However, the disadvantage of this mode is that it will restrict the thinking of students, requiring students to follow teachers closely and take part in the discussion environment constructed by the teacher. If students do not follow their teachers, the effect is very poor; on the contrary, if students' thinking exceeds that of the teachers, the learning initiative will be frustrated. But as the range of classes in China is great, the teacher cannot attend to every student, so "PP→DS" mode is compatible with Chinese mathematics lessons.

It can be observed that "PP→DS" mode contains several sub modes.

4.1.1"PP→DS→PP→DS"

After presenting the problems, the teacher discusses solution methods, and then presents another problem and discusses solution methods. This mode appears in both lessons of SH1 (4/10) and SH3 (1/10).

Following is a part of a live recording about the "PP→DS→PP→DS" mode in the 3rd lesson of SH1.

T: So boys and girls, let's look at this question together. It says, draw the following points on the rectangular coordinate plain according to their respective coordinates. So let's look at point A. Look, what's the abscissa of point A?

T: //Three.

E: //Three.

T: The ordinate is…

T: //Two.

E: //Two.

T: So look at this boys and girls. Which quadrant does point A belong to?

E: The first quadrant.

T: The first quadrant. So now let's look at this. Firstly, its abscissa is three, right? So we mark this point as N. This point is N. Okay. Now we should draw a line perpendicular to the x-axis from point N. We've just discussed that point A is in the first quadrant. Look at this, we draw a line perpendicular to the x-axis from point N, (...) or if the point is in the fourth quadrant, if the point is in the fourth quadrant,

what should we do? Ah, look at this; we have to draw it as a straight dotted line. First of all, how to do this? Draw a line perpendicular to the x-axis. Then think about it, we have just discussed that the ordinate is two. So we get the point on the y-axis and mark this point as M. Then we draw a line perpendicular to the y-axis from M.

[Teacher is writing on the blackboard and students are making notes]

T: *Okay, then these two perpendicular lines (…) So, this point is A. After confirming point A's location, we can write down the coordinates. The abscissa is three and the ordinate is two. So we have confirmed the location of this point. Then, think about it. How can we find point B's coordinate using the same method?*

00: 11: 30

T: *How's the location of point B? Okay, let's look at this together. The abscissa two… negative two and one over two. Okay, let's do it together. Look, which quadrant does it belong to?*

E: *Second… fourth.*

T: *Ah, the abscissa is positive and the ordinate is smaller than zero. So in the same way, we have to find… the point on the x-axis. Oh, find the point, (…) then we can get… if we indicate it by letter G, we can draw a line perpendicular to the x-axis form G.*

[Teacher is drawing on the blackboard and students are jotting down the notes]

T: *Okay, a perpendicular line. Now look at this, the ordinate is negative two and one over two. So it lies in between negative two and negative three on the y-axis. Therefore, this point is negative two and one over two. Then we can mark it by a letter. If we use Q to represent it, then what's our next step? What should we do from point Q?*

T: *//Draw a line perpendicular to the y-axis*

E: *//Draw a line perpendicular to the y-axis.*

T: *Okay, so there are two perpendicular lines and they intersect*

at one point. This point is the location of B. Now we can mark the coordinates. The abscissa is four and the ordinate is two and one over two. Okay. Let's continue. How about point C? Point C's abscissa is…

T: //Zero.

E: //Zero.

T: How about the ordinate?

T: //Three.

E: //Three.

T: So, what must the point's location?

T: //On the y-axis.

E: //On the y-axis.

T: Good. So it's this point, right? This point… so… look at this. Oh, how about point D? Its abscissa is negative two and the ordinate is zero. Then which axis does it lie on?

E: //x-axis.

T: //x-axis. So on the x-axis, its abscissa is three and its ordinate is zero. This point is okay… so next let's look at point E…

00: 14: 00

T: The abscissa is negative one and the ordinate is negative two. Boys and girls, look, point E is in…

E: The third quadrant.

T: Which quadrant? The third. The abscissa is smaller than zero, and how about the ordinate? It's also smaller than zero. So we can do it by the same method we've just used, right? Oh, so let's say the point on the x-axis is negative one and we have to give it a letter like L here. Boys and girls, look, what should I do from L to the x-axis?

E: //Perpendicular line.

[Teacher is drawing and students are jotting down the notes]

T: //Draw a perpendicular line. Okay. Look the ordinate is negative four, so we must be able to find negative four on the y-axis. If we mark this point as S, then boys and girls, what should we do from S?

T: //Draw a line perpendicular to the y-axis.

E: //Draw a line perpendicular to the y-axis.

T: Draw a line perpendicular to the y-axis. Look, now the two perpendiculars intersect each other. This point (…) then its abscissa is negative one and its ordinate is negative four. Okay, so look at this. Now we have marked the five points on the coordinate plain for this question. Let's continue, read question two. [Teacher presents the transparency]

These living recordings also illustrate that Chinese mathematics lessons have characteristics of great capacity to instruct a great quantity of information at a quick pace.

4.1.2 "PP→DS→P"

After presenting the problems, the teacher discusses solution methods, and then allows students to practice. This mode appears in the lessons of both SH1 (6/10) and SH3 (3/10).

"PP→DS→P" mode has two sub modes: "PP→DS→P→HS" and "PP→DS→P→DS". That is after practice, the teacher can summarize about this type of problem, or discuss the practice, depending on students' reactions. If students do well during practice, there is no need to discuss solutions and only emphasis and summary are enough, otherwise, the teacher can discuss the solutions of the practice problems with the students.

4.1.3 "PP→DS→HS"

After presenting the problems, the teacher discusses the solutions with the students, and then summarizes this kind of problem. This mode appears in the lessons of both SH1 (5/10) and SH3 (1/10).

In a word, "PP→DS" is a traditional mode in Chinese mathematics lessons. Thus both SH1 and SH3 utilize this mode. Compared with SH3, SH1 utilized it more, and is more traditional.

Mode 2: "PP→WP"

"PP→WP" mode appears in lessons of both SH1 and SH3. Following are the statistics about the times and the percentage of "PP→WP" mode in the ten lessons of SH1 and SH3.

Table 5

Statistics on Appearance Times of "PP->WP" Mode During Ten Lessons of SH1 and SH3

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
SH1	0	0	0	0	1	0	0	0	0	0
SH3	1	0	3	0	1	3	3	0	1	1

Table 6

Statistics on Appearance Percentage of "PP->WP" Mode During Ten Lessons of SH1 and SH3

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
SH1	0%	0%	0%	0%	25%	0%	0%	0%	0%	0%
SH3	50%	0%	43%	0%	33%	100%	75%	0%	50%	33%

It can be observed from Table 5 and Table 6 that "PP→WP" mode appears in lessons of both SH1 (1/10) and SH3 (7/10). For SH3, the 6th lesson utilizes this mode exclusively (100%). The majority of the 7th lesson (75%) utilizes this mode. Parts of the 1st, 9th (50%), 3rd (43%), 5th and 10th (33%) lessons utilize this mode.

According to the above analysis, we can observe that the "PP→WP" mode exists in Chinese mathematics lessons, and its frequency appearance varies depending on teachers' styles. The difference of "PP→WP" and "PP→DS" is the function of the teacher. In "PP→DS" mode, the teacher plays the main part, requiring students to follow teacher's thinking and take part in the discussion; In "PP→WP" mode, the teacher plays the role of conducting, providing an environment to make students study by themselves and solve problems. In "PP→WP" mode, the teacher's functions are relatively weak, leaving more space to the students, allowing them to study by themselves. However, since in this mode, the teacher has little control of students, students may go in a wrong direction, which requires the teacher to interact with every student. Although in China, the number of students in a class is large and it is very hard for a teacher to pay attention to each student, it is the trend for Chinese mathematics teachers to utilize this mode.

It can be observed that "PP→WP" contains several sub modes.

4.2.1 "PP→WP→PS"

After presenting the problems, the teacher makes students solve problems via group or independently, and then explains the solutions.

The characteristics of this mode is discussion and interaction between teacher and a single student during the process when students solve the

problem via group or independently. If the teacher is satisfied with the solution which is made by the students in a group or independently, he can make students explain their ideas.

4.2.2 “PP→WP→DS”

After presenting the problems, the teacher makes the students solve problems via group or independently, and then discuss as solutions methods.

The characteristics of this mode is discussion and interaction between teacher and a single student during the process that students solve the problem via group or independently. If a teacher observes that students cannot make out the solutions via group or independently, he can discuss solutions with students together.

In a word, in “PP→WP” mode, the teacher plays the role of conducting, making students the main part of the mode. This is being accepted and utilized by Chinese mathematics teachers gradually.

Mode 3:”PP→PS”

“PP→PS” mode appears in the lessons of both SH1 and SH3, but not as frequently. Following are the statistics about the times and the percentage of “PP→PS” mode in the ten lessons of SH1 and SH3.

Table 7

Statistics on Appearance Times of "PP→PS" Mode During Ten Lessons of SH1 and SH3

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
SH1	0	0	0	0	0	0	0	0	0	1
SH3	0	0	0	0	0	0	0	0	1	2

Table 8

Statistics on Appearance Percentage of "PP→PS" Mode During Ten Lessons of SH1 and SH3

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
SH1	0%	0%	0%	0%	0%	0%	0%	0%	0%	25%
SH3	0%	0%	0%	0%	0%	0%	0%	0%	50%	67%

It can be observed from table 7 and table 8 that: "PP→PS" appears in lessons of both SH1 and SH3, but not as frequently. The statistics show that "PP→PS" mode appears in the 10^{th} lesson of both SH1 and SH3. The10^{th} lessons of SH1 and SH3 are exercise lessons. Thus "PP→PS" usually appears in the exercise lessons. This mode also appears at the end of the 9^{th} lesson of SH3. According to the above analysis, we can deduce that the "PP→PS" mode usually appears in the exercise lessons or when the main part of the knowledge has been taught. In these cases, students have acquired all the knowledge in that lesson and can make explanations when the teacher proposes problems.

It can be observed that "PP→PS" mode has several sub modes:

(1) "PP→PS→HS" mode. After proposing mathematics problems, the teacher lets the student explain ideas and solutions, and then make summaries.

(2) "PP→PS→DS" mode. After proposing mathematics problems, the teacher lets the students explain ideas and solutions, and then discusses solutions with the students' altogether.

Mode 4: "PP→WS→DS→WS→DS"

This mode only appears in the 1^{st} lesson of SH1, which illustrates that in the Chinese junior students' lessons, teachers mainly focus on solving problems and pay less attention on the relationship between problems.

In a word, Chinese mathematics lessons focus on "solving problems" and teachers have many different ways to solve problems.

References

Clarke, D. J. (2003, April). *The structure of mathematics lessons in Australia*. Symposium conducted at the annual meeting of the American Educational Research Association, Chicago.

Clarke, D., & Suri, H. (2003, Month??). *Issues of voice and variation: Developments in international Comparative Research in Mathematics Education*. Symposium conducted at the annual meeting of the American Educational Statistics Conference

Clarke, D., Emanuelsson, J., Jablonka, E., & Mok, I. (Eds.). (2006). *Making connections: Comparing mathematics classrooms around the world*. Rotterdam: Sense.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K., Hollingsworth, H., Jacobs, J., Chui, A., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E.,

Etterbeck, W., Manaster, C., Gonzales, P., & Stigler, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study.* Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Jablonka, E. (2003, Month??). *The structure of mathematics lessons in German classrooms: Variations on a theme*. Symposium conducted at the annual meeting of the American Educational Statistics.

Leung, F. K. S. (1995). *The mathematics classroom in Beijing, Hong Kong and London* [J]. Educational Studies in Mathematics.

Lopez-Real, F.J., & Mok, I.A.C. *Is there a Chinese Pedagogy of Mathematics Teaching?* Published in CERCular, nos. 1 &2 of 2001

Mesiti, C., Clarke, D., & Lobato, J. (2003). *The structure of mathematics lessons in the United States.* Symposium conducted at the annual meeting of the American Educational Statistics. .

Shimizu, Y. (2003). *Capturing the structure of Japanese mathematics lessons as embedded in the teaching unit*. Symposium conducted at the annual meeting of the American Educational Research Association, Chicago.

.Shimizu, Y. (2002). *Discrepancies in perceptions of lesson structure between the teacher and the students in the mathematics classroom*. Symposium conducted at the annual meeting of the American Educational Statistics.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press

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