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An Insider's Perspective: "Variation Problems" and Their Cultural Grounds in Chinese Curriculum Practice

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Different cultures with their own advantages and disadvantages, rather than oppositional, are complementary and interrelated. As mainstream of international research is frequently from a 'Western' perspective and under a 'Western' criterion, to avoid a fragmented view, in this paper, we attempted to give an "insider's perspective" of an "indigenous" pedagogical practice. This "indigenous" practice, appearing rarely in the West, is typically regarded as a natural strategy to deepen the understanding in local curriculum as a daily routine. The practice, namely, variation problems, are known widely in Chinese mathematics curricula as "One Problem, Multiple Solutions" (OPMS, 一題多解，varying solutions), "Multiple Problems, One Solution" (MPOS,多題一解, varying presentations), and "One Problem, Multiple Changes" (OPMC,一題多變，varying conditions and conclusions). This paper critically looks into the "indigenous" practice, its theoretical implications, and its cultural grounds, including the goal of examination, the goal of the national curricula, and the goal of pedagogy. It might enable us to see which parts of educational processes the different educational systems can learn from each other in a more integrative perspective.

Key words: problem solving, problem design, curricula development, cultural background.

Background

In the recent twenty years, variation practice has attained considerable attentions to the importance of variation as necessary conditions for deep learning (e.g. Marton & Booth, 1997). There is a robust literature that variation practice is also regarded as an important tool of comparison in mathematical pedagogy (e.g. Yakes & Star, in press) and as a critical means for facilitating examples and exemplification (e.g. Watson and Mason, 2005). Among these studies, it is interesting to note cultural differences in the use of variation are rather confused, which is typically expressed by Rowland (2008): "variation practice in structured exercises varies considerably from country to country and from text to text". This indicates that it would be rather difficult to characterize cultural features and textual features of variation practice.
Interestingly, a number of studies (e.g. Gu, Huang, & Marton, 2004) consistently identified that variation practice offers some advantages in Chinese mathematics education. For example, Gu ,Huang, and Marton (2004) argued that, by adopting teaching with variation, even with large classes, students still could actively involve themselves in the process of learning. The "paradox of Chinese learners" might originally be a misperception by Western scholars due to the limitation of their theories. Marton (2008) argued that there is a specific Chinese Pedagogy in Chinese culture after working on the topic for over ten years.

The studies above shed light on how to explain the learning significance of variation practice. However, these studies drew attention to the "indigenous" practice is not enough. Encouragingly, Sun (2011) revealed the "indigenous" variation approaches in structured examples and their roles in the Chinese mathematical textbook. The variation approaches are known widely in Chinese mathematics curricula as "One Problem, Multiple Solutions" (OPMS 一題多解，varying solutions), "Multiple Problems, One Solution" (MPOS 多題一解, varying presentations), and "One Problem, Multiple Changes" (OPMC一題多變，varying conditions

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and conclusions) in China. Sun, 2011) found that in the topic of fraction division in a most popular textbook in China 87.5% examples are OPMC and 57.5% of examples are OPMS. Cai and Nie (2008) argued that the use of this type of variation problem is widespread in China. In a survey, the teachers were asked to point out how often they use each of these problem-solving activities in the classroom. The results showed that all of the teachers used these problem-solving activities. Over a half of the teachers surveyed used OPMS, MPOS, and OPMC very often in their instruction (Cai & Nie, 2008). This practice seems rather popular in China. Reader may wonder why this practice is favored by Chinese teachers and what is the rationale for this practice?

Research questions

Since most international research is conducted in 'Western' culture, from a 'Western' perspective and under a 'Western' criterion, in this paper, we attempt to give an "insider's perspective" of variation problems with hopes to understand this pedagogical phenomenon and its cultural grounds in detail and to reflect the nature of curricula practice in a different cultural setting and from a more integrative perspective. We will focus on the following questions: 1) What is variation problem and its theoretical basis? 2) Why and how Chinese use variation problem

Variation problems and Theory Basis

It is interesting to note that Chinese routine lessons or exercises are made up with, not isolated problems, but problem sets, which called *Bianshi* (变式 in Chinese，where *Bian* stands for "changing" and *shi* means "form", can be translated loosely as "variation" in English.) or Pudian ("paving a road;" 铺垫 in Chinese). This strategy easily traced to any single teaching material (such as textbooks or teaching plans) at school and any single learning material (such as student exercises or worksheets) done after school in China. We will illustrate the task design with the two examples.

One Problem with Multiple Changes (OPMC)

Here is a typical "prototype" example of OPMC in the Chinese textbook, in which the concept of division is always introduced in the Chinese textbook below as: $4\times6=24$, $24\div6=4$, and $24\div4=6$ (Figure 1).

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Figure 1: **The example introducing the division concept using OPMC in the Chinese textbook.** (Mathematics Textbook Developer Group for Elementary School, 2003, Vo2. 1, p.24)

Within the problem set, there are two concepts of multiplication and division behind three similar problems made with 4, 6, and 24. Example problem: How many trees do 6 lines need so that each line can have 4 trees? Variation problem 1: How many trees will each line get if we plant 24 trees in 6 lines? Variation problem 2: How many lines do we plant if we plant 24 trees in order so that each line has 6 trees?

Clearly, the intent of OPMC is to enable students recapitulate the "general "relationship of mutiplication and division, and the meaning of "equal" from the problem set 4×6=24, 24÷4=6, 24÷6=4 , similar to learning the meaning of division from a single problem, 24÷6=4. The problem set hinges on, not the objects of the division procedure, but exemplifying the understanding of "equal" relationships, namely, discerning the general relationship of multiplication and division, and then building a comprehensive structure. The task draws students into a "space of relations" as opposed to directing attention to the object itself.

One Problem with Multiple solutions (OPMS)

Here is a typical "prototype" example of OPMS in the Chinese textbook. In the variation problems below, $\frac{2}{11}\times 3=\frac{6}{11}$ is designed to naturally introduce a solution system of fraction multiplication (Figure 2). Within the "problem set", there are three solutions made available. The first one is using the "count" solution in addition. The second solution is that of fraction addition, and the third is that of fraction multiplication. Clearly, OPMS requires different solutions and provides a platform wherein students and teachers can reflect, interrelate, and generalize among three solutions.

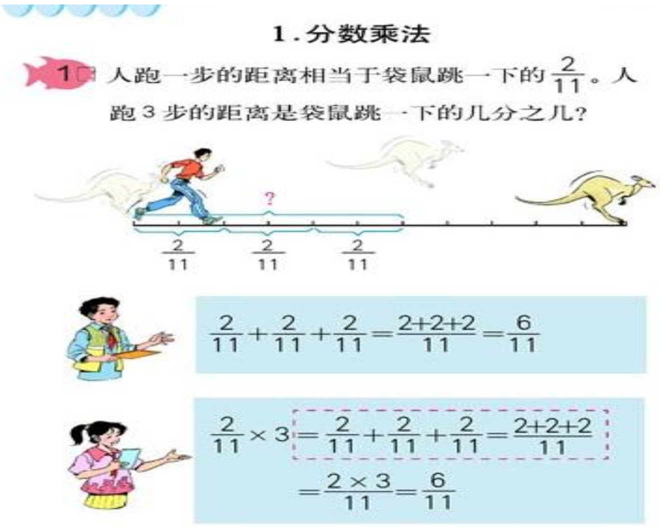

Figure 2. **The example introducing the subtraction concept using OPMS in the Chinese textbook.** (Mathematics Textbook Developer Group for Elementary School, 2006, Vo6. 1, p.8)

In contrast with variation problems, priority of “contextualization problems” is generally regarded as the common curricular trend in the West (Clarke, 2006). However, “contextualization problems” in the interest of facilitating engagement mainly provide with examples of the same concept and method of one solution, which might miss the chance to make timely connections between concepts and methods. In this light, variation problems suggest a way in which way Western counterparts could learn from content-orientation curricula in China. Clearly, variation problems are two-edged sword which could lead to more learning challenges compared to “contextualization” problems because variation problems require the use of multiple concepts, solutions and conceptual development targeted.

Theoretical Basis

Curricula practice does rely on very different fundamentals e.g. mathematical view, philosophy-orientation, and cognition-orientation. In this section, we will re-examine the theoretical implications of variation problems in three perspectives: its mathematical characteristic, “indigenous” learning theory, and “indigenous” philosophy.

Regarding the mathematical characteristic. Researchers have documented the fact that the different cultural views on mathematics influence mathematics education in different ways. The use of variation problems or contextualization problems might reflect the different view on the characteristics of mathematics: Is mathematics abstract or concrete? Mathematics educators from both the East and the West will surely say that it is both, but it is the position on the continuum between the two extremes that divides an Eastern view and a Western view.

The analysis of the variation problems of OPMS, OPMC, and MPOS aims to present more opportunities to deepen abstract thinking (i.e. the structure, strategies, or concepts) than does an isolated problem or contextualization problems, which reflects Chinese view on mathematical characteristic of abstract. The contemporary Western view in recent decades focuses more on the concrete contextual features of doing mathematics (e.g. problem solving, investigations). It seems Western counterparts have gone too far towards the concrete features extreme. They might need to re-affirm the importance of the abstract characteristic of mathematics in the feature of the concrete context. Reversely, it seems Eastern counterparts have gone too far towards the abstract characteristics extreme. They also need to re-affirm the importance of the feature of the concrete context in stressing the characteristic of abstract mathematics.

In this light, the analysis above suggests that the different curricula trends are helpful for mathematics educators from different traditions that can judge these activities and exploit them for different curricula purposes. The essential point is what goal of curricula is needed rather than stressing use of "contextualization" problems or variation problems.

From the learning theory perspective. Many readers may argue that the variation approach above may be confusing and that a sequential organization with time gaps ("one-thing-at-a-time") should be preferred. The issue of variations in problem sets directly reflects the old Chinese proverb, "no clarification, no comparison" (沒有比較就沒有鑒別), rather than "to consolidate one topic, or skill, before moving on to another," a notion broadly used in most textbook development throughout the world (Rowland, 2008). In contrast, this "one-thing-at-a-time" design would clearly provide fewer opportunities for "making connections" compared to those of contemporaneous variation approaches. The "one-thing-at-a-time" design might possibly reflect a hidden concept where making a connection could naturally happen. In this context, the curricula role of making connections could either be relatively neglected or taken for granted. According to variation theory (Marton and Booth, 1997) which emphasizes "simultaneousness," the "one-thing-at-a-time" design might miss the chance of discerning critical aspects between two or more topics. More importantly, the contemporaneous variation approach, to a greater extent, emphasizes a "general relationship" more than others .Thus, OPMC, OPMS and MPOS aim to discern and compare the invariant features of the relationship among concepts, solutions and contexts, and provide opportunities for making connections, since comparison is considered the pre-condition to perceive the structures, dependencies, and relationships that may lead to mathematical abstraction. The emphasis on "general relationships" above reflects the soul of variation theory, emanating from the work of Marton who argues that "learning will take place through discernment of variation in simultaneous events"; "variation is a necessary condition for effective discernment. Variation / nonvariation can enhance or limit the possibility of learning. Such tasks draw learners into a "space of relations" rather than into the problem itself.

From an "indigenous" philosophy perspective. Different practices are based on different deep-rooted cultural backgrounds. The curricula rationale to extend a lesson or to organize exercises by using variation problems is related to "indigenous" philosophy, which could be traced back mainly to the ancient "general consensus" developed from the system of Daoism (Chinese: 道家) and Confucianism (Chinese: 儒家). For example, the following view from the Chinese *I-ching* theory (the Book of Changes), one of the oldest classic Chinese texts, subscribes to "discerning unvaried law beyond the varied (in Chinese 變中發現不變)" and "using 'invariability' in response to 'variability' (以不變應萬變);" the central idea of the evolution of events as a change process and the acceptance of the inevitability of change further reveal the ideologies behind the implementation of variation problems. A simple way to appreciate variation thought is to consider it as being based on the spirit of deep understanding. Based on the analysis above, in Figure 4, we attempt to summarize a framework to describe the ideas and functions of variation problems in mathematics teaching.

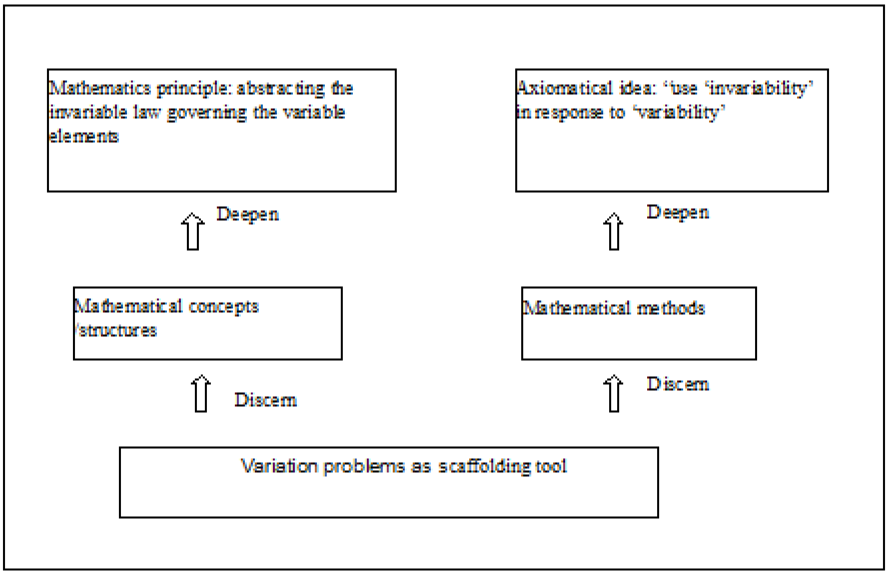

Figure 3. The functions of variation problems played in Chinese mathematical curricula.

Why and How Chinese Use Variation Problem

Traditions of teaching and learning are deeply embedded in history and culture. One of the intentions of education is to support the traditional continuity of structure and function of a special environment. By examining the elements that "breed" variation problems in China, we could understand the practice and its cultural foundation in a deeper manner, which would also provide a platform to reflect on culture in its own educational settings that we often take for granted, and eventually learn from each other (e.g. successes and failures).

Chinese Culture

Mathematics occupies a central place in the curricula of nearly all countries of the world, yet there may be subtle differences in the importance attached to mathematics as a school subject in different cultures. The different perceived roles of mathematics may affect the way mathematics education is organized in different cultures. Because competition in employment, education, and social mobility are crucial, mathematical examination plays a crucial role in China. Moreover, among all the subjects, mathematics is the best means to sort students as they go up the education ladder due to "examination culture". Indeed, vital exams in mathematics are required in order to have a suitable differentiation ratio (The Illustration of 2003 National College Entrance Examination (NCEE) for Science Majors in China, 2003). The examination problems in the NCEE are required to be "from textbooks, but above textbooks" (words from NCEE). As a result, there is a big gap between textbook problems and examination problems, which teaching have to link. Accordingly, the "economical" and "efficient" method of teaching is developing special curriculum with problems linking textbook problems to examination problems, explicitly or implicitly. This system requires mathematics teaching not to simply use the formulas, examples, and exercises but to develop "special experience", which help students to identify within minutes the original problems from which the examination problems are taken or to recognize the category of the problems and the methods to solve them (students are generally required to finish at least 100 problems in two hours for each normal examination). So exam problems themselves require learning have to develop a relational understanding through the process of varying representations, conditions, conclusions, and the deductive process in order that the variable or invariable relations can be discerned.

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Generally, educators in the West treasure intrinsic motivation, and regard extrinsic motivation from examination pressure as harmful to learning (Leung, 2001). For a long time, examinations have been commonly criticized. However, the top-performing countries in the Third International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA), such as Singapore, Korea, Japan, Hong Kong, and the Netherlands, were more likely to have high stakes examination systems. The significance of examination might go beyond tradition. The role of examinations as a means of quality control in the educational system is neglected. According to some studies (e.g. Phelps, 2001), examinations work a mechanism to enforce the curricula and the instructional system. The more quality control measures are employed in an educational system, the greater is the students' academic achievement (e.g. Phelps, 2001). The discoveries are all remarkable even in states and provinces. In this sense, examinations enable variation practice to play a connective function in the educational system, which indicates a balance not only between extrinsic and intrinsic motivation but also a balance between the intended curricula (textbooks) and the attained curricula (exams).

China is well known as a country wide in area but relatively imbalance in educational resources; there is a "national achievement standards" as a "gate-keeper" for entry to high school or university each year. This exam aims to maintain curricula quality among the different regions or states. In this sense, variation problems, targeting examinations, can play an important "control" role in "adjusting" education quality in the different regions. So exams create a balance not only between intended curricula (textbooks) and attained curricula (exams) but also between local and national curricula. The analysis above suggests the vital point is the roles exams are playing in shaping the teaching quality rather than contemplating how to avoid exams.

Two Bases

Studies of different cultural traditions often reveal differences in the goals of education. In China, a unified national curriculum standard (the national general outline, 教學大綱), "two bases", namely, basic knowledge and basic skills as the foundation of continuous learning, is strictly stipulated by the Ministry of Education (Ministry of Education, 1999), and clearly embodied in the national textbooks. Under the national curricula standard, teaching is strictly carried out according to "three unifications," namely, the unified national general outlines, the unified national textbooks and the national reference books. This is different from the system in the US where local governments have the autonomy.

Because the national curricula standard is considered as the compulsory "core curricula", all stakeholders of education (e.g., teachers, parents, schools, etc.) are required to work well together to carry out this educational goal. The related professional training, resource, and assessments are demanded to make such enactment possible. From this perspective, variation problems could be regarded as a curricula implementation tool encircled by the invariable "two basics", which are characterized by "dispersed and progressive difficulties". The similar view was stated: "Teaching and exercise unfolded by variation problems activities are considered as the pathway to the "two bases" " (Zheng, 2003). Although school mathematics syllabi might be similar in different societies, the syllabi of implementation could be very different. The analysis above suggest the vital point is, not whether the system has curricula standards or not, but whether implementation is consistent with intended standards.

Pedagogy Knowledge

Education is mainly conditioned by the environment. There are more students in each classroom in China than in other countries. Therefore, teaching in China is very different from teaching in those foreign countries where classes have a relatively fewer number of students. In this environment, it would be helpful to use variation problems activities to cater to individual differences. The example below might illustrate how variation problems can cater differences in a large classroom setting:

Factorization of polynomial: x^2+6x+8 (Level 1)
What is the possible value of a and b, such that the polynomial can be factored?
x^2+ax+6? (Level 2) ; x^2+ax+b? (Level 3) ;x^3+ax+b? (Level 4); x^n+ax+b? (Level 5)

It is clear that variation problems include the "factorization" concept development of five levels. Each student should learn the "factorization" concept and extend their understanding from surface to structure through the five stages. Finally, they can have the chance to extract the concept of "factorization," conceptualize the "'theory of factorization" from a progressive series of sub-problems. It provides a platform for students with different levels of cognition to find their own starting point accordingly rather than those of a single level.
According to "the genetic law of cultural development," which claims that the cultural development of children appears on two planes (Vygotsky, 1978), an activity first takes place interpsychologically as the interaction between individuals, and afterwards it occurs within the child as an intrapsychological achievement. Based on this explanation, the group of variation problems can be regarded as a flexible curricula mediating both at the individual and the classroom levels. This not only helps students gradually deepen basic knowledge beyond the variation problems, but it also helps to cater to the differences in the classroom. This analysis enables us to understand how Chinese classroom practice could contribute to thinking development given the large class size without group activities. The crucial point is what scaffolding role in the content playing in the teaching and learning setting rather than how big the class size is.

This example inspired us to reflect on the role of knowledge. With the quick growth of the internet, it is believed that the role of a teacher should be that of a facilitator of learning rather than the source of knowledge learned in the West. However, it is easy to find that, in this case, without knowledge of Viete's Theorem, a teacher could not facilitate teaching this kind of task with correctly assessing the answers. It would be impossible to design and adjust the difficulty level to meet understanding in different levels. Such a task indicates why subject matter knowledge is considered the pre-condition to facilitate to teaching, namely, content knowledge takes priority over pedagogy content knowledge in China. The belief, as the local proverb says, is "a teacher may not give a student a cup of water unless he/she has at least a barrel of water 倒出来一碗水，需要有一桶水".

Summary

To avoid the "one shot" study, which does not form a coherent picture, this paper examines variation problems and its cultural foundations in a more systematic way. It is interesting to note that the variation approach as a well-known practice in China, but is rarely known in the West. Different cultures, with their own advantages and disadvantages, rather than oppositional, are complementary and interrelated. As most of international research is frequently conducted from a 'Western' perspective, in this paper, we try to give an "insider's perspective" of variation problems and its cultural underpinnings. It might enable us to see which parts of the educational processes the different educational systems can learn from each other, which hopes to bring about an understanding of the essence of curricula practice in the different settings in a more integrative manner. It will be more likely to develop a process of self-reflection on the traditional ways that we often take for granted , and support the development of more sophisticated, broader pedagogies adapted to the demands of each classroom setting, and less constrained by culture.

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