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Enhancing Students' Mathematical Conjecturing and Justification in Third-Grade Classrooms: The Sum of Even/Odd Numbers

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This study is intended to explore how third-grade teachers designing tasks for conjecturing the sum of even (odd) numbers for students engaging in mathematical justification. The teachers participating in a teacher professional program which is designed to support teachers incorporating conjecturing into mathematics contents scheduled in textbooks and implementing into classrooms for promoting students' mathematical argumentation. Six teachers at grade 3 to 6 participating in the study were encouraged to create conjecturing tasks for teaching instead of plodding through a textbook page. Each teacher was encouraged to design at least a task for conjecturing in a semester. The main data collected for the study included video tape of lesson, lesson plan, and students' worksheets collected in the classrooms. The study reported in the paper focuses on how two teachers incorporated conjecturing into the odd (even) numbers scheduled in the textbook and how the third-graders engaged in the sum of odd (even) numbers and justifying their conjectures.

Keywords: *conjecturing, mathematical argumentation, odd and even numbers, third-grade.*

Mathematical proving is not introduced until students enter into junior high school. This arrangement in curriculum might results into high school students, undergraduates, future or inservice teachers performing poorly on argumentation or proof in school mathematics (Barkai, Tsamir, Tirosh, and Dreyfus, 2002, Ko, 2010; Stylianides et al., 2007). For instance, inservice upper elementary school teachers do not believe a single counterexample is sufficient to refute a universal statement (Barkai et al., 2002). These studies or innovative curricula recommend that students should have early opportunity to incorporate conjecturing and proving into their mathematical learning (Barkai et al., 2002; NCTM, 2000; CCSC, 2010). The current standards advocate that mathematical proficient students should have the opportunity of justification to help them rely on their own knowledge and understanding, not on justification by authority (CCSC, 2010, MOE, 2008). Nevertheless, primary teachers of Taiwan seldom create opportunities for engaging their students in conjecturing activities in classrooms.

In recent years, researchers draw increasingly attentions to mathematical argumentation at primary school (Carpenter, Franke, & Levi, 2003; Sowder & Harel, 1998). For instance, Carpenter, Franke and Levi (2003) focus on the conjecture and justification for generalizing statement of a number pattern or relationship. The conjecturing, generalizing, and justifying are the ideas leading students to think about mathematical ideas and is a basis for mathematical argumentation. Here, justification encompasses a set of arguments that students us to show that a conjecture is true. Thus, mathematical argumentation includes why the statements are true more than explaining the statements (Knuth, 2002). Given this, looking for a number pattern or relationship could be a possible mathematics content for elementary school students engaging in the activities of mathematical argumentation. Thus, this is one of the reasons of the sum of even (odd) numbers as the mathematics topic of the study reported here.

Conceptual Framework

Justification used by students can be identified as different levels. Sowder and Harel (1998) identify three levels of justification used by high school and college students as: external based proof, empirical proof, and analytical proof. External proof is when students rely on outside sources such as, teachers, parents, or books. Empirical proof is when students base their reasoning on an example or series of examples. Analytic proof is described as arguments that incorporate mathematical reasoning and are of a more general nature.

Carpenter, Franke and Levi (2003) also categorize mathematical argumentation into three levels of: (1) appeal to authority, (2) justification by example, and (3) generalizable arguments. Appeal to authority refers to students relating their reasoning or argument to a rule or procedure that was taught or told to them by someone with authority. Their argument does not contain any mathematical justification. Student justifications that are embedded in specific numbers are justifications by example. In a generalized argument a student presents a logical argument that applies to all cases included in the conjecture. These arguments may be verbal, symbolic or even concrete. Students at this level need to explicitly understand that the process they use can be generalized to all numbers. For example, when justifying that the sum of an even number and an odd number is odd, a student may use pairs of blocks to construct an even number and pairs of blocks plus one block to make an odd number.

The justification used by elementary school students is mostly at the level of justifying by example. However, students’ levels of justification can be improved if teachers support students in discussing about mathematical reasoning or through questioning both correct and incorrect results (Flores, 2002, Keith, 2006). The contention is evidenced by Valentine's work with six-graders on justification and proof of classroom conjectures for the commutative property of multiplication (Valentine, 1999). Valentine indicates that the majority of her students not only progresses in the notion of what constitutes proof, but also the students moved away from justification relying on authority to generalizable arguments. The implication of empirical studies is that the quality of students' mathematical argumentation can be promoted if teachers provide the elementary school students with the opportunities of generating conjecturing and justifying the conjectures.

Justification is closely related to conjecturing. Conjecturing launches justification. Justification is a vehicle to enhance students' understanding of mathematics concepts and to support students to explore why things work in mathematics and explain their disagreements in meaningful ways (Hanna, 2000). Both conjecturing and justification could promote mathematical thinking and launch mathematical inquiry. Hence, in order to promote students' justification, taking conjecturing as approach is considered for the study. The focus of the study to support primary teachers incorporating conjecturing into mathematics contents scheduled in textbooks and carrying out classrooms, in particular, on designing tasks for conjecturing.

Not all tasks lead to conjecturing, and different tasks lead to different kinds of conjecturing (Cañadas, et al., 2007). For instance, tasks for conjecturing and tasks for proving (Cañadas, et al., 2007). Lin and Tsai suggest that it is a good start for the teachers who are novice in designing the tasks for proving with a false statement instead of a true statement in helping students learning to prove. Furthermore, different topics might lead to different level of justification (Lin & Tsai, 2013). The conjecturing can be various types, for instance, a conjecture can be made based on the observation of a finite number of discrete cases, in which a consistent pattern is observed (Cañadas, et al., 2007). The conjecturing can be divided into several stages. Cañadas & Castro (2005) offer seven stages for conjecturing based on finite discrete cases: (1) observing cases, (2) organizing cases, (3) searching for and predicting patterns, (4) formulating a conjecture, (5) validating the conjecture, (6) generalizing the conjecture, and (7) justifying the generalization. The analysis of a conjecturing is the base of supporting teachers designing the tasks for conjecturing.

Due to the students or teachers who are unfamiliar with the conjecturing or justifying, selecting mathematics topics from textbook for incorporating conjecturing into should be take into consideration. Thus, this study aims to provide a preliminary understanding of third-grade students' conjecturing and justifying conjectures through odd and even numbers.

Mathematical Justification of Odd (Even) Numbers

The odd and even numbers are learned at lower grade level in most of the countries. The study on justification of odd and even numbers in a second-grade class (Keith, 2006) reveals that students' justification of an odd or even number is built on the definition of odd number and even number. The definitions made by the second-graders in two three ways. One is by using the blocks. The second grade students split blocks (thousand cubes, hundreds flats, tens rods, and ones cubes) into two equal groups starting with the thousands, then hundreds, tens, and ones. The number is even if the groups each has the same amount of cubes. If one of the groups has an extra cube, then the number is described as odd. The second definition used by students is by sharing equally. Even number does not have any left over when divided by two, while odd number cannot be divided into two groups that are equal without splitting or having one left over. The third definition is that even is when you have an amount that two people can share and each person will have the same amount. It will be fair. With an odd number you will have one left over after dividing by two when all numbers are kept whole.

Likewise, the third-grade students in Ball's class (1993) give three definitions with different words. For instance, (1) even numbers have two in them (she circled the hash marks in groups of two) and an odd number is something that has one left over; (2) even numbers are a multiples of two and odd numbers being multiples of two plus one; and (3) how many groups of two an even number has.

Regarding to the sum of odd (even) numbers, Keith (2006) concentrates on 14 second-grade students' the conjecturing and justification. In the beginning, all the 14 students use examples to prove why they believe a conjecture is true. They are satisfied that examples provide enough proof. As the class engaging in discussions about what constitutes enough proof, the students' mathematical argumentations become more general and more convincing. Furthermore, all 14 students are able to transfer their generalized thinking to a problem situation.

To better understand how teachers incorporate conjecturing into mathematics contents covered in the textbook and its effect on students' understanding of conjectures and justifying the generalization, the following two research questions are proposed to be answered.

1. How did teachers incorporate conjecturing into on odd (even) numbers in third-grade classrooms teaching?
2. How did the third-grade students make conjecturing and justify the sum of odd (even) numbers for all cases?

Methods

Settings: Classrooms and a Teachers' Learning Community

The study involves in two settings. One is two classrooms at grade 3. The other is a place where teachers in a teacher professional learning community have routine weekly meeting. The learning community including six teachers and a mathematics educator was set up in a three-year project called as Teacher Professional Development via Task Design and Implementation for Conjecturing (TDIC). The focus of the project is to help teachers to create conjecturing tasks for engaging students in the activity of mathematical argumentation. The learning community held weekly meetings throughout the school year. The two settings are intertwined each other. After designing the tasks for conjecturing, which is discussed in the teacher learning community, the teachers carried out the tasks in the classrooms to examine if the tasks work or not. Likewise, after implementing the conjecturing tasks in the classrooms, what/how students worked on the tasks was brought to the learning community to discuss and revise.

The participating teachers were encouraged to create conjecturing tasks for teaching relying on the instructional objectives of the textbook instead of plodding through a textbook page. The task design for conjecturing was supported by the teacher learning community. Teachers were encouraged to select the mathematics contents to be incorporated into conjecturing. It seems that it is rare to have the same topic to be taught in different classrooms, even in the same grade. To understand the extent to which a conjecturing task can provoke students' conjecturing and justifying the conjectures, it is necessary to have more classes to examine if the task works or not. This is one of the reasons of selecting two third-grade classrooms reported in the paper. One is Jing's class and the other is Ken's class.

The other three reasons of selection the task for the study were based on: (1) the task of odd (even) numbers was implemented in two classrooms. It is tried out in Jing's class first, then immediately discussed and suggested by the learning community, and finally re-revised and re-implemented into Ken' class. When taking together, the two classes offered a good picture of how a conjecturing task highly tying with textbook was incorporated into the third-grade classes. The task provided a picture of what constitutes conjecturing and justification in third-grade classrooms.; (2) in order to help students developing mathematical argumentation, conjecturing the sum of odd (even) numbers engaging can be a representative topic for conjecturing as an instructional approach; and (3) conjecturing about the sum of odd (even) numbers would provide a good site to begin engaging in the activity of justification.

Participants

The subjects reported in the paper as part of the second year of the data collected in TDIC project were the 49 students in two third-grade classes. Ken's class was comprised of 14 boys and 10 girls, while Jing's class was comprised of 13 boys and 12 girls. Ken's class was located in a suburban area and Jing's class was located in a urban area. Ken and Jing have been participated in my previous teacher professional development for supporting teachers in teaching as learned-centred approach at least ten years. Furthermore, Jing has been involving in the tasks for proving for one year, but designing the tasks for conjecturing was her first year. This is Ken's first year of the involvement of the tasks for conjecturing and he never involve in designing the tasks for proving before.

Designing Tasks for Conjecturing

Each conjecturing task was created with a ADDIER model: Analysis of teaching materials in textbook, Developing potential tasks for conjecturing, Designing and reporting to the team of the community, Implementing it into classrooms, immediately Evaluating by a follow-up professional dialogues of the professional team, and Revising the task for next day teaching or for another class in the same grade. Each task was designed for inquiring five main processes of conjecturing: (1) constructing finite cases and organizing the cases, (2) observing, searching for a pattern, formulating conjectures, categorizing conjectures and checking the correctness, (3) validating the conjectures, (4) generalizing conjectures, and (5) justifying the generalization. Each of formulating, testing, justifying, and generalizing, and verifying conjectures went through from individual, group, to whole class work.

The students in each of the two classes were grouped heterogeneously in groups of 4 or 5. There were six groups in each class. Students worked individually or small group on solving and recording solutions to the tasks for conjecturing. Within the two classes, mathematics conversation revolved around students' conjecturing, explanation, justification for how a task was inquired.

Data Collection and Data Analysis

The data sources and artefacts for the study consisted of the tasks created by the two teachers, teachers' textbook analyses, observation sheets from four lessons related to odd and even numbers; photocopies of students worksheets; audiotapes of classroom observations, and two meetings of the professional dialogues.

The data of classroom observations were analysed by two graduate students. They looked for evidence of mathematical argumentation in each step of conjecturing and sorted students' work into levels of justification, as represented by the students' recording on their papers. They looked for the differences and similarities in the levels of justification used by the students in the two classes.

The study reported in the paper focuses on what a task for conjecturing looks like and how third-grade students in the two classes constructed the meanings of odd numbers and even numbers and conjectured and justified the sum of odd (even) numbers through engaging in the conjecturing task.

Results

Teachers' Intention to Incorporate Conjecturing

An activity in one textbook page for students understanding the meaning of odd and even numbers is given by a 10×10 table with 1 to 100 in order. Students are expected to discover a pattern of "*An odd number whose digit in ones place could be 1, 3, 5, 7, 9., while an even number whose digit in ones place could be 0, 2, 4, 6, and 8*". Jing was not satisfied with the textbook page, she stated in a weekly meeting of analysing textbook as follows:

> The textbook page asks students to mark all even numbers in 10×10 table, and it is followed by looking for a pattern. It is risk that the discovery (conclusion) is made only based on a single case. I am afraid that this pattern to be discovered could cause students' misconception. For instance, students would judge incorrectly 236 as odd number, since there are three digits in 236. Furthermore, students do not agree 126 is an even number, since not all the three digits 1, 2, 6 are even numbers (Weekly meeting, 26/03/2014).

Table 1

Tasks for Conjecturing the Sum of Odd (Even) Numbers

Objective of Task 1: To construct the meaning of an odd and an even number	Stages of Conjecturing
1-1 Each of you write 2 different numbers (0~100) in a given card. 8 numbers you created must be distinct in your group.	Constructing cases
1-2 Categorizing the numbers you created.	Organising and looking for patterns
1-3 What is an odd number and an even number? Write them into a statement.	Formulating a conjecture
Objective of Task 2: To understand the properties of the sum of odd(even) numbers	
2-1 Each of you draws 2 numbers randomly from the 8 numbers you created, and then adds them up.	Constructing cases
2-2 Write the addition number sentence on your own worksheet. Odd number is marked in red and even number is marked in black.	Organizing the cases in a group
2-3 What did you discover? Write it down.	Looking for a pattern, Formulating conjectures
2-4 Which of your discovery remains true for more cases?	Verifying the conjecture
2-5 How do you describe each of your conjectures if it is generalized for all cases?	Generalizing
2-6 How do you convince your friends that your statements are always true?	Justifying

Excepting a single case, even number plays a significant role of learning other mathematics concepts in the following grades. For instance, even number is a prior knowledge of multiples and factors of a number at grade 5. Taking the two reasons into account, Jing and Ken agreed on incorporating conjecturing into odd (even) numbers. The conjecturing tasks generated by Jing, revised by the group members, and revised by Ken were given in Table 1.

Students' Definitions

In Task 1-2, after creating 8 numbers in each group, students in the two classes were aware of odd (even) numbers as a criteria of sorting the numbers into two categories. Table 2 shows that there were 4 groups in Jing's class and 2 groups in Ken's class categorizing the two categories by odd and even numbers. Students in the two classes were asked to give a definition for odd and even numbers and write on their own worksheet. Six definitions for odd and even numbers were given by the two classes as shown in Table 2.

Table 2

The Definitions of Odd and Even Numbers Given by Students

Categories	Definitions of odd and even numbers	Group #
(1)	Odd numbers will not have friends and even numbers have friends.	A6
(2)	Odd numbers are single and even numbers are double.	B3
(3)	Odd numbers are 1,3,5,7,9,…, and even numbers are 2,4,6,8,0…	A3,A5,B2
(4)	The digits in ones of odd numbers are odd, while the digits in ones of even numbers are even.	A4
(5)	Odd numbers are from 1,+2,+2…keep going, and even numbers are from 0,+2,+2…keep going.	B1, B6
(6)	(a) Odd numbers have one left over when divided by 2 groups and even numbers will not have one left over. (b) Odd numbers are 2 goes with together at one each time and have left one over, and even numbers will not have any left over.	A1,A2,B4, B5

A1 represents group # 1 in Jing's class B1 represents group #1 in Ken's class

The category (1), (2), and (3) of defining odd and even numbers from students' perspective are related to the interesting characteristics and category (4) is the oddness or evenness of the number is determined by the ones. Of the 12 groups in the two classes, 6 groups wrote definitions correctly as shown in category (5) and (6) of Table 2. The two descriptions in category (6) are distinct in that (1) is share division and (2) is measurement division.

Following the cases they constructed, three definitions of odd (even) numbers given by the third-grade students with verbal without suing a concrete representation as follows: (a) share division: odd numbers have one left over when divided by 2 groups and even numbers will not have one left over; (b) measurement division: odd numbers are 2 goes with together at one each time and have left one over, and even numbers will not have any left over; and (c) odd numbers are from 1,+2,+2…, and even numbers are from 0,+2,+2….

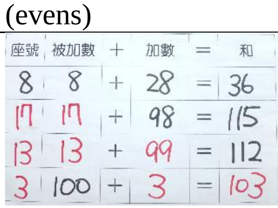
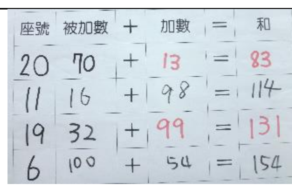
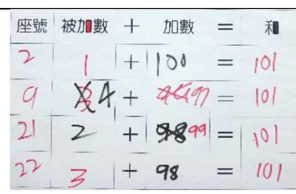
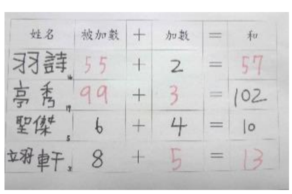
Creating Number Sentences for Observing

To help students conjecturing on the properties of sum of odd (even) numbers, Jing asked each student in a group to draw randomly 2 numbers from eight numbers generated as shown in Table 2 task 1-1. Students were asked to write the 2 numbers with addition into a number sentence. Afterwards, students were asked to compile the four sentences from each group. The number sentences were shown in Table 3 (a), (b), and (c).

Based on Jing's practices, Ken made a revision on the use of students name on the worksheet instead of students' seating number displaying in the worksheet for compiling the number sentences, since the seating number (in the first column of the Table) would mixed up with the numerals in the number sentences.

Table 3

Number Sentences Students Construed for Searching for the Sum of Odd (Even) Numbers

	Numbers sentences students generated for searching for the sum of odds (evens)		
Jing's class	 (a)	 (b)	 (c)
Ken's class	 (d)	 (e)	 (f)

From the data in Table 3, the odd numbers marked in red and odds marked in black colour required in the task 2-1 are truly advantage of looking for the patterns of adding an odd number and an odd will equal an even numbers, such as 13 + 99 = 112, according to red and red turn into black, shown in Table 3(a). Comparing to Jing's class, the use of students' names is clearer in Ken's class, as shown in Table 3(d), 3(e), 3(f). The result indicated mutual learning between teachers.

Conjectures Students Made

Tasks 2-1 and 2-2 in Table 2 were designed for students discovering the patterns of the sum of odd (even) numbers. The first four conjectures students formulated were accepted in each of the two classes as shown in Table 4.

Table 4

Five Conjectures Students Made For the Sum of Odd (Even) Numbers

Conjectures from students		Group #
English version	Chinese version	
(1) odd + odd = even	103年3月19日第（二~2）題 三年乙班第（1）組座號（1.7.16） 我們發現的數學想法 奇數加奇數[illegible]偶數。	A1, B2, B3, B4, B6
(2) odd + even = odd	103年3月19日第（二~2）題 三年乙班第（5）組座號（5.12.24.15 我們發現的數學想法 奇數十偶數一定會=奇數。	A1, A5, B1, B3, B4
(3) even + even = even	103年3月19日第（二~2）題 三年乙班第（4）組座號 4.10.18.23 我們發現的數學想法 偶數+偶數=偶數。	A4, A5, B1, B2, B5
(4) even + odd = odd	103年3月19日第（二~2）題 三年乙班第（三）組座號（20.11.19.6 我們發現的數學想法 偶數加奇數=奇數。	A1, A3, B5, B6
(5) If the sum is constant, then addend is greater and add is smaller.	103年3月19日第（二~2）題 三年乙班第（6）組座號（2.9.22.21 我們發現的數學想法 被加數越大加數就應該要一樣。 和相同時，被加數愈大，加數愈小。	A6
A1 means group # 1 in Jing's class	B1 means group #1 in Ken's class	

Except of the sum of odd (even) number adds odd (even) number, Jing's students in group 6 (A6) also discovered that "when the sum is constant, if addend is getting greater and add is getting smaller", as shown in row 5 of the Table 4. To validate the conjecture, the teachers used another number sentence from other groups to examine if the conjecture remains true. For instance, the conjecture "*odd + even = odd* " made from 17+98=115 in Table 3(a) made by group 1 in Jing's class (A1) was validated by a new number sentence 3+98=101 from group 6 (A6).

Generalizing and Justifying the Generalization from Students

Task 2-5 in Table 2 was designed for students to generalize four conjectures about the sum of odd (even) numbers for all cases by asking students about "*whether or not each of the conjectures was true for all numbers?*" and "*How do you describe each of your conjectures if it is generalized for all cases?*".

All the following generable conjectures generalized to all cases were from Ken's students. Statement (1) was given by group 1 and 5, while statement (2) was given by group 2 and 6. Statement (3) was given by group 3, while the statement (4) was given by group 4. However, Jing did not ask students to generalize the conjecture (5) in Table 4 for all cases, even it is a true statement. It is due to Jing's unawareness of putting the stage of a conjecture to be generalized in her mind. Consequently, she skipped this stage of conjecture. Through debriefing Jing's teaching in the weekly meetings of teacher professional team, the researcher addressed the importance of a conjecture to be generalized for all cases. Thus, Ken's added the task 2-5 and the task 2-6.

Statement (1): For all odds, when adding an odd and an odd number the sum will always be even.
Statement (2): For all odds and evens, when adding an odd and an even number the sum will always be odd.
Statement (3): For all evens, when adding an even number and an odd number the sum will always be even.
Statement (4): For all evens and odds, when adding an even and an odd number the sum will always be odd.

To enhance students justifying the conjectures they made, students in the two classes were asked to answer the question "*How do you convince your friends that your statements are always true?*". We found that the definitions students made for an odd number or an even number in the earlier stages became the warrants for their justifications. Their justification was not only given by a few examples, instead, they use their definitions to justify the generable conjectures. For instance, for justifying the statement (1), students in group 6 (B6) in Jing's class stated as:

> "Odd numbers will not have friends, but both two odd numbers become friends when they get together. Therefore, when adding odd numbers the sum will always be even." (A6's worksheet).

Students in group 4 (B4) in Ken's class justified the statement 1 by the definition they made in Table 2 (6) as follows:

> "Whenever you add an odd and an odd numbers it will equal an even number because there is 1 extra after you divide and odd number by 2." (B4's worksheet).

Students in group 6 (B6) in Ken's class justified the statement 1 by the definition they made in Table 2 (5) as follows:

> "Whenever you add an odd and an odd numbers it will equal an even number because an odd number is 1+2+2+…, and odd + odd = 1+2+2+…+1+2+2+…=2+2+2+…." (B6's worksheet).

Approaching to the end of the lesson each, the whole-class discussion in each of the two classes was summarized by the two teachers with four statements. It is ended by a thoughtful question hiding in Ken's mind for the whole class discussion. The question was that "*Is the number 1234 an odd or an even number?*" The following excerpt was drawn from the whole class discussion:

128 ken: Is the number 1234 an odd or an even number?
129 S4 : Odd, no.
130 S24: Even.
131 Ken: Why?
132 S7 : Because 1234=1000+200+30+4, here 1000 is even, 200 is even, 30 is even. 1000+200 is even plus even, it will be even. Even plus 30, an even, will be an even, so that 1000+200+30 is an even. Even plus 4 is an even. So that 1234=1000+200+30+4 will be an even.
133 S16: I found out, found out …a number with four digit numbers can be expanded as how many thousands, how many hundreds, how many tens, and how many ones. The sum of the first threes is even. Thus, the number is an even or an odd number, it depends on if the digit in ones is even or odd.
134 Ken: This finding is just the pattern discovered from 10×10 Table in the textbook page which shows "the digits in ones of odd numbers are odd, while the digits in ones of even numbers are even".

The four statements of the addition of odd (even) numbers were extended by the students in Jing's class to the subtraction of odd (even) numbers, as wrap of the lesson. The two teachers Jing and Ken did not realize the significance of the sum of odd (even) numbers until they taught the lesson, even though they have learned about the properties of the sum of the odd (even) numbers in junior high school.

Discussion

The results suggest that the third-grade students' justification for the sum of odd (even) numbers for all cases presents a logical verbal argument using the definition of odd (even) numbers beyond than justifying by an example or series of examples. The distinct definitions of odd and even numbers they made became the warrants for justifying the conclusion of the sum of odd (even) numbers. The justification performed by the third-grade students' reveals the level of justification students made was advanced to the highest level of generalizable arguments, suggested by Carpenter et al. (2003). The students did not have to go through from the first two levels: appeal to authority and justification by examples. The finding of the study does not become as one of the literatures, which the justification used by elementary school students is mostly at the level of justifying by examples (Carpenter, et al., 2003; Flores, 2002, Keith, 2006). However, this study supports Keith's claim (2006) that students' justification was improved if teachers support students in engaging in conjecturing (Flores, 2002, Keith, 2006).

One possible reason of promoting third-grade students with the highest level of justification was the two teachers providing their students the opportunities of engaging the five stages of conjecturing: constructing cases, giving the definition of odd (even) numbers with their own words, formulating four conjectures about the sum of odd (even), generalizing, and justifying the generalization. The learning from the first three stages including constructing cases, formulating the conjectures, validating the conjectures became the warrants for justifying the generalization.

The second reason was the selection of mathematical topic of odd (even) numbers for students justifying the sum of odd (even) numbers. The knowledge related to odd (even) numbers is readily to intrigue students' or teachers' interesting. For students, they were excited in making the conjectures for the sum of odd (even) numbers based on looking for the patterns of the number sentences. For teachers, the knowledge for teaching justifying the sum of odd (even) numbers is not as complicated as other topics.

The third reason as the essential is with respect to the nature of the tasks. The tasks was sourced from the instructional objectives. This contributes to teachers' willing to incorporate conjecturing into classroom. In addition, it lent itself ready to the same grade teachers design and discuss together. Accordingly, they mutually supported and learned from each other. However, it is challenge for the teachers to identify where and what the topics can be incorporated conjecturing into the textbook page. How to design a task for conjecturing for promoting students' mathematical argumentation will become a focus for further study. The issues are suggested to be the focus of the teacher preparation programs and teacher professional development programs.

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