Examining US Elementary Teachers' Perceptions of and Comfort with Students' Mathematical Mistakes

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Developing a growth mindset is essential for students' academic and lifelong development. How teachers view and manage students' errors in mathematics classrooms shapes students' mindset towards making mathematical mistakes. This study examines teachers' perceptions of students' mathematical mistakes and their comfort levels in addressing those mistakes during instruction. The hour-long, semi-structured interviews included questions to investigate how teachers described their views on student mistakes and how they felt when they tried to address a student's mistake in the classroom. We interviewed seven elementary teachers. The results showed that teachers believed mistakes are essential for "proactively teaching," supporting student learning, and lesson planning. However, teachers expressed that they did not feel comfortable addressing student mistakes. The factors that appeared to contribute to their low comfort levels were fighting against the instinct to correct the mistakes for students, concerns about students' negative emotional reactions, and low efficacy in addressing student mistakes. Implications and suggestions are discussed.

Keywords: Mathematical mistakes, perception, comfort level, elementary teachers, growth mindset

For decades, results from international assessments (e.g., TIMSS and PISA) showed that the mathematics achievement of elementary school students in the United States has lagged behind their peers in East Asian countries (Honey & Mumford, 2014; Mullis et al., 2012). Comparative studies conducted between the United States and East Asian classrooms (e.g., China and Japan) have shown significant differences in teachers' approaches to students' mistakes (Santagata, 2004; Schleppenbach et al., 2007; Stigler &

Hiebert, 1999). In China, teachers appeared to believe that "failure is the mother of success" and tended to teach under that belief (Wang & Murphy, 2004, p. 120). Studies have shown that Chinese teachers prioritized creating a safe space for students to make and critique mathematical errors publicly (e.g., An & Wu, 2012; Boaler, 2015; Fang, 2010; Stevenson & Stigler, 1994; Stigler & Perry, 1988; Wang & Murphy, 2004). Similarly, Japanese teachers often challenge students with complex tasks, allow students to make mistakes, then encourage them to reflect on their mistakes to gain fuller and deeper mathematical understanding (Stigler & Hiebert, 1999; Stigler & Perry, 1988).

Alternatively, teachers in the United States seem to avoid engaging with students' mistakes. Although teachers express that students' mistakes are important (Schleppenbach et al., 2007), Santagata (2004, p. 160) noticed that "US teachers' concerns about students' self-esteem is likely to play a critical role in their decision to reduce the complexity of the problems as they guide students through their solution." Similarly, these concerns tend to lead teachers to avoid addressing students' mistakes publicly and directly (Santagata, 2004). Researchers observed that in responding to students' mistakes, US teachers were more likely to respond with statements while East Asian teachers asked more follow-up questions probing the thinking underlying the errors (Schleppenbach et al., 2007; Stigler & Hiebert, 1999; Stigler & Perry, 1988).

Traditionally, mistakes tend to be perceived as a reflection of a lack of intelligence. Recognizing that this perception of mistakes leads to an avoidance of discussing mistakes, Boaler (2015) advocated changing the message about mistakes to a more positive perspective. However, before asking teachers to make changes, we must first understand what "message" United States teachers would convey about mistakes. Thus, in this study, we examined seven US in-service teachers' perceptions of students' mistakes and their comfort level in addressing students' mistakes. Our research questions are: (a) What are elementary teachers' perceptions of students' mistakes? and (b) How comfortable do elementary teachers feel about addressing students' mistakes?

Review of Relevant Literature on Students' Mistakes

Students make various mistakes in solving mathematics tasks (see students' diverse error in solving a single task in Liu & Jacobson, 2022). The literature in mathematics education has widely documented the value of students' mistakes. Mathematics mistakes are considered "as important and as significant as correct answers" (Schwarzenberger, 1984, p. 159). They are "springboards for inquiry" into mathematical concepts (Borasi, 1994, p. 166), a door to understanding students' thinking (Ball, 1991; Fennema et al., 1996), and a prompter of productive mathematics discourse (Blunk, 1998; Boaler, 2016; Rittenhouse, 1998; Smith & Stein, 2011; Webb & Mastergeorge, 2003).

Because mistakes can help to extend learners' thinking and potentially "tell us more about what might be happening in a pupil's mind than any number of correct answers" (Schwarzenberger, 1984, p. 165), they are seen as important sites for students' learning (Hiebert et al., 1997), teachers' diagnoses (Liu et al., 2020) and building teacher-student as mathematically productive relationships (Liu et al., 2022). Mistakes, or errors, as they are sometimes referred to, are conceptualized in two ways, spontaneously arising and intentionally planned or planted. Spontaneously arising mistakes are those that occur during the solving of a problem which may be due to a range of reasons, including students' inattention and misunderstanding of a mathematical concept or procedure (Palkki & Hästö, 2018). Unlike spontaneous errors that are typically made by students, intentionally planned errors are usually initiated for serve instructional purposes. For example, teachers may plant an error to highlight a common mistake or initiate a discussion about different conceptions of a mathematical idea to strengthen students' thinking (Star et al., 2015). While both types of errors create opportunities for fostering a complete understanding of mathematics ideas (Tsamir & Tirosh, 2005), they are often not used by teachers in the United States in these ways. The set of beliefs underlying the avoidance of using mistakes include believing that mistakes are indicators of faulty instruction, signals of not being smart (Greeno et al., 1994); sharing students' mistakes may embarrass or confuse them; and they can be distractions that do not warrant whole-class attention (Bray, 2011). These beliefs held by the students, the teacher, or both are likely to initiate feelings of shame and incompetence and make it possible that neither would willingly participate in error-sharing. Thus, to ensure that students are provided with every opportunity for deep learning, it is essential to shift this deficit perspective on mistakes.

Promoting Positive Views of Student Mistakes

Supported by research in neuroscience, Dweck (2008) and Boaler (e.g., 2015, 2016) underscored the power of making mistakes and experiencing productive struggle in learning mathematics. In particular, making mistakes and realizing that one has made a mistake could create potential brain sparks that promote mental growth (Moser et al., 2011). Thus, making mistakes may provide more profitable opportunities for students' cognitive development, which are above and beyond solving problems correctly. As such, students must also have opportunities to make mistakes spontaneously and be able to share these mistakes and get productive feedback. Boaler (2016) argued that taking risks without being afraid to be wrong is central to being successful in life. Thus, it is important that teachers promote a culture where mistakes are seen as an essential part of the learning process and where they are openly discussed. Students' participation in this kind of environment supports both the mistake-maker and other students (Boaler, 2016). Hiebert et al. (1997) also argued that mistakes are sites for students' mathematical learning and

emphasized the importance of teachers establishing a classroom culture that honors this. What's more, the standards for preparing teachers of mathematics (Association of Mathematics Teacher Educators, 2017) explicitly advocated the practice of "intentionally foster growth mindsets among students about learning mathematics" with a given example that "well-prepared beginners acknowledge mistakes as critical for learning and help students view mistakes as important in the learning process and for engaging in mathematics"(p. 13).

Factors Affect US Teachers' Response to Student Mistakes

Research (e.g., Palkki & Hästö, 2018; Tulis, 2013) suggested that teachers in the United States tend to view students' mistakes differently from their counterparts in other countries. Stigler and Perry (1988) explored the mathematics learning in Japanese, Chinese, and American classrooms and found that the teachers in Japan shared their students' incorrect solutions most frequently for whole classroom discussions. They tended to engage all students in discussing the nature of the mistakes and possible remediation with students' erroneous examples on the board. Teachers in China also preferred to display student mistakes publicly to discuss them. In general, mistakes are considered as "an index of what still needs to be learned" in Chinese and Japanese classrooms (Stigler & Hiebert, 1999, p. 92).

Drawing on data from the TIMSS 1995 Video Study, Stigler and Hiebert (1999) revealed that US teachers tended to view school mathematics as a set of mathematics facts and procedures and perceive student mistakes as failure in learning and a sign of ineffective teaching. Align with these views, they found that US teachers expected students' mathematical products to be error-free. As such, they often provided students with too much information during the problem-solving process and tended to suppress students' mistakes rather than create opportunities for students to grapple with them.

An alternative explanation for these differences in teachers' perceptions of mistakes across countries can be attributed to their beliefs about students. For example, Santagata (2004) found that teachers in the United States educational system tended to be deeply concerned about students' self-esteem. Teachers in the United States tend to hold the view that pointing out students' mistakes can negatively impact students' feelings and identity, as they believe only positive reinforcement can build students' self-esteem (Santagata, 2004). Thus, they tend to be unwilling to discuss students' mistakes, as a way to avoid possible negative impact (Ding, 2007); instead, they focus more on motivating students by making mathematics fun and praising students (Santagata, 2004).

We noted that although the field has encouraged a positive view of mistakes with literature explicitly advocating the use of student errors (e.g., Boaler, 2016; National Council of Teachers of Mathematics, 2014), current empirical work on teachers' perceptions of the use of students' mistakes is sparse. This study aimed to address this gap.

Methods

Participants

Seven elementary teachers (five females: Lori, Willow, Sara, Megan; Jean; and two males: Adam, Barney, all names are pseudonymous) from three schools in the same district in the Midwest of the United States participated in this study. Teachers were certified to teach kindergarten through 6th grade and were considered generalists, meaning they were responsible for teaching the core subjects of mathematics, English/language arts, science, and social studies. Six were general education teachers, and one (Barney) taught a class of students with special needs. This class typically did not have more than five students at a time. They all taught in schools situated in low socio-economic communities that served high populations of students of color. Their teaching experience spanned from 3 to 25 years. Most teachers identified as elementary generalists, and the remainder were comfortable being labeled as mathematics teachers (Cross Francis et al., 2018).

At the time of this study, the participants were participating in a two-year, state-funded professional development (PD) program. Prior to the start of data collection, the participants completed eight workshops and one coaching cycle. All teachers volunteered to participate in the program and the study. The PD program was designed to support teachers in transforming their practices to align more closely with student-centered instruction. Teachers met with the professional developers once per month for PD workshops and were involved in monthly instructional coaching cycles. Workshop sessions involved discussion of student work focusing on unpacking the thinking of correct and incorrect responses; thus, mistakes were considered important in the context of understanding students' thinking.

Data Source and Analysis

The primary data source for this study was 30 to 60- minute, semi-structured interviews. Interview questions focused on teachers' thoughts about the value and use of students' mathematics mistakes and their practices in attending to them. Interviews were audio-recorded and transcribed. We employed thematic analysis to analyze teachers' interview transcripts inductively (Braun & Clarke, 2006). First, one transcript was read by the first two authors of the paper, and codes were generated that reflected the teacher's statements about the value and use of students' mistakes. Coders then met and discussed the codes as used in the transcript. The consensus was reached about the codes that best reflected the teacher's thoughts and voice. Followed by a similar coding process, coders coded all the other six teachers' responses. Coders then compared, contrasted, and aggregated all the codes to identify common themes together. Next, teachers' statements were reorganized by the common themes in a table which included columns labeled "Theme," "Code," and "Teachers." Coders then read the table row by row to validate the

alignment between the theme, codes, and teachers' statements. Based on the table, we described each theme and reported the results. To ensure trustworthiness, we invited the third author to read the coding results and transcripts independently to ask critique questions. All three authors arrived at a consensus. We report the findings below.

Results

Teachers' Perceptions of Student Mistakes

All teachers stated that students' mistakes were important, indicating their positive view. For example, Lori said, "As the teacher, I think that it's incredibly important." Similarly, Adam also pointed out that "mistakes are extremely important." Barney, Willow, and Sara underscored their perspectives by stating "very important" to describe the value of knowing students' common mistakes. Adam further stated, "A mistake is almost as important as getting an answer correct, currently." Although they all agreed that students' mistakes were valuable, there were various perspectives across teachers about why they viewed them as important. These perspectives are discussed below.

Important for "Proactively Teaching"

Several teachers addressed the importance of knowing the common mistakes and anticipating students' mistakes for "proactively teaching." Megan expressed that "It could help you with your planning if you're already aware of the kind of mistakes students can make… It can help you prepare for how to help them through it." Willow further explained that gaining knowledge of students' common mistakes could help her foresee her students' mistakes and recognize them during the lesson. She stated that "recognizing if it's a common mistake, and some of them, it's like you almost know where they're going to make the mistake ahead of time, so, you kind of proactively teach the lesson." Willow highlighted this point by sharing her experience of tutoring a sixth grader. When she worked with a sixth grader as a second-grade teacher, she realized that she had no idea of the sixth grader's common mistakes. Although she knew the content and was able to solve the problems, without knowledge of common student errors, she felt less equipped to adequately support the student.

> And if I'm sitting there struggling with it, and that hasn't happened so far, but it's in the back of my mind. The teacher normally gives me the work a week ahead of time so I've got time to go in and review it, but I haven't been in the classroom so I don't know what kind of mistakes they're headed towards.

Similarly, Barney recognized the value of knowing students' mistakes for proactive teaching. He acknowledged that he started to learn more about students' mistake patterns through the PD program. Yet, he still struggled in the classroom as he was not fully able to anticipate all students' mistakes.

> I'm starting to see patterns in students' mistakes and anticipate them… I don't think I'm quite there yet … at the beginning of a lesson of being like, oh this is where they're going to, this is what they'll have trouble with. So, I'm having to notice and then adjust more, rather than, kind of coming into already knowing what a common mistake might be.

As he stated, without having knowledge of potential student errors ahead of the lesson, when he encountered students' mistakes, he had to adjust spontaneously, which did not always bode well for deploying the best teacher move. However, Barney realized that he could leverage what he learned about students' errors in earlier lessons within a unit that would support later lessons allowing him to teach more proactively. He stated that he could "adjust throughout the week based on… [the] mistakes that students made [in the current lesson]."

Important for Supporting Student Learning

Several teachers emphasized the informative aspect of students' mistakes for supporting student learning. Sara expressed that she thought "…making sense of [students' mistakes] helps them to learn… I think learning through those mistakes of children shows me how they think." Jean explained the importance of knowing where students are confused in order to revisit those muddy points. She said:

> I need to know where they're confused so that I can have something to clarify their confusion. But if I don't know what they're confused about then, I'm not going to know how to help them understand better.

Similarly, Lori expressed that "being able to identify where the mistakes are coming from, it allows me to go back and figure out how I can best help them." Barney also demonstrated how he used students' mistakes for fostering more effective teaching for understanding by saying:

> I would try to nurse [guide] them towards that, but then they wouldn't really have an understanding of it. Now I'm realizing that I need to listen in the lesson, pay attention to the mistakes, and then use those as examples to get from where we are to something that's a little bit better.

However, although teachers realized the value of understanding students' mistakes, determining how to support students appeared to be challenging

based on the case Jean shared about her revisiting of the concept of fraction comparison. She stated:

> So, if you have two different denominators then you're not thinking of the whole. You have to make them similar. But, anyway, so what I would do is I would take the length of the fractions sticks and show them, "OK, here is 1/16" and show them how to get a common denominator so that we're dealing with the same fraction.

In her statement, "if you have two different denominators, then you're not thinking of the whole," Jean appears to recognize the student's mistake (having two different denominators) and offers remediation, "anyway, what I would do is I would take the length of the fractions sticks and show them." However, her approach to addressing student difficulties appeared to be more of a procedural one than promoting cognitive engagement and knowledge building. Holmes et al. (2013) distinguished between exploring misconceptions instead of mistakes. They noted that when the error is superficially addressed, there may be a missed opportunity to increase students' conceptual understanding of the topic. However, when teachers target the misconception underlying the mistake more, students can achieve a deeper meaning of the focused concepts.

Important for Lesson Planning

Anticipating students' common mistakes before teaching was a challenge for these teachers. In this regard, the teachers valued students' mistakes as useful information in deciding what content to teach next and how to teach it. Among teachers' responses, they believed that knowing students' mistakes would reduce the occurrences of the need to "reteach, redress, review." For instance, Lori stated:

> [Student mistake] tells me whether or not I'm ready to move on to a different idea or a different concept or to take it to the next level, or if I need to go back and reteach, you know, as a whole group, if I need to pull a small group and work with them. I mean, it kind of determines whether or not they're ready to do anything else next.

Similarly, Willow commented that "[When I grade], if a significant number of the kids have made the same mistake, then I'll go back and readdress that with the whole group." Jean also stated a similar practice: "If I see that they're struggling with a concept that I thought they would get quickly, then I'm going to revisit that concept. [Otherwise], I'm going to approach it differently the next day."

Teachers' Comfort Level in Using Students' Mistakes

Misaligned with teachers' highly positive perceptions of student mistakes, overall, teachers' comfort levels were not high in addressing students' mistakes. Three teachers had low confidence, one had a low-to-medium level, two teachers had medium-to-high confidence, and one stated it depended on the grade level she taught. Their underlying reasons for feeling uncomfortable addressing students' mistakes were captured in three themes: fighting against that "instinct" of considering students' mistakes negatively; concerns about students' feelings; and lacking self-efficacy related to students' mistakes.

Fighting that Instinct of Considering Mistakes Negatively

Barney reflected that the social norm "that lessons should be very tight and moving very quickly" influenced his instructional decisions and how he responded "when mistakes pop up." Barney stated he had to "fight against that instinct" to avoid the negative and ineffective situations where "they [mistakes] end up getting swept under the rug or quickly telling them what the answer is without any of them [students] understanding" and "[his] uncomfortableness of staying with the mistake and trying to dig deeper to figure out exactly what's going on." The term *instinct*, reflective of his own educational culture, aligns with the academic "performance culture" Boaler (2016, p. 12) described. Several teachers mentioned this.

Concern about Students' Feelings

Being sensitive to students' feelings was important to most of our participants. Willow said directly, "I mean, I don't want to hurt their feelings." Concern for students' feelings towards their mistakes being utilized in class influenced how and when teachers used them. Teachers focused on the reinforcement of positive self-esteem through praising and caring language, which was internalized by teachers as part of professionalism (Santagata, 2004). They perceived that highlighting students' mistakes would undo this. However, avoiding addressing mistakes is more likely to reinforce students' negative perception of making mathematical errors as it communicates that mistakes are not to be discussed (Boaler, 2015, 2016). In this regard, developing a growth mindset around failure and mistakes seems to be a systemic cultural struggle.

Efficacy Related to Addressing Errors

Feeling capable of addressing students' mistakes appropriately influenced teachers' comfort level. Lori, who had a medium to high comfort level, stated, "I'm usually able to identify where it is that they're struggling and be able to figure out how it is that I can help them get back on the right track." Confidence in her diagnostic ability and ability to employ effective strategies to address students' learning difficulties contributed to her comfort in addressing students' mistakes. However, Barney, who had a low to medium

comfort level, felt that he lacked expertise in anticipating and attending to students' mistakes. It sometimes created situations where he had to support multiple struggling students simultaneously. Teachers' efficacy beliefs in this regard appeared to be connected to teachers' self-evaluation of their ability to teach a challenging math concept, how well they can anticipate and accurately analyze student thinking underlying the students' mistakes, and how many available strategies they have to address different students' cognitive needs. The literature suggests that elementary teachers in the United States struggle with understanding mathematics content deeply (e.g., Ma, 2020). Thus, they tend to be apprehensive about encountering students' mistakes, knowing the likelihood they would struggle to readily diagnose its source and be able to support their students in resolving the error effectively.

Discussion

We drew three central insights from our findings: (a) the US teachers in this study acknowledge the importance of students' mathematics mistakes, but in superficial ways, (b) positive perceptions of students' mathematics mistakes do not necessarily indicate a high comfort level in addressing them, and (c) addressing students' mistakes is influenced by context and knowledge of students.

Our findings indicated that all teachers in the study valued students' mathematics mistakes. They identified the importance of students' mistakes from three major perspectives - important for lesson planning, supporting student learning, and proactively teaching. This finding paints a different picture than what is currently stated in the literature; in particular, that mistakes are devalued by teachers in the United States (Boaler, 2016). This shift may be due to the participants' exposure to perspectives on students' mistakes as valuable through books and journals for practitioners (e.g., Boaler, 2015, 2016; Lim, 2014), what is advocated by teacher organizations (e.g., National Council of Teachers of Mathematics) and the materials that were used in the PD program they were involved.

Although teachers' perceptions of students' mistakes were positive, their understanding of the value of students' mistakes seemed pretty superficial. Teachers thought mistakes were important for lesson planning. Yet, they mainly focused on the initial steps in lesson planning, that is, "what I should teach next." This aligns with Holmes et al.'s (2013) observation that teachers "generally identify problems that were missed by the majority of students and then do nothing more than just re-work these problems during the next class session." Teachers saw student mistakes as an indicator of poor understanding of math content and used mistakes to determine whether to move on, reteach, or review content. None of the teachers mentioned using students' mistakes as launching pads for inquiry-based instruction or for provoking deeper mathematical thinking (Boaler, 2016; Cobb et al., 1992;

Smith & Stein, 2011). This may be because the teachers lack the knowledge or skills to identify students' underlying misconceptions (Holmes et al., 2013) although they could easily recognize when students' solutions misaligned with the correct one. Thus, we argue that the teachers' lesson planning perspective is more closely reflective of the performance culture (Boaler, 2016), where mistakes indicate adverse learning outcomes (Stigler & Hiebert, 1999).

In addition to valuing student mistakes, teachers expressed low to medium comfort in addressing mistakes. US teachers' levels of discomfort align with Stigler and Perry's (1988) observation, "American teachers feel more comfortable parsing the students who perform well than discussing the errors that can occur in the course of problem solving" (p. 52). However, our study documents the conflicts teachers experienced utilizing students' mistakes. Our analyses show that classroom norms and teachers' efficacy in handling student mistakes appeared to contribute to teachers' comfort levels in attending to students' mistakes. For example, Barney expressed that he had to overcome the "instinct" of telling students the correct answer, which was hard for him as the expectation was that lessons would progress efficiently and distractions, like unpacking mistakes, should be avoided. From his case, we realized that instinctively and culturally, teachers are more likely to see students' mistakes as a sign of failure, but professionally and instructionally, they need to remind themselves that students' mistakes are growth points. Navigating through this tension is often complicated and unsuccessful for teachers.

Another aspect of culture involved how society viewed children. As researchers have argued, students are viewed as inherently good by parents and teachers; thus, reinforcing students' positive self-esteem through praising and caring language was internalized by teachers as part of professionalism (Santagata, 2004). This may explain teachers' concern about students' feelings when their mistakes are addressed publicly. For example, Lori expressed that she avoided sharing students' work as it might embarrass them, which may indicate that in Lori's mind, making students' mistakes public would cause shame and embarrassment, from which she could not envision any public good. This suggests that students in the United States may also have a negative perception of making mistakes which is likely to be reinforced consistently in their classrooms (Boaler, 2015, 2016). In this regard, developing a growth mindset around failure and mistakes seems to be a systemic cultural struggle.

In addition to the social and cultural factors, teachers' self-efficacy beliefs in addressing student mistakes played a crucial role in determining teachers' comfort level in dealing with students' mistakes. This self-efficacy appeared to be connected to teachers' self-evaluation about how well they can teach a challenging math concept, how precisely they can anticipate students' common mistakes, how accurately they can analyze student thinking underlying the mistakes, and how many available strategies they have to

address different students' cognitive needs. These kinds of self-evaluation align with the *Standards for Preparing Teachers of Mathematics* (AMTE, 2017, p. 19) which suggests that teachers should "first try to see mathematical situations through their students' eyes rather than immediately correcting mathematical errors or demonstrating their approaches." The teachers might know it is inappropriate to correct the students' errors but cannot decenter and understand students' mathematical thinking, especially at a moment when they interact with students. The literature suggests that US elementary teachers struggle with a deep understanding of mathematics content (e.g., Ma, 2020) and knowledge of students' thinking. Thus, teachers are apprehensive about encountering students' mistakes knowing they may struggle to readily diagnose its source and be able to support them in resolving the error effectively. As such, it would seem that the core task ahead for mathematics education researchers and teacher educators is not to convince teachers that mistakes should be valued; instead, to support them in developing the breadth and depth of knowledge and the efficacy needed to create mistake-friendly environments.

One limitation of this study is that we only interviewed a small subset of teachers, the teachers who participated in a PD program, which limited us to making grand claims about US teachers in general. Another limitation is that we did not analyze and report on our observations of teachers' responses to students' errors in classroom teaching to examine if their perception and comfort level aligned with their classroom teaching. We are working on a paper that systematically examines these teachers' responses to students' mathematical mistakes across five coaching cycles which lasted for one academic year as a supplementary to this limitation.

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