

The Development of Chinese Students' Understanding of the Concept of Fractions from Fifth to Eighth Grade

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The purpose of the present study was to investigate fifth to eighth grade Chinese students' understanding of the concept of fraction. One hundred and ninety nine students were tested using a questionnaire that required them to make judgments about fraction size and to order a set of given fractions. The participants were also asked to justify their responses. Primary findings revealed that students progressed along three conceptual levels in grasping the concept of fractions (Stafylidou & Vosniadou, 2004): at level 1, a fraction is viewed as consisting of two independent numbers; at level 2, a fraction is viewed as indicating part-whole relations; and at level 3, a fraction is viewed as indicating the ratios between the numerator and the denominator. Results also indicated that, from fifth to eighth grade, Chinese students made significant progress in understanding the concept of fraction. Misconceptions about fractions such as the whole-number bias, the misunderstanding of unit 1 are also discussed in the paper.

Key words: *fraction concept, misconception, whole number bias*

As a number concept, fractions indicate proportional relations between part (numerator) and whole (denominator). Mastery of this concept was considered a milestone in students' number development (Ni, 1999), and has an important effect on understanding the nature of continuity and divisibility of number. However, the concept of fraction was found to be the most complex and abstract concept in the primary grades (Bulgar, 2003; Saxe, Taylor, McIntosh, & Gearhart, 2005) and was difficult to learn (Hartnett & Gelman, 1998; Mack, 1995; Smith, Solomon, & Carey, 2005). Even adults

cannot understand the concept well (Gelman, 2006). Recently, researchers generally agreed that fractions can be conceptualized as a set of interrelated meanings: part-whole, ratio, operator, quotient, and measurement (Charalambos & Pitta-Pantazi, 2007). The part-whole meaning emphasizes the relationship between the part and the whole, and fraction represents a comparison between the numbers of parts of a partitioned unit to the total number of parts in which the unit is equally partitioned. The ratio meaning is defined as a comparison between two quantities, so, it is a comparative index, rather than a number. The operator meaning emphasizes fraction as functions applied to some number, object, or set. A fraction can also be represented as result of a division situation, and that is the quotient meaning. Furthermore, for the measurement meaning, a fraction is considered as a certain number, which can be located on the continuous number line, rather than a combination of two whole numbers.

Children cannot simultaneously understand those five meanings. Compared to others, the part-whole meaning is easier to understand because this meaning can be assimilated with a prior knowledge of natural numbers (Ni, 1999). Moreover, the part-whole meaning is usually emphasized in fraction teaching, and teachers often use the part-whole meaning to introduce the concept. For example, $3/10$ means three apples of ten apples. In contrast, the measurement meaning is difficult for children to understand because it requires that children view a fraction as one number (instead of two independent numbers) and as a point on the continuous number line. Children need to undergo a conceptual changing process to accommodate this meaning (Gelman, 2006). Specifically, children must expand their concepts of numbers from a discrete system, which is a characteristic of nature numbers, to a continuous system, which is a characteristic of rational numbers. The differences between natural numbers and fractions often cause students to make errors when carrying out calculations involving fractions.

Therefore, an important question is one relates to how students acquire accurate concept of fractions and to what degree the acquisition process involves conceptual change needs to be addressed. Based on Piaget's cognitive constructivist theory and the concepts of assimilation and accommodation, Vosniadou (1994, 2001, 2003) developed a conceptual change theoretical framework to explain how students develop science concepts. According to the theoretical framework, the knowledge acquisition process involves enriching existing conceptual structures and reorganizing existing knowledge. Learning that entails reorganization of existing

knowledge is much more difficult than that which entails enrichment. In other words, accommodation is more difficult than assimilation. It is very likely that children create misconceptions in the process of reorganization when they attempt to change existing concepts. This theoretical framework can explain processes that children use to acquire novel knowledge of astronomy, physics, chemistry, and biology (Vosniadou & Vamvakoussi, 2006). In a similar vein, the conceptual change theory can also provide good explanations for comprehending the conceptual journey that students undertake to tackle the mathematics concepts such as fractions (Stafylidou & Vosniadou, 2004), negative signs (Vlassis, 2004), and the concept of number (Merenluonto & Lehtinen, 2004).

Specifically with the concept of fraction, Stafylidou and Vosniadou (2004) contended that the reason why children have difficulty in understanding fractions is because before learning fractions, they have established solid knowledge of natural numbers. It is the existing knowledge of natural numbers that induces a robust tendency in children to use a discrete-unit counting scheme to interpret fractions. Stafylidou and Vosniadou (2004) described a conceptual hierarchy with fraction, which can be divided into three levels (descriptions of these levels were discussed later in the method section). However, whether that conceptual framework is applicable to Chinese children needs to be verified. Cross-cultural studies showed that Chinese students often outperform U.S. and Europe students on international tests in mathematics including fractions concepts (Fang, Tong, & Liu, 1988; Stevenson, Lee, Chen, Lummis, Stigler, & Han, 1999; Wu, Li, & Haffner, 2006). Some scholars believe that differences among different languages in representing and conveying concepts are important factors in affecting fraction learning (Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999). The English fraction-naming system differs from those of the Asian languages. For example, in Korean, Chinese, and Japanese, the notion of fractional parts is explicitly embedded in fraction names. For instance, in the Chinese and Korean languages, one fourth is roughly translated “of four parts, one,” which explicitly conveyed the part-whole meaning. In contrast, in English, $1/4$ is read as “one fourth” or “a quarter,” which does not reflect the part-whole meaning. As supported by research on learning (Miura et al., 1999), the fraction names which reflect transparent part-whole relations facilitate East Asian children’s understanding of fractions. Therefore, the part-whole meaning and even others meanings are easy to understand for Chinese students.

There have been various studies in China about fraction learning and teaching including studies on students' understanding of the multiple meanings of fraction (Ni, 1999; Zhang, Liu, & Wang, 1982), the status of fraction teaching (Su, 2007), and research reviews (Yang & Liu, 2008). However, seldom have studies been done to explore Chinese students' fraction concept with the explanatory framework developed by Stafylidou and Vosniadou (2004). The purposes of the present study include a) investigate whether the explanatory framework applies for the development of Chinese students' fraction concepts, b) examine how fifth to eighth grade students' fraction concepts distribute at different developmental levels of the explanatory framework, and c) identify typical errors and fraction misconceptions for Chinese students and provide corresponding diagnostic suggestions.

Method

Participants

One hundred and ninety-nine students participated in this study: 52 fifth graders (27 boys and 25 girls, mean age 11 years and 3.6 months); 47 sixth graders (33 boys and 14 girls, mean age 12 years and 7.2 months); 46 seventh graders (24 boys and 22 girls, mean age 13 years and 6 months); 54 eighth graders (28 boys and 26 girls, mean age 14 years and 9.6 months). The students came from a private school in north China that serves middle class parents, and this school sets in primary school, middle and high school in one.

Questionnaire

The questionnaire that was designed by Stafylidou and Vosniadou's (2004) was adopted to examine Chinese students' understanding of fractions (see Table 1). This questionnaire consisted of two sets of questions: in the first set, students were asked to write the smallest and biggest fraction they could think of and justify their answers. In the second set, they were asked to order and compare fractions (e.g., $5/6$, 1, $1/7$, $4/3$), and to give an explanation for their answers.

Codings

We adopted the explanatory framework that was developed by Stafylidou and Vosniadou's (2004). According to the framework, children first think that the numerical value a fraction is represented by two independent numbers. Specifically, when they order or compare the fractions, they only consider the size of the numerators or denominators. This level was called *Explanatory Framework of Fraction as Two Independent Numbers* (Stafylidou & Vosniadou, 2004). Furthermore, there are two different types of performances nested in this level: a) children believe that the value of a fraction increases when the numerator or the denominator increase; b) on the contrary, children consider that the value of a fraction increases when the numerator or the denominator decrease.

Table 1

The Two Sets of the Questionnaire

	Questions
Set I	Question 1: Write the smallest fraction that you can think of, and why do you think this is the smallest fraction? Question 2: Write the largest fraction that you can think of, and why do you think this is the largest fraction?
Set II	Order fractions. Please order the numbers $5/6$, 1 , $1/7$, $4/3$ from the smallest to the biggest one and write down why did you placed them in this order. Compare five pairs of fractions. Please choose the biggest fraction from each pair of fractions: $4/5-2/5$; $4/15-4/7$; $5/8-4/3$; $2/7-5/6$; and $2/3-4/9$. How did you make this comparison?

Though both types of performances are incorrect, they reflect two types of conceptual understanding. Specifically, Children who make the first type of error and will make grammatical errors accordingly. For the fractions questionnaire, there are two response categories in set I. In response category 1, children believe that the larger the numerator or denominator is, the larger the value of a fraction is. For example, they believe that $1/1$ is the smallest fraction, whereas $10000/100000$ is the biggest fraction. On the contrary, children who are grouped in response category 2 consider that as the numerator or denominator decrease, the value of a fraction increases. For example, children think the smallest fraction is $10000/100000$ and the biggest fraction is $1/1$. In set II, there are three response categories according to the performance of children. In response category 1, when the numerator or denominator of a fraction increases, the fraction itself increases, and the unit

is the smallest of all fractions. In response category 2, when the numerator or denominator increases, the fraction increases, but children consider the unit as the biggest of all fractions. In response category 3, children also believe that as the numerator or denominator increases the fraction increases, but they do not order the unit.

At the second level, which was called *A Fraction is a Part of a Whole*, children view the numerator and denominator of a fraction as representing a part-whole of a natural object. Children at this level usually use discrete objects to represent units, and think that a unit is bigger than all fractions. In other words, there is no use of improper fractions (e.g., $11/6$). This level is further divided into three subcategories according to Stafylidou and Vosniadou's (2004). In subcategory 1: *Naïve Part of a Unit*, children seem to have grasped some concept of the part-whole aspect of a fraction, but, they still maintained some beliefs from their prior knowledge. For example, some children think that the biggest fraction is $1/100$, and the smallest fraction is $1/2$, because "when a pie is divided into 100 pieces or 2 pieces, 100 pieces is bigger". In subcategory 2: *Advanced Part of a Unit*, children often use the part "that we take" to represent the numerator, and view the denominator as the parts into which the unit is divided. Therefore, children believe that all fractions are smaller than a unit. For example, some children think that $1/10$ is the smallest fraction, and $9/10$ is the biggest fraction, because "when the unit is divided into 10 pieces, we take at most 9 pieces". In subcategory 3: *Sophisticated Part of a Unit*, children obtain more sophisticated knowledge such as the concept of improper fractions. Reflected in testing performances, for set II, children in this category can correctly order the fractions and the units by using the method of reducing all fractions to a common denominator. For set I, however, they still think the unit is the biggest fraction. A fifth grader represented $11/10$ as "11 pieces of a pie with 10 pieces," which indicates that he still tried to use part-whole concept to interpret fractions. Specifically, there are 2 response categories in set I. In response category 3, the students believe that a fraction is a part of a whole and that the value of a fraction increases as the size of the numerator approximates the size of the denominator. For example, according to a seventh grader, "the smallest fraction is $1/1000$, because the unit is divided into 1000 pieces, and I take only one of them." while the biggest fraction is " $999/1000$, because the unit is divided into 1000 pieces, and I take almost all of them". In response category 4, children for the first time conceptualize the biggest fraction to be the fraction whose numerator is a bigger number than the denominator. In set II,

there are two response categories. In response category 4, children consider that as the numerator or denominator decreases, the value of fractions increase, and they put the unit at the end believing that it is bigger than any fraction. Response category 5 includes responses that give the correct ordering of the fractions except the unit.

The third level was called *Relation between Numerator/Denominator*. Children's concept of fraction is radically changed. They now consider fraction as a relation between the numerator and denominator. Specifically, "they consider a fraction smaller than the unit when its numerator was smaller than its denominator and bigger than the unit when the numerator was bigger than the denominator" (p. 513, Stafylidou & Vosniadou, 2004). The third level is divided into two subcategories. Subcategory 1: *Relation of Two Numbers without Infinity*, in which children do not refer to the unbounded infinity at all, in other words, they believe that there is a certain smallest and biggest fraction respectively. Subcategory 2: *Relation of Two Numbers with Infinity*, in which students develop a more complete knowledge of the fraction. These students absolutely understand the quotient and measurement meanings, and they believe that the smallest or the biggest fraction is an unbounded infinite number. There are two response categories in set I. In response category 5, children think that there is a unique smallest fraction but not a biggest fraction, whereas in response category 6, children believe that there is no smallest and biggest fraction. In set II, there is only one response category. Children who are grouped in response category 6 can correctly order the fraction and the unit.

Based on the above explanatory framework, students' performances and answers were grouped initially into one of those three levels, and further into the six subcategories. A category 7 was created and included all the responses (e.g., "I don't know it is so" and "I think so") that could not be categorized in those identified categories.

Data analysis

We conducted data analysis with SPSS 16.0. Specific analysis includes a) frequency and proportion calculation for determining level distributions of students' fraction concept among the different categories of the explanatory framework, and b) *Exact Kruskal-Wallis* test for examining level distributions among different grades.

Procedures

All participants completed the questionnaire in a group during their class time, and participants at each grade were divided into two sub-groups so that the number of participants at each sub-group was less than 30. Most students finished the questionnaire in approximately 20 minutes. Two trained graduate students in psychology supervised the testing.

Results

Students Distribution in the Explanatory frameworks of Fraction

Table 2 presents the frequency/percentage of students distributed in the various categories described above. Generally speaking, across all grades, 10% of students reached the first explanatory framework, 23.6% of students reached the second explanatory framework, and 58.8% of students reached the third explanatory framework. Specifically, the number of lower-grades students in the first explanatory framework is larger than the number of higher-grades students: 19.2% of fifth students were grouped in the first explanatory framework, but only 3.7% of eighth students were grouped in this level. On the contrary, the number of higher-grade students in the third explanatory framework is larger than the number of lower-grade students: 74.1% of eighth graders reached the third explanatory framework, but only 34.6% of fifth graders did so. We performed the *exact Kruskal-Wallis rank test*. The exact K-W test results showed that grade was a statistically significant factor ($H = 24.83, p < .001$). According to the performance of students, we can see the older students understand the concept of fractions more accurately.

Set I: Smallest / Biggest Fraction

Table 3 presents the frequency/percentage of students distributed in the various categories described above in set I. Students were asked to answer which fraction is smallest or biggest. As demonstrated in table 3, there are 49.7% of students were classified category 6, which is the highest level. In category 6, the percentage of students from fifth grade to eighth grade are 13.5%, 76.6%, 41.3% , and 72.2%, respectively. In contrast, there are only 11% of students were classified the lower categories of 1 and 2.

Set II: Comparison of Fractions

Table 4 shows the frequency/percentage of students distributed in the different categories in set II. For this set, students were asked to compare and order fractions. Overall, 79.4% of students could correctly order the fractions and the unit. The percentage of students who could do so from fifth to eighth grade is 82.7%, 70.2%, 82.6%, and 77.8%, respectively. The results indicated that students can correctly adopt the principle of ordering fractions.

Table 2
Explanatory Frameworks for the Numerical Value of a Fraction

Explanatory frameworks	5th Grade	6th Grade	7th Grade	8th Grade	Total
A. Two independent numbers	10(19.2%)	3(6.4%)	5(10.8%)	2(3.7%)	20(10.0%)
(1) Two independent numbers whose value increases as the numerator (or the denominator) increase	4(7.7%)	2(4.3%)	2(4.3%)	2(3.7%)	10(5.0%)
(2) Two independent numbers whose value increases as the numerator (or the denominator) decrease	6(11.5%)	1(2.1%)	3(6.5%)	0(0%)	10(5.0%)
B. Part of a whole	19(36.6%)	5(10.6%)	14(30.4%)	9(16.7%)	47(23.6%)
(1) Naive part of a unit	0(0%)	0(0%)	2(4.3%)	3(5.6%)	5(2.5%)
(2) Advanced part of a unit	3(5.8%)	5(10.6%)	1(2.2%)	3(5.6%)	12(6.0%)
(3) Sophisticated part of a unit	16(30.8%)	0(0%)	11(23.9%)	3(5.6%)	30(15.1%)
C. Relation between numerator/denominator	18(34.6%)	35(74.5%)	24(52.2%)	40(74.1%)	117(58.8%)
(1) Relation of two numbers without Infinity	11(21.2%)	5(10.6%)	5(10.9%)	2(3.7%)	23(11.6%)

(2) Relation of two numbers with Infinity	7(13.5%)	30(63.8%)	19(41.3%)	38(70.4%)	94(47.2%)
D. Mixed—could not be categorized	5(9.6%)	4(8.5%)	3(6.5%)	3(5.6%)	15(7.5%)
Total	52(100%)	47(100%)	46(100%)	54(100%)	199(100%)

Table 3

Numerical value of fraction-Set I : Smallest/ biggest fractions frequency and percentage of responses per grade

Categories of responses	5th Grade	6th Grade	7th Grade	8th Grade	Total
1.The value of a fraction increases when the numbers that comprise it increase	0(0%)	1(2.1%)	2(4.3%)	2(3.7%)	8(4.0%)
2. The value of a fraction increases when the numbers that comprise it decrease	1(9.6%)	0(0%)	5(10.9%)	2(3.7%)	14(7.0%)
3. The value of the fraction increases as the size of the numerator approximates the size of the denominator	13(25.0%)	0(0%)	2(4.3%)	2(3.7%)	17(8.5%)
4. The value of the fraction increases as the size of the numerator becomes bigger than the size of the denominator	16(30.8%)	5(10.6%)	9(19.6%)	3(5.6%)	34(17.1%)
5. There is no biggest fraction (infinity only for	1(1.9%)	2(4.3%)	2(4.3%)	2(3.7%)	7(3.5%)

big fractions)					
6. There is no smallest/biggest fraction (infinity for all fractions)	7(13.5%)	36(76.6%)	19(41.3%)	39(72.2%)	99(49.7%)
7. Mixed-could not be categorized	7(13.5%)	3(6.4%)	7(15.2%)	4(7.4%)	20(10.1%)
Total	52(100%)	47(100%)	46(100%)	54(100%)	199(100%)

Table 4

Numerical value of fraction—Set II: Comparison of fractions frequency and percent of responses per grade

Categories of responses	5th Grade	6th Grade	7th Grade	8th Grade	Total
1. The value of the fraction increases as either the numerator, or the denominator increase—the unit is the smallest of all fractions	0(0%)	2(4.3%)	0(0%)	2(3.7%)	5(2.5%)
2. The value of the fraction increases as either the numerator, or the denominator increase—the unit is bigger than all fractions	1(1.9%)	2(4.3%)	0(0%)	1(1.9%)	5(2.5%)
3. The value of the fraction increases as either the numerator, or the denominator increase—ordering without the unit	0(0%)	1(2.1%)	0(0%)	0(0%)	1(0.5%)

4. The value of the fraction increases as either the numerator, or the denominator decrease—the unit is bigger than all fractions	3(5.8%)	1(2.1%)	0(0%)	0(0%)	4(2.0%)
5. Correct ordering only for the fractions	0(0%)	5(10.6%)	2(4.3%)	6(11.1%)	11(5.5%)
6. Correct ordering for the fractions and the unit	43(82.7%)	33(70.2%)	38(82.6%)	42(77.8%)	158(79.4%)
7. Mixed-could not be categorized	5(9.6%)	3(6.4%)	6(13.1%)	3(5.5%)	15(7.5%)
Total	52(100%)	47(100%)	46(100%)	54(100%)	199(100%)

The Different Performance between Set I and II

There are differences in children's performances between set I and set II. The number of students (158 students) who correctly ordered fractions in set II is larger than the number of students (99 students) who correctly answered the questions in set I. The difference, however, does not indicate that there is non-conformity in our coding between set I and set II. The reasons rather lie in the different conceptual facets that were tested in set I and set II. Specifically, the questions in set I emphasized on concepts such as infinity; whereas the questions in set II emphasized operational fluency. To further pursue this line of thought, we firstly chose the students who had correct answers in set I, then explored their performance in set II. Results show that from fifth to eighth grade, the percentage of students who correctly ordered the fractions was 85.7%, 75%, 89.5%, and 87.2%, respectively. Then, we chose the students who could correctly order the fractions in set II, then explored their performance in set I. From fifth to eighth grade, the percentage of students who had a correct answer in set I was 14%, 81.8%, 42.1%, and 81%, respectively. From the different patterns of data, we can see that although some students could order and compare fractions fluently, they still had difficulty in mastering the concept of fraction.

Discussion

Developmental Levels of Explanatory Framework of Fraction

Findings from our study confirmed the three developmental levels of explanatory framework of fraction (Stafylidou & Vosniadou, 2004) with Chinese students. As their Greek counterparts, Chinese students in the initial explanatory framework *Fraction as Two Independent Natural Numbers* demonstrated two almost contradictory beliefs. One is the belief that “the value of the fraction increases as the value of the numerator (or denominator) increases” and the other is the belief that “the value of the fraction increases as the value of the numerator (or the denominator) decreases”. While the former belief is consistent with the laws of natural numbers (the larger the number, the bigger the value it represents), the latter belief is a misconception. For example, a student reasoned that “the bigger the numbers a fraction has, the smaller it is,” or “the smaller the numbers a fraction has, the bigger it is.” This reasoning indicated children’s incomplete understanding of the knowledge of “the more the number of partitions, the less each partition has.”

The second explanatory framework *Fraction as Part of a Unit happens* seems to be a transitional phase between the initial framework and the ultimate framework. The concept that fraction is parts of a whole is not adequate enough for students to solve all the problems, especially the improper fraction problems. However, when children begin to learn the concept of fractions, Chinese teachers usually teach the part-whole meaning, and mostly use discrete objects modeling fractions which may hinder students’ further development to the ultimate stage.

The ultimate framework is to interpret fractions as quotient or ratio that can be compared, added, subtracted, multiplied or divided. In our sample, 58.8% of the students used this framework to solve problems. This proportion is much larger than that of the initial and secondary framework (20% and 23.6%).

Misconceptions of Fraction

All children's conceptual frameworks develop from their experiences and are built upon chunks of naïve ideas. However, frequently, their intuitive understanding of the world around them does not agree with the scientific

explanation. Scholars and teachers cannot focus only on whether the answer is correct or wrong, but also pay attention to the logic of the misconception and how children change their concepts from naïve ideas to scientific concept. Chinn and Brewer (1993) discussed the situation when children acquired the new knowledge. They found that children may have three reactions to new knowledge: a) children may ignore, reject, or exclude the new knowledge, therefore the misconception is maintained; b) they may also reinterpret the new scientific concepts according to their prior framework, or change their own theory superficially, but actually they do not change their misconception; c) children may change their existing knowledge and incorporate the new scheme. In our study, these three conditions were embodied in three conditions. The first condition: children only focused on the numerator or denominator of fractions when they ordered or compared the fractions. It seemed that they ignored the new learning that teachers taught them. This misconception derived from the qualitative difference between the knowledge of fractions and the prior knowledge of natural numbers. The second condition: although children regard the fractions as having part-whole meaning, they still use the principles of natural number to order the fractions, which is called whole number bias (Ni & Zhou, 2005). Whole number bias is defined that when beginning to learn the fraction concept, children often use previously formed single-unit counting scheme to assimilate it. The third condition: children who acquired the multi-concepts of fractions can solve the problem about fraction correctly.

Moreover, we also found that some students had the misunderstanding of unit 1. In the concept of fractions, 1 represents not only a certain number, but also represents a unit. For the former meaning, 1 is the smallest natural number, but the largest number of the proper fraction; for the latter meaning, 1 is not a number; it is a unit or a whole. Because of complicated meaning, children may make mistake easily. For example, a fifth grader said “1 is the smallest fraction and the largest fraction”. In addition to this, some students did not know the value of the unit, and therefore they cannot justify which fraction is larger. In particular, when the unit of $\frac{4}{5}$ is 1 (natural number) and the unit of $\frac{2}{5}$ is 2, $\frac{4}{5}$ is equal with $\frac{2}{5}$. Therefore, students have difficulty in understanding the concept of unit. The difficulty is universality in learning the concept of fractions (Chinn & Brewer, 1993). Many students blurred the boundaries between 1 as a whole number and 1 as a unit of fraction, ignoring that 1 is an absolute value and thusly, they made mistakes in solving problems. In summary, the unit 1 is the key concept in learning fractions, and it is the

bridge between the natural number and the rational number.

Performance on Ordering and Comparison Tasks

The result also showed that Chinese students were more familiar with the calculation and comparison rules than the concept of fraction numbers: those who got the wrong answer in set I were able to correctly answer the ordering problems in set II; the proportion of 85% right answer rate is much higher than the proportion of students reaching the ultimate framework of understanding, which is 47.2%. Our study results were consistent with those by Moss & Case (1999), Su (2007) in which students were better at calculation than in the understanding of the fraction concept. Moss and Case (1999) suggested that the difficulty in learning fractions was because instruction paid more attention to how students can correctly solve the problem rather than helping them understand the concepts and meaning of the problem deeply. Su (2007) also believed that students grasped the rules of operation more expertly than understanding the meaning of the fraction.

The number of 1 is re-mentioned in this part as it is commonly acknowledged as the key number in the learning of fractions. When making fraction comparisons, 1 can be seen as a reference point. However, only those with a framework at the third level were able to use this strategy to save time and energy. The reason is that they truly understood that fractions numbers are in fact numerical values.

Educational Implications

The present study generates three specific implications for the teaching of fractions.

First, teachers should provide multiple representational models and help students to form an accurate and solid conceptual understanding of fractions. For the current teaching practice, the part-whole meaning of fractions is over-emphasized but the other meanings (e. g., ratio, operator, quotient), especially the measurement meaning, are largely neglected. Although it is logical to introduce fraction with its part-whole meaning by using region, line segment, discrete object models, it is necessary to further develop students' conceptual understandings by exposing them to the other representational models. For instance, the number line model is an ideal instrument for representing the measurement meaning of fractions (Ni, 1999). Zhang and

colleagues (Zhang, Sun, Huang, Huang, & Tang, 2009) have found that the number line model integrates whole numbers, fractions, and decimal fractions into one system and therefore has great potential in promoting children's understanding of rational numbers, and therefore should be emphasized in teaching.

Second, teaching fraction calculation should build upon students' conceptual understanding but not on rote formula application. As advocated by National Council of Teachers of Mathematics (2006), sixth grade students should use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explaining why they work. In China, the primary school mathematics curriculum standard (Ministry of Education of China, 2001) also suggests that teachers create meaningful problem solving situations to cultivate students' understanding of the rationales behind arithmetic operations.

Lastly, teachers need to help students understand and differentiate the concept of 1 as a number and the concept of 1 as a unit. In order to construct the correct concept of fractions and solve fraction problems correctly, students need to understand that the unit 1 may be composed of one object or multiple objects.

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