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Experimental Study on Effect of Different Mathematical Teaching Methodologies on Students' Performance

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This study designed four teaching methodologies based on Chinese mathematics teaching situation. It represents four teaching philosophies in mathematics. Investigations using questionnaires filled by teachers and sample study method accomplished by students learning mathematics were employed to detect the factors affecting teacher's teaching philosophy and evaluate their effects.

Key words: Mathematical philosophy, exampling method, labeling method, diagram method, equation method.

Question and Researching Purpose

It seems that treating students as the principal part of teaching is a common view. 'New Lesson Standard' tried to prove this theory through concrete examples and demands. However, on the one hand research shows long term vocational training made teachers organize their own and sealed 'time mark' mathematical view, which led teachers to make a decision about their teaching methods and changed the mathematics view from scientific modality to educational modality (Ernest, 1991). On the other hand, students apprehend mathematics without philosophical effects and they totally treat mathematics on the aspect of psychological sense (Anderson, Famell, & Sauers, 1984; Tang & Lo, 2004); As a result，there is a underlying conflict of choosing learning and teaching methods between students and teachers. This research aimed at illustrating the conflict through analyzing and packing up four different teaching methods of one typical knowledge point.

Experimentation Design

A survey was made for teachers based on the four modes and designed example case aimed at students in the sense of psychology. Through these two aspects, this survey gave an example to the otherness between students and teachers choosing different methods.

Four Mathematical Teaching Methods Delegating Different Concepts

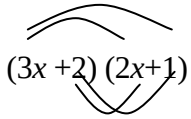
Formula $(ax+b)(cx+d)=acx^2+(ad+bc)x+bd$ can be designed as a mode treating for

four teaching methods as below:

Exampling Method

Calculate：$(3x+2)(2x+1)$

Explanation：Solve the question by the chart below



This method is the recommended teaching method for the current middle school mathematics textbook. It has conspicuous advantages for it provides both the algorithm and the calculating steps. Further, there are many similar applications in the textbook such as factoring through cross-multiplication, using the root axis method to find out solution interval of inequality and so forth. Both methods can make use of these intuitional sketch maps and this is why teachers are in favor with this method.

Labeling Method

Calculate: $(3x+2)(2x+1)$,

Explanation: Signing every terms name in the parentheses

Outer term	Inner term	Inner term	Outer term
(3 *x*	+ 2)	(2 *x*	+ 1)
First term	Outer term	Inner term	Outer term

Removing the parentheses，we can get：

$$3x\times 2x + 2\times 1 + 2\times 2x + 3x\times 1$$

so, $(3x+2)(2x+1) = 6x^2 + 7x + 2$

One characteristic of such a solution (Anderson, Famell, & Sauers, 1984) is marking the target and making it friendly to read. Thus, the solution of this question can be concluded into an oral law, "qian qian jia hou hou, li li jia wai wai", which is a regular method in teaching, remembering by heart easily. For example, when teaching students how to solute Quadratic Inequality, under the circumstance of inequality more than zero，we can make an oral law like "da yu di, xiao yu xiao di", promoting memory for the solution. In another case, when teaching complex number formula,

$$(a+bi)(c+di)=(ac-bd)+(bc+ad)i,$$

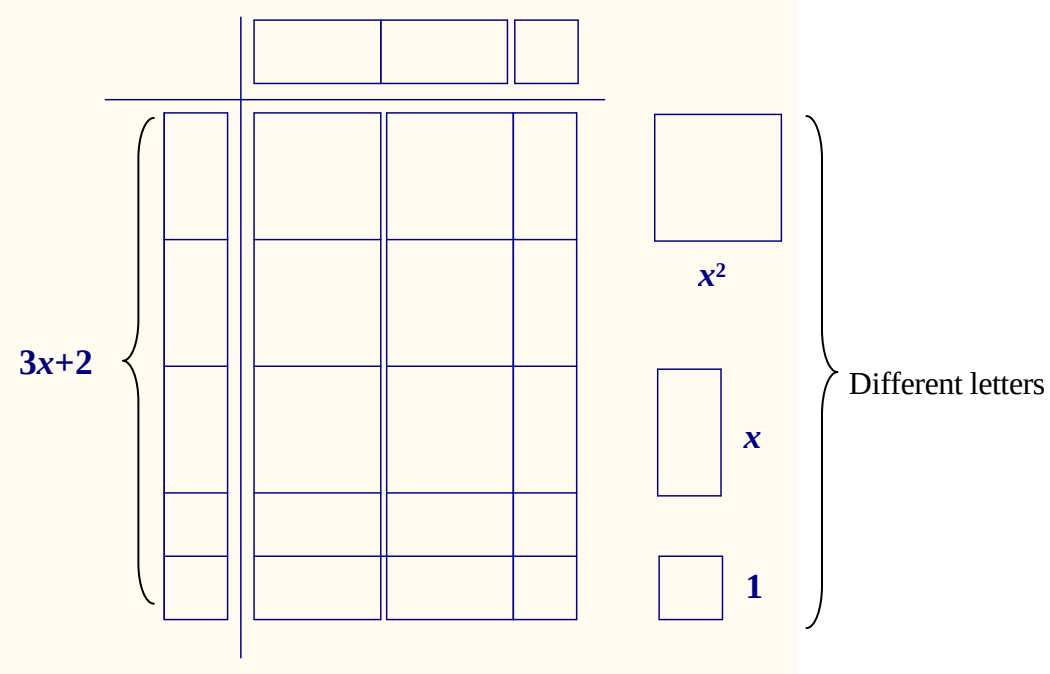
the PEP teaching reference book noted "qian qian jian hou hou, li li jia wai wai", in order to help students remember the formula. Needless to say, all of these labeling methods are produced through teachers' valuable experience in double- base-training.

Diagram Method

For example：$(3x+2)(2x+1)$

As below，use different diagrams representing

$2x+1$



There are 6 **big squares，7 rectangles and 2 little squares above.**

As a result, we can deduce as below： $(3x+2)(2x+1)=6x^2+7x+2$.

This method, advocated by the mathematical teaching standard of Chinese compulsory education, is extensively used in the experimental teaching materials of ECNU and BJNU. And Stein (1997) named this teaching method dealing with symbol calculation for highness cognitive teaching. In fact, this method derived from 'ou-tin complementary principle', which has an important status in Chinese archaic mathematics. And Liu Wei, a Chinese ancient mathematician, illustrated this method profoundly in noting *Textual Collation and Correction of the Chapter of Gou Gu of Nine Chapters on the Mathematical Art*. What's more, 'ou-tin complementary principle' is the most easily understanding way in proving the Pythagorean proposition (Kang, 2002).

Equation Method

Given formula $(ax+b)(cx+d)=acx^2+(ad+bc)x+bd$,
how to calculate $(3x+2)(2x+1)$?
Explanation: According to the formula, we can get
$a=3$，$b=2$，$c=2$，$d=1$,
$(3x+2)(2x+1)=3\times2x^2+(3\times1+2\times2)x+2\times1=6x^2+7x+2$

Equation method, edited by ourselves, matches another teaching standing point emphasizing abstract symbol calculation and deductive and reasoning training in mathematics.

Teaching Methods that Teachers Chosen

Participants in the survey are 24 students that had taken related courses and had graduated from mathematics and science school in a southwest university in China.

Among those 24 teachers, nine are teach in elementary schools and 15 are teach in junior high schools and the shortest teaching age is three years and the longest is 12 years.

For the purpose of real result, before beginning the survey, the author explained the four methods on the blackboard in detail and impartially, without evaluating the four methods' advantages, disadvantages or the difference on the aspect of teaching. At the beginning of the survey, we emphasized the hypothesis of the research on the teachers that this would be a new lesson of polynomial multiplication and it would be the first time for students contacting this knowledge.

Diagram Method Ignored

In the question that "which of the four method have you never met?" nine from the 15 junior high school teachers answered they have never met diagram method and four from the 15 said they have never met labeling method. In the question that "which method do you usually use in teaching？" all the teachers answered the exampling method.

In the question that "Please choosing two methods form the four methods to teach the expansion of $(3x+2)(2x+1)$", ten chose exampling method and diagram method, five chose exampling method and labeling method and three chose exampling method and equation method.

When we asked the teachers to arrange the four methods from the most like to the least like, we found that ten put exampling method in the first, three put diagram method in the first, two put the equation method in the first and seven put the diagram last, four put the labeling last, three put the equation method in the last, one put the exampling method last. In addition, among the nine elementary school teachers, also five put the diagram method last.

This result showed that no one accept the diagram method. One possibility may be old textbooks never advocate the diagram method. What we are interested in the most is besides the recommendation from textbooks and the customs formed in teaching, what are other factors could influence on the result showed above?

Equation Method Getting Attention

The survey above is based on the teacher's standpoint. The following survey adds factors from students: under the circumstance that teaching this knowledge in the class of students lacking of good mathematical basis，ten chose the applying exampling method, two chose the labeling method, two chose the diagram method and one chose the equation method. Under the circumstance that teaching this knowledge in the class of students good at mathematics，seven chose the equation method, six chose the exampling method, three chose the diagram method and one chose labeling method.

Form the survey we can deduce that when teachers choose the teaching methods they were made a discreet considerations. It is actually easier to use the exampling method for students who lacking of good mathematical basis to understand the expansion as well as to form calculating skills. And since it is easier for students who good at mathematics to

remember the formula and understand abstract symbol operation, teachers prefer to choose the equation method aiming at training students' symbol operation skill.

Teachers lack of "Mathematics Magic" View in Their Mathematical Philosophy

From the series of surveys above we can deduce that it is exampling method and equation method that teachers usually used on their daily lives. After analyzing these two methods, it's easy to explain why teachers make such a decision, choosing different methods from exampling method to equation method orderly, depending on students from lowest mathematical level to highest level.

The exampling method gets the expansion through multiplicative distribution law and its essential concentrates on algorithm and the level matching the logic of mathematical calculation. When comparing to diagram method, it has a character of possesses more algebra calculation.

The equation method first demands students accept the formula

$$(ax+b)(cx+d)=acx^2+(ad+bc)x+bd,$$

and then using the method of undetermined coefficient to get concrete calculation（for example$(3x+2)(2x+1)$. It seems include 'axiomatization' idea as well as an abstract symbol operation process. Method of undetermined coefficient, however, is another important method emphasized in high school algebra education, relating to solve the equation, with 'stronger mathematics taste'.

The diagram method expresses algebra formula using area, providing an algebra operation background, relating geometrical area calculation to algebra calculation (Anderson, Famell, & Sauers, 1984). The most important point is diagram method illustrates how magic mathematics is: why does every single expansion can be formed from small rectangle into a big rectangle? Why does algebra calculation can be expressed by area? The magic of mathematics is regarded as the important energy goading human being explore mathematics (Livio,2002). However, given that the mystery contact between algebra and geometry cannot turned calculating skill at once, teachers consider this method to be unnecessary.

Students' Reflection on the Four Methods

This study used the example research technique of cognize psychology (Mo, 2000) and selected four solutions as samples that match four test questions to each solution, judging the degree and level of the four solutions by the students' test result. To find out which solution can be most advantageous for students understand the skill of expansion such as$(ax+b)(cx+d)$.

Experiment Process

The experimental object is students of grade six, who know some knowledge about area and can be able to calculate areas of square and rectangle. There are no minus in the sample and test questions. The students were from a District from a south city in city, 30 from A elementary school, 22 students from B elementary school, 17 students from C

elementary school, and 27 students from D elementary school, who are all over the average level at their grade of school. Before all participants participating in this survey, they should pass a test at first. Each school students participated in one different solution's test questions.

Testing content consisted one example and four test questions which were used for each school, but using different solution methods were used for different schools. This experiment aimed at testing the controllable degrees of four methods of expansion of$(ax+b)(cx+d)$: each student had a two-page test paper that the first page contained the example and the second page included the four test questions. A mathematics teacher, who was trained for the experiment target and the teaching method, explained the example data. The whole testing time was 18 minutes. First, using 3 minutes telling students a short mathematics story in order to make the students concentrate on the teacher, and then spent about 5 minutes to teach the solution method. Finally, the students were provided 15 minutes to finish the four test questions.

Table 1

The Summary of the Correct Rate of the Test for the First Time

	Exampling method	Labeling method	Diagram method	Equation method
Question 1	60	40.91	88.24	22.22
Question 2	60	36.36	47.06	11.11
Question3	60	22.73	76.47	14.81
Question 4	73.33	31.82	70.59	3.70
The least square mean	63.45^{a}	32.76^{b}	71.68^{a}	12.07^{c}

Analyze of Experimental Result

Testing result indicated that the diagram method and the exampling method almost have the same correct rate and this rate is much higher than the labeling method or the equation method. We made statistic of the correct rate of the testing objects (See table 1），and made arcsine conversion for the correct rate. Then we made variance analyzing whose model includes the question and solution effect. The result shown in the table 1 is that not the testing questions but the solution method which had a significant impact on the correct rate. The result of Duncan multiple comparisons shows the students receiving the diagram method or the exampling method got the highest correct rate which is much higher than the other two methods. But there was no conspicuous difference between these two methods.

Discussion of Experimental Result

The diagram method is easier for students to accept and remember this new knowledge and only the exampling method's effect is close to the diagram method. It is obvious that the labeling method and the equation method can not match with students' cognize rule. Research regarded the diagram method as highly cognitive teaching function,

the exampling method（or labeling method）as lowly cognitive teaching function. The main difference between them is that diagram method gives student a relationship between images (area) and symbol operations, but exampling method (labeling method) provides a physical arithmetic. Teachers accepted exampling method generally, but high level diagram method exerts the same influence on helping students and this standing point has been proved in this research.

Conclusion and Implication

These four disposal methods of text symbol operation represent four current concepts of mathematics teaching. Although the equation method was designed by us, teachers maintained that this method can representing mathematics request and training good student's calculating skill. Furthermore, the reason that the diagram method was ignored because teachers believed this method had no direct help form students' calculating skill. However, students' choice has significant difference from the teachers'. Through analyzing the samples, we concluded that the diagram method and the exampling method have the same effect on students learning.

The conclusion of the research：First，teacher's teaching methods come mainly from textbooks. In other words, textbook's teaching requirements determine teacher's choice. For examples, exercises and schoolwork in the textbook can all intensify the choice of teaching method. Second, the social education atmosphere forces teachers choosing a quick teaching method to guarantee students learn things quickly. This teaching method, however, always has no relationship with comprehension, but has relationship with imitation. What is more, teachers tend to choose methods in favor of abstract symbol operation or reasoning，but seldom take student's intuitive sensation into consideration. Finally, under the current teaching requests, teachers form mathematical philosophy which regards mathematics as a kind of form and hope students have ability to apply formal symbol operation quickly. Teachers create ways to help students remember knowledge and deal with some intuitive and associated teaching methods. For example, in using diagram method, teachers consider that it cannot has a rapid and significant impact on training students' instant operation skill, as a result, they refused to use the intuitive methods.

This study suggested that given examination of cultural mathematical (teaching) philosophy has a conspicuous difference with student's cognizing character; and teachers are the leaders in teaching, if they do not change their mathematical philosophy, it would be impossible for them to ensure students' main status in teaching essentially. Furthermore, this might not lead making the student-oriented and democratic culture of the classroom.

Textbook editions should regard high-level cognizant activity as a teaching mission and sufficiently take student's cognitive level into consideration, but should not regard follow-up work as a new lesson. At the same time, teacher's dynamic role is obvious. Once teachers formed their own mathematical philosophy, they would apply their own comprehension to teaching.

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