Case Study of Cooperative Learning in Mathematics: Middle School Course Design

Hua Cheng
Xianyang Normal University, China

Cooperative learning in mathematics (CLM) course design places emphasis on analysis and problem solving of complicated issues and knowledge that is of the nature of extensive coverage of the interconnected reasoning path. The key to CLM lies in its "cooperativeness". CLM design should align its contents properly with students' epistemic baselines. Teachers' activities in CLM should be focused mainly on promoting students' active involvement, while highlighting both students' in-class and out-of-class cooperation. In order to achieve the expected results, CLM should also be applied in combination with other teaching methods.

Key words: cooperative learning of mathematics course; class teaching; thinking participation.

Cooperative learning, which is hailed as the "most important and most successful teaching method reform in the last decade" (Ellis & Fouts,1997) and as one of the teaching methods that the new curriculum reform advocates, is increasingly winning favor in primary and secondary schools in China. While cooperative learning theory is better understood and relevant research contents are further enriched, the focus of the domestic researcher shifts from theoretical introduction of international state-of-the-art in this area to a further step of exploratory study by applying this theory to its local context and practice in China, while emphasis is increasingly drawn on its influence on subject construction. "Full-time compulsory education mathematics curriculum standards (trial version)," highlights: "students should be encouraged to discover the unknown independently and be willing to cooperate". Educational value varies depending on different subjects, therefore answers to questions such as, how to understand the cooperative learning in mathematics classroom, and what subject is suitable for cooperative learning in mathematics, deserved to be investigated due to their value in reality. This paper will discuss the issue based on a survey (case study) and propose a number of ways to improve teaching quality.

Case Study on Cooperative Learning

Case Description

The whole set of all mathematics teachers, up to twenty seven, from a top notch middle school of Xi'an city, were surveyed in this case study after mathematics oriented cooperative learning had been utilized in this school for two years. A self-designed questionnaire was used in interviews. Open questions were randomly chosen from a prepared list and used in the questionnaire and answered anonymously. Twenty seven questionnaires were issued and 100% of them were collected while all were valid statistically. Statistical analysis was conducted after returned forms were sorted based on different categories.

How to Choose Cooperative Learning

Table 1 lists different percentages of various types of questions that are chosen for cooperative learning practice. It shows that a high percentage usually is accompanied by types of questions such as “questions worth deep investigation”, “questions involving complex or difficult knowledge”, and “questions that can be solved in different ways” and a much lower percentage with other types of questions. The result suggests that mathematics teachers interviewed in this case tend to choose the topics that invite complex knowledge or problem solving skills for cooperative learning practice.

The results of the interviews show rationality and reality driven features.

i) Cooperative learning allows students to benefit from idea exchange when they tackle problems that require complex knowledge or problem solving skills;

ii) Students tend to deviate on complex issues and cooperative learning in this context, This contributes to communication and therefore helps broaden the individual’s views in solving problems. It also invites inspirational sparks among students.

iii) If simple knowledge is the target of cooperative leaning, cost of CLM does not worth the gain, and the learning efficiency would be largely compromised;

iv) Reflection of “score driven” reality. High score on exams have for a long time been the key focus of domestic education. Effectiveness of learning is usually measured by whether or not a high score can be achieved by efficiently solving problems in exams. As a result, mathematics teachers pay more attention to contents that contribute directly to higher exam scores. Our results show that teachers in this case study are able to take advantage of cooperative learning by choosing proper learning topics while placing more emphasis on its role of achieving better score on exams.

Table 1

Percentage of Various Types of Questions

Question types	Percentage.
Questions worth deep investigation	44%
Questions involving complex or difficult knowledge	41%
Questions that can be answered in different ways	33%
Questions difficult to be solved independently or involved knowledge hard to be self-learned	19%
Key knowledge	11%
Questions that can be solved by team work	7%
Questions involving knowledge of which students might have their unique insights	3%
Easy questions that can be answered by the average via self-learning	3%

How to Organize Cooperative Learning

We conducted a statistical analysis of some key words that occur frequently in answers given by interviewees to question such as “What are the steps when implementing cooperative learning?” It reveal**s** that mathematics teachers in the case study emphasize more “cooperativeness”, “communication”, “discussion”, and “thinking independently” rather than “teacher guidance”. It also shows that "thinking independently" takes second place.

Investigation also shows that it is quite common that a "students' role" is improperly regarded as superior to a "teachers' role" in cooperative learning. This may be due to a number of reasons. The traditional teaching approach may have been over-corrected, but it is more likely that teachers have been lacking a proper approach in guiding students. Mathematics is fundamentally self-contained, strict and abstract and traditional teachers tend to adopt a teaching pattern that places students in a passive stance. In contrast, cooperative learning appears more ephemeral and varies more in terms of how it is organized in real life practice and in terms of related knowledge being presented, and therefore teachers, who, if rarely, had ever been exposed to the cooperative learning environment, tend to be less skillful or less productive when guiding students.

Analysis also shows that interviewees highly endorse "thinking independently". The following reasons may explain this.

i）It is due to this fundamental feature of mathematics. Mathematics is a discipline that represents more logical reasoning than others and it involves learning subjects' more internal activities such as thinking.

ii) It is due to the influence of the traditional way of mathematics knowledge acquisition. Some scientists (de Bary，1983）found that Chinese students are required to study and think more independently when it comes to mathematics learning and this is a phenomenon heavily influenced by traditional Chinese culture .

iii) It is due to the features of cooperative learning. Cooperative learning without engagement of active thinking tends to result in poor efficiency.

iv) It is due to China's special exam centered cultural background. Students need to solve problems independently during exams and no one else can help in such a context, therefore, they need to constantly put "thinking independently" into practice as if they were on the front line in order to keep their competitive advantage in such matter.

In summary, "thinking independently" is equally important in cooperative learning practice, and it not only reflects the Chinese tradition in teaching methodologies, but also echoes Chinese teachers' experience in their teaching practice.

Analysis of CLM

The above case study unveils some interesting observations regarding CLM. Johnson & Johnson (Year) gave their own definitions for cooperative learning. Different from other sciences such as art, mathematics is more programmable, more discipline centered, and stricter with less attention placed on the emotional aspects of human beings. Then what needs to be taken into account to cover this missing part?

The Use of CLM

The teaching methodology serves the contents and the first issue to be discussed is the contents of cooperative learning. Some researchers believe that cooperative learning is suitable for more complicated or higher level epistemic targets and also suitable for learning tasks involving emotions, attitudes, and values. However, in our context, a more specific statement is needed to offer a better guideline for mathematics teaching practice in our middle schools.

We believe that cooperative learning in mathematics is suitable for topics involving large-scale conceptualization, and multi-tiered reasoning. With more scale or with more tiers more advantage will be gained from cooperative learning. For examples, “study conditions for Congruent triangles”, which requires one to investigate which condition regarding relations among three angles and three lines , need to be satisfied. Students need to first carefully classify the situations into nine types that respectively include cases with one line, one angle, two lines, two angles, and one line plus one angle. In this example, students learn by team vs. team discussions and students are able to manipulate the learning target by trying to give peer/opposite examples, visualizing concepts and doing experimental research. This allows research to be conducted in depth. Cooperative learning in this example shows clearly its advantage for topics involving large-scale conceptualization, and multi-tiered reasoning where the learning process can be made more vividly with colorful spotlights.

Independent Thinking

Capability of solving problems independently should be encouraged in mathematics cooperative learning. When students solve problem independently, they tend to overwhelmingly rely on their own biased personal experience in order to find the solution while missing or rarely rendezvousing a different perspective on the same issue. It is often seen that even when alternative solutions come across their minds they tend to be overlooked due to the existence of the first solution already in mind. However, in cooperative learning, a team composed of subjects with different mathematics knowledge experience, background and thinking patterns will help each other take advantage of the complimentary views of others in the epistemic process, benefiting from multichannel communication and further polishing their capability in problem solving.

We believe that under the circumstances of cooperative learning, thinking independently and cooperative communication intermingle and nurture each other. It is especially important to see the independency of thinking in the processes of students’ representing, listening and discussing. In fact, mentors tend to reserve a certain amount of time for students to be able to think independently prior to communication; however they often overlook students’ independency during discussion.

For example, it is often seen that average students take advantage of achievements made by merit students and they tend to follow them. Merit students usually are those who first propose solutions and therefore the chance of independent thinking for the average students becomes automatically blocked. This compromises the teaching quality largely due to the varying possible gains obtained by these two different types of students. Many reasons account for this:

i) It can be due to the contents of overcomplicated problems used in cooperative leaning. When the subject is complicated and very abstract, students can be bored, and when tasks involved have a less clear structure, average students are likely to be very anxious about the situation’s uncertainty;

ii) Skill levels of different students can be heterogeneous and differentiation in terms of students’ knowledge structure and varying tiers in absorbing knowledge will damage the synchronization between different students in terms of the maturity or correctness level of their logical reasoning prior to cooperative discussion.

A variety of factors account for weakened independent thinking. Whether or not students are able to keep their thinking independent determines, to a large extent, whether cooperative learning is able to achieve its desired quality. However, it is a challenge to assure that all students, in the same class, with varying skill levels, are able to think independently to meet our satisfaction.

The Power of Inspiring Sparks and Reciprocal Learning

Nurturing students' rationalism, rational mentality and skill of logical thinking is the main educational goal of mathematics. Whether or not students are able to think actively is also an important benchmark for mathematics cooperative learning practice. For instance, in a case where two neighboring students are assigned a task to measure the perimeter of a circle, if there is only cooperation while thinking or reasoning is missing, it is not qualified to be called cooperative learning any more. In contrast, if students cooperate in a way that they discuss how to measure the perimeter of a circle, e.g. by experiment such as rolling a round-shaped card, which was colored by ink on the edge on the surface of a paper, and by further measuring the length of the footprint of the card, and they further debate on how to prevent it from rolling away and how placing the card against a ruler will help achieving a correct result, then, this is sufficiently qualified to be called CLM. Therefore, the cooperativeness meant by cooperative learning mainly refers to the activities that can trigger inspiring sparks, where students experience complementary learning from each other and help one another, rather than the physical activity of "cooperation". What is meant by "cooperativeness" is the cooperativeness in the sense of mental activity rather than cooperation which is superficially just a form like people gathering. Any cooperation without sufficient mental involvement or serious and hard mental work is not cooperative learning according to ours definition.

Zhang (2006) (no reference) believes that cooperation is of three basic types that are respectively comprehensive cooperation, cooperation based on job division, and cooperation enabled by communication. Mathematics learning that mainly involves mental activities and tasks that need to be committed depending on an individual's independent thinking falls into the third category. Based on our case study and interview, we realized that CLM in a classroom takes the third type – cooperation enabled by communication as the main approach and it is consistent with what is believed by Zhang. In summary, knowledge acquiring or question discussions involving a large scale of complexity and of a multi-tier nature, are more suitable for CLM. In cases where students find it difficult to study independently or explore independently based on their AS-IS skill set or epistemic level, it is better to engage in cooperative learning as this will achieve a better result in such a way that sparks from different individuals will help all participants train themselves to think in broadened dimensions, where more thinking approaches can be referred to and can contribute, and a training style that leads to fully interactive units of team intelligence will benefit all students' capabilities in problem solving.

Constructivist View

Based on a previous case study, we suggest the guidelines below may help achieve a better educational result.

Line with Students' Epistemic Baseline

Contents in CLM need to be in line with students' level. When too much emphasis is placed on chasing the complexity and playfulness of mathematics problems, cooperative learning tends to be over- interpreted into patterns, over simplified and deviates too much from mathematics' own fundamental nature.

Take "how to teach: the sum of degrees of interior angles for triangles". Let us imagine the scenario below. Students were divided into groups to measure the sum of degrees of interior angles for triangles and "found out" that the sum is 180°. Students then conducted an experiment where they approached each angle of a triangle shaped paper and tore the paper into three pieces. Further on, students found out that all pieces can be rearranged in a way that three angles were merged into one strait line, namely a 180° angle, by placing them staying immediately one after another side by side, and this inspired students to draw virtual lines in order to prove related theorems/laws. When the law, the sum of degrees of interior angles for triangles remains 180°, was unveiled, mentors might present a question, such as: Whether or not the sum of the degrees of interior angles for polygons follow a certain rule, in order to encourage students to further study via cooperative learning.

Course design in the above-mentioned fictitious scenario is exquisite, however, one important flaw is that it overlooked students' epistemic baseline. Most students would have been familiar with the law of the sum of interior angles of triangles. Stating that the result was "discovered", in the experiment would be overkill in terms of wording for such a situation; it might not be attractive enough for lower level middle school students to perform an experiment such as paper tearing, which was just a repetition of what was done already years ago. Also, although the initial intention of the paper tearing experiment here is to guide students, by creating a side-by-side angle situation where the sum of has angles has a result of 180 in order to find a way to draw a virtual line in order to help in problem solving, it overlooked this fact: graph puzzling games (such as situation created as angles being immediately together side by side) can relate to an open discussion and it is not guaranteed that all possible cases will result in an ideal virtual line construction. It appears that the course design, weaving the course contents by creating a smooth transition from discussion of triangle interior angles issues to discussion of polygon interior angle issues, is quite scientific and shows a superior structure, however it might not be true due to the following reasons. The way of thinking for issues about the interiors of polygon is different from what is true about triangles. It requires a logical pattern of generalization, from special instance to general type and from the concrete to the abstract. This is not suitable as CLM material and is too difficult for lower grade middle school students to perform during a short CLM class. This is an example where CLM design is not properly in line with students' epistemic baseline or levels and therefore a perfect assumption does not guarantee an equally logical implementation in practice.

Motivating Students' High Level Mental Activity

The involvement level of students' proactive thinking is closely related to the manner of how teachers guide. We believe that although guidance should cover skills and methods relevant to cooperation, emphasis should be placed on how to enhance students' involvement in high level thinking, and on how to help students achieve results sooner while remaining at this higher thinking level.

Take the teaching material "features of an isosceles triangle" and imagine a scenario of CLM as follows. Someone would like to introduce the concept of the features of the isosceles triangle using CLM. He first asked students to fold a paper in the shape of an isosceles triangle following its center axle and his intention is to first let students guess and then proof their guesswork to better understand the concept. But because folding the paper itself is just a different way of expressing of the drawing of virtual line, it literally bypassed the thinking process for one solution by indicating the solution of how to prove itself. This type of CLM obviously takes away the opportunity for students to think more proactively at a more advanced level. A better way would have been to ask students to observe first and then to assume that the two corner triangles are equal before exploring solutions to proof and learn its related features via CLM. Drawing a virtual aiding line is the key for the solution in this case, however teachers should prevent the situation from happening where merit students may first shout out excitedly with multiple solutions such as by drawing a geometrical symmetric axle line, a virtual line in the direction of height, as well as an aiding line that equally divides the top corner angle etc., and thereby suppress the chances of the others. Rather teachers should have guided students in a manner that motivates and inspires students, who are at varying levels to all think proactively. A good example may have been to enlighten them by asking questions such as: How to prove two angles are equal? Which part of knowledge may provide a tool to prove this? These questions help them relate the problems to the law of "peer angles are equal to each other in congruent triangles".

When a hint such as "however there is no situation where two congruent triangles exist" is given to students, the worst performing students would even think a step further by dividing a triangle into two using a virtual line and then think of what might be the best way to do so in order to prove the hypothesis. The folding paper experiment, however, might have been taken as an interesting way to verify the law as the last step rather than the first step. Teachers in CLM should try to create a platform to help all to improve their solution finding capabilities and their ability of thinking independently, rather than letting some take advantage of what has been achieved by others who are more proactive in the experiment.

In summary, the art of teaching lies in better inspired sparks of students' thinking, better motivated and controlled thinking capability, and it should prevent turning a high level task into an oversimplified program or steps of a simple procedure.

In-and Out of Classroom Cooperation

The contents of mathematics usually feature continuality, expandability and application-orientation, and it is challenging to achieve the goal of making students understand comprehensively and thoroughly in a one-time CLM instance. Considering the fact that In-class cooperation tends to be restricted by limited duration, space, extent of student devotion and many more factors, in-classroom cooperation and outside-of-classroom cooperation should be treated as equally important. When outside of classes, students are able to think independently by taking advantage of self-arranged time periods, and it makes interactions in the classroom more thorough. Also teachers are more confident in their guidance and it further contributes to students' better awareness of the importance of cooperation.

CLM outside-of-classroom experiences can be categorized into two types, namely prior-to-class cooperation and after-class cooperation. In the first one, an overall task is divided into a few subtasks where each member takes one. For instance, subtasks for "area and algebra equation" can be as follows

(a) Relate area calculation to algebra equation: $(a-b)^2 = a^2 - 2ab + b^2$;

(b) Relate area calculation to algebra equation: $a(b+c)=ab+ac$, $(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$;

(c) Relate area calculation to algebra: equation: $(a+b)^2-(a-b)^2=4ab$, Relate area calculation to Pythagorean theorem $a^2+b^2=c^2$;

(d) By relating area calculation to algebra equation, solve the problem: Can you construct a giant square by aligning 5 smaller squares each with a different lateral size?

CLM prior to class can be adopted as a way to warm-up, or to facilitate so that students have an overview of the material before the real class starts. CLM in the manner of so that discussion after class should applied more extensively. Advantages of the above are:

1) Unsolved problems during the class, or extensive discussion of related issues, cannot be accomplished during the class, but can continue to be explored. When teaching “the sum of the degrees of the interior angles of triangles”, the contents about “the sum of degrees of interior angles for a polygon” can be left for after-class CLM study;

2) Experimental tasks or further investigation can be arranged for after class CLM. For example, after teaching “mosaic”, assign after class CLM jobs to students to study the distribution patterns of floor tiles.

3) Ask students to a arrange collection of mathematics problems and have them find test exams themselves to help them review what they have learned. CLM has the advantage that it is not restricted to limited in-class time, plus, the contents and style of CLM are more colorful and tend to be friendlier when students generate their own topics of interest.

Class with Large Numbers

CLM utilizes the benefit that students are able to study more proactively with more motivation and it helps students share resources more efficiently, however, its usage does have limitations in some areas. Even upper grade middle school students who have rapidly developed their capabilities of abstract thinking (which even supersede other aspects of their development) to a large extent still reply mainly on their intuition for learning. Since mathematics is fundamentally an abstract world and its internal logic utilizes sequences among its patterns, it is not pragmatic to rely too heavy on efforts of CLM in order to achieve the teaching goal of mathematics; CLM tends to occupy more time and therefore if it is abused key points will be largely diluted due to limited efforts or time. Hence CLM is not suitable to be applied in class with large numbers and it only fits the circumstances where a limited domain is involved and is combined with other teaching approaches, e.g. circumstances combined with teachers’ instructive presentations, demonstration and students’ independent exploration. On oral teaching approach that places students on the passive side is beneficial at efficiently highlighting the nature of mathematics as a knowledge system but tends to set the boundaries too early and therefore restrict students’ breadth of thinking dimension, while CLM fills in the gaps created during lectures or oral teaching.

CLM is just one of many teaching approaches and it does not guarantee students’ motivation and proactive learning. Any approach is able to develop students’ thinking capability and the key is to choose the right contents, apply them in the right proportion, and utilize the proper way of guidance. Teachers need to predict: What problems are those that can be solved by students independently? What can be solved in the classroom through CLM? What needs to involve elaboration and teachers' demonstration? What approach needs to facilitate this elaboration and what is the best timing? Based on the teacher's predictions, proper lessons should be prepared.

For topics where CLM applies, concentration should be focused on the difficult components, the key knowledge and the contents where confusion tends to occur. Teaching approaches for mathematics should be colorful, and the best should be the ones that are able to combine and apply proper basic approaches in the right context.

Conclusion

CLM enriches methods and procedures for students' mathematics learning and at the same time brings up many new challenges. In China, faced with some special cultural and domain related background issues, the practice of CLM in reality is far more complicated than an academic discussion of "how to". Solutions can only be found via experimental research in reality, through practice. Teachers may solve problems better, learn from/inspire each other, and cooperate more efficiently by analyzing each other's special treatment of teaching material contents, selected approaches and system design, and by further investigating the differences in-between these aspects.

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Author:
Hua Cheng
Xianyang Normal University, P.R.China
Email:ch7723@163.com