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Using the Evidence-Based MSA Approach to Enhance Teacher Knowledge in Student Mathematics Learning and Assessment

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The purpose of this research was to contribute to a better understanding of how to enhance teachers' knowledge in student mathematics learning and assessment through implementing the Model, Strategy, and Application approach (MSA) in a teacher education program. Participants were 23 classroom teachers in a graduate program in mathematics education at a university in Southern California. The MSA approach was used weekly to support the participants' learning and understanding of the three components of the MSA approach in each course of two advanced mathematics methods courses. The participants learned to develop the MSA assessment items at their grade levels and provided the assessment to evaluate their students' mathematics proficiency course 2. They were also trained in coding and analyzing their students' mathematics learning using the MSA rubrics. The results show that both groups of elementary and secondary teachers improved their knowledge of assessment, knowledge of student thinking, and knowledge of student strengths and weaknesses, and there was no significant difference in their knowledge between two groups at the end of the program.

***Keywords**: Teacher knowledge, pedagogical content knowledge, model, strategy, application, student mathematical learning, student thinking, assessment*

According to the National Council of Teachers of Mathematics (NCTM, 2000), "Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies" (p. 17). Knowledge of students' mathematics learning furnishes specific insights that help teachers gauge how well students understand mathematical concepts, recognize error patterns and understand possible misconceptions behind the errors, and develop strategies to correct the misconceptions. It is sought that classroom teachers engage "in inquiry to deepen their understanding of students' thinking" (The National Research Council [NRC], 2001, p. 389) through careful analyses of their students' work. However, many recent studies indicate that U.S. elementary and middle school teachers possess limited knowledge of assessing student mathematics learning. In addition, there is a lack of useful and practical instruments for assessing student mathematical abilities, and a structured assessment tool can be developed in such a way as to gather valid and reliable information about students' mathematical abilities (Wuttke & Wolf, 2007) in classroom teaching practice.

The National Research Council's report *Adding It Up* calls for achieving mathematical proficiency in adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NRC, 2001). To help students achieve mathematics proficiency, classroom teachers must have assessment tools and know how to use them to assess students' mathematics learning (NCTM, 2000). The quality of mathematics tasks and questions for students to understand mathematics concepts, build procedural fluency, and develop strategic competence in word problem applications become critical for the field of mathematics education to improve student mathematics proficiency (NRC, 2001; Small, 2009) on the ongoing base. In teachers' daily teaching practice, ongoing assessment must be provided in order to assess student mathematics proficiency and effectiveness of the instruction (Blythe, 1998). Classroom teachers can benefit from the analysis of student mathematics learning from their ongoing assessment (Corno, 1996; Corno, 2000).

The goal of this study was to provide classroom teachers an opportunity to learn, develop, and use a structured teaching and assessment tool via an evidence-based approach of Model-Strategy-Application (MSA) (Wu & An, 2006) to assess their student mathematics learning in the three areas of mathematics proficiency in order to provide effective mathematics instruction.

The research questions in this study were: What do classroom teachers gain from learning the MSA approach and applying it in assessing their students' mathematics learning? Is teachers' knowledge of student mathematics learning and assessment improved by implementing the MSA approach in the teacher education program? What are the differences in the teacher learning in the MSA approach between elementary and secondary teacher groups?

Theoretical Framework

Teacher Knowledge of Assessing Student Mathematics Learning

A study by An, Kulm and Wu (2004) addressed the need for enhancing teachers' pedagogical content knowledge that connects content and pedagogy in four dimensions: building on students' ideas in mathematics, engaging students in mathematics learning, addressing students' misconceptions, and developing student thinking about mathematics. Their model of teachers' pedagogical content knowledge asserts the knowledge of student misconceptions and thinking as central to pedagogical content knowledge. Various studies suggest that teachers with a better understanding of children's thinking develop a profound mathematical understanding (Sowder, Philipp, Armstrong, & Schappelle, 1998). Knowledge of students' mathematics learning requires specific knowledge that allows teachers to know how well students understand mathematics concepts, understand possible misconceptions and their patterns, and to develop comprehensible strategies to correct misconceptions. Teachers should "engage not only in inquiry about how to apply knowledge about students' thinking in planning and implementing instruction, but also in inquiry to deepen their understanding of students' thinking" (NCTM, 2000, p.389).

Although there is growing evidence that knowledge of students' mathematics learning is grounded in practice (Falkner, Levi, & Carpenter, 1999), too little of the extant research probes what type of knowledge of students' mathematical learning is essential and how to support teachers to enhance such knowledge. It is necessary to explore an effective approach to enhance knowledge of student mathematical learning in a structured way.

Using the MSA Approach for Students' Mathematical Learning

The MSA approach is a structured way of teaching mathematics and assessing student mathematics learning, which is comprised of three components in assessment: 1) assessing student conceptual understanding by analyzing their various visual **models**, 2) gauging student procedural fluency from their using different computational **strategies**, and 3) evaluating student strategic competence of problem solving from their knowledge and skills of real world **application**. The essential components of the MSA assessment demonstrate diverse ways of assessing student learning (Wu, 2008**).** The NRC (2001) and RAND (2003) provided five indicators of student mathematics proficiency: conceptual understanding, procedural fluency, problem solving, adaptive reasoning, and productive disposition. The essentials of the proficiency strands are aligned with the components of the MSA approach that includes mathematical conceptual understanding, procedural fluency, and strategic competence in real world applications; adapted reasoning is embedded in these three components. Furthermore, when students have attained conceptual understanding, procedural fluency, and real world application knowledge and skills, they will have also developed beliefs regarding mathematics as sensible, useful, and worthwhile (RAND, 2003). Therefore, the MSA approach is consistent with the indicators of mathematics proficiency as defined by NRC (2001) and RAND (2003); it is not only a fundamental framework for teachers to assess students' mathematics learning, but also a guiding principle for teaching mathematics in a balanced way as addressed in the California Mathematics Conceptual Framework (California Department of Education, 2006).

According to the NCTM Assessment Standards (1995), it is important to use multiple indicators for student assessment. “One assessment or one type of assessment should not be the sole measure of a student's achievement, because it is not likely to give an adequate picture of that student's learning. Nor should any one assessment be used to make decisions of any consequence about a student's educational future" (Koelsch, Estrin, & Farr, 1995, p. 11). The MSA assessment supports classroom teachers to gauge their students’ learning in three ways, namely: conceptual understanding, procedural fluency, and problem solving in real world application.

To learn how to apply the MSA approach to assessing their student mathematics learning, teachers must 1) learn and be able to create various visual **models** to build conceptual understanding, 2) learn and be able to develop **strategies** in procedural fluency to master basic and complex computational skills in an accurate, fast, and flexible way, and 3) learn and be able to build strategic competence in word problem **application**s. Learning, teaching, and assessing via the MSA approach will enable teachers to not only build a solid foundation of mathematics content and pedagogical content knowledge and develop a systematic approach of teaching mathematics in a balanced way but also to master multiple ways of understanding student mathematical learning (Wu & An, 2006).

Methods

Subjects

The participants were 23 graduates in a graduate program in mathematics education in a university in Southern California. They were also certified classroom teachers, four females and 19 males, 13 at the elementary level and 10 at the secondary level, from eleven school districts. The criteria of inclusion for subjects were: (a) to be taking a graduate assessment course in fall 2010, and to be taking two graduate advanced mathematics methods courses in the MSA approach in spring 2011and fall 2011; (b) to have a valid teaching credential and, (c) to have had at least two mathematics content courses at the college level.

Procedures and Data Collection

In this study, the participants took three related courses: Assessment course, Advanced Mathematics Methods Course 1 and Advanced Mathematics Methods Course 2. The participants had an opportunity to learn various assessment techniques in the assessment course. The MSA approach was taught in all mathematics content areas at the elementary level in the Advanced Mathematics Methods in Teaching 1 and at the secondary level in the Advanced Mathematics Methods in Teaching 2. The participants actively engaged in learning how to assess student learning by doing the MSA weekly (MSAW) individually first and then discussing appropriate models, strategies, and applications in each MSAW in a specific mathematics task in the whole group setting in two advanced methods courses. In the Advanced Mathematics Methods in Teaching 2 they also learned how to develop the MSA assessment items in five content areas and use the MSA rubrics to score and analyze their student MSA items. In addition, they used SPSS to analyze a correlation been student MSA scores and their CST scores, and analyze possible associations between their student MSA scores and demographic factors. All participants were assessed using a pre-survey and post-survey of student mathematics learning, their views on their knowledge growth, and reflection in learning in each course. The teachers were surveyed again on their views of growth of their knowledge at the end of the second year.

Instruments

Various instruments were used to assess the participants' progress of learning and using the MSA approach: A pre-survey consisting of nine MSA problems was provided for the course Advanced Mathematics Methods in Teaching 1 and a post-test consisting of five MSA problems and two student thinking problems was provided in the course - Advanced Mathematics Methods in Teaching 2. In addition, a survey consisting of 25 problems of knowledge of student thinking, a survey consisting18 pedagogical content knowledge questions and 23 learning reflection questions, and a survey of 29 learning reflection questions and 11 learning experience questions were provided during the program in different courses. In this study, four questions regarding teacher learning in the MSA approach in the post-survey in course 1, seven questions in course two, and five questions at the end program were used for data analysis (see Appendix A). A total of 14 MSAWs were provided for each course. Each MSAW had one word problem that required participants to solve the problem in three ways: Modeling, strategy of computation, and creating and solving a similar word problem (see Appendix B). In the course of Advanced Mathematics Methods in Teaching 2, the participants worked with their grade level group to design a set of five MSA problems that were aligned with the state content standards in the areas of number, algebra, geometry, measurement, and statistics and probability. They provided the MSA assessment to their students, scored and analyzed their student MSA work using the MSA Rubrics that were developed by An and Wu (2010) (see Appendix C). The same MSA Rubrics were used for scoring and analyzing the teachers' MSAW weekly in this study.

Data Analysis

Both qualitative and quantitative data analysis methods were used in this study. To answer research question 1, a qualitative data analysis method was used to measure teacher learning from doing their weekly MSAWs and the MSA assessment using questions from post-surveys in courses 1 and 2. The open-ended responses to teacher learning on the surveys were coded, categorized, and compared for data analysis (Lincoln & Guba, 1985).

Research question 2 examined changes in teacher knowledge. Paired t-tests were used to determine if there were statistically significant differences in teacher learning on knowledge of student thinking, student strengths and weakness in mathematics learning, assessment, coding and analyzing student MSA work, between courses 1 and 2. To answer research question 3, the General Linear Model (GLM) was used to analyze differences in teacher knowledge growth between elementary and secondary teachers. The reliability and validity of the study were ensured by using triangulation of data, member checks, and peer examinations. The combination of both quantitative and qualitative methods is supported by numerous studies (Cronbach & Associates, 1980; Fielding & Fielding, 1986).

Results

The results of this study show that in-serve education can support classroom teachers to learn and develop a measurable and practical model that can be used to assess their student mathematical conceptual understanding, procedural fluency, and strategic competence in word problem applications through sequentially designed courses and coursework.

Classroom Teachers' Learning from the MSA Approach

At the end of each advanced mathematics methods course, the teachers answered open-ended questions on their learning from the MSA Approach in their post-surveys. This study reports the results of analyzing their learning of MSAWs and MSA assessment.

Learning from doing MSAW weekly. Table 1 shows the results of the analysis reflected in the categories and examples of teacher responses regarding their learning from MSAWs in course two. The main parts of teacher learning from MSAWs are modeling and problem solving, followed by mathematics content knowledge and other learnings.

Here is an example of a teacher response from course 1: "I enjoyed working on the MSAW assignments because it gave me an opportunity to see how other people might solve the same problem. It was also very helpful to see other people's models." The MSA approach also enabled the teachers to realize their weaknesses in mathematics content knowledge, as Carla, an elementary teacher indicated, "I need more practice in geometry. I need to be knowledgeable in high level mathematics."

Learning from doing the MSA assessment. Table 2 shows teachers' responses to their learning from doing the MSA assessment with their students. The main learnings indicated by the teachers were teacher knowledge of students, student thinking, student strengths and weaknesses, designing and analyzing assessments.

Table 1
Teacher Learning from Doing MSAWs

Category %	List three things you learned from doing MSAWs	
Modeling 28%	• Learned to solve and model solutions to address different learning styles • Similar figures – using models to solve geometry • New methods on how to model particular problems • Relating modeling to strategies • I think I also got better at showing models for my work • Model problems with words and symbols	• Learn how to model problems • Modeling algorithms • Modeling in different ways • How to model different problems • How to model the task
Problem Solving 25%	• The challenge of solving word problems • Learned to analyze my own problem solving skills • Strategies to solve geometry problems • Finding out that there are multiple ways to solving problems • I learned to look at problems in different ways • Seeing and learning different ways of solving problems by watching my classmates	• Multiple ways of solving problems • Different approaches to same problem • Different approaches to problem solving • New strategies for solving problems
Understand Concept 12.5%	• Deeper math concepts • Problem understanding • Flexible about using visuals in explaining concepts	• Review of math concepts and strategies • Reviewing math concepts
Math Content 12.5%	• Geometrical relationships between shapes • The ones with the shapes are probably my favorite • Introduction to higher math concepts	• Practice math skills • Practice different types of problems
Real Life 5%	• Make you connect your work (learning to everyday life)	• Putting math related to content in real life situations (applying)
Other 17%	• Solutions thru elementary, middle school, and high school view points • Learned to rewrite solutions differently • It was good way to start off with some of the more challenging ones	• Thinking process • Create similar problems • Organization of task • Collaborating with others

Table 2

Teacher Learning from Doing the MSA Assessment for Students

Category %	List three things you learned from doing the MSA Assessment	
Misconceptions 5%	• Misconceptions were very easy to spot	• Common Misconceptions
Student Thinking 13%	• Creating word problems that demand a higher level of thinking • It was helpful to see that my students don't know how to explain their thinking by using models. It helped with my planning for future lessons	• Student thinking • Thinking process • It helped me understand my students' thinking process
Student Strengths & Weaknesses 11%	• I learned to identify student problem solving weaknesses and strengths, and it is useful to learn how to teach • Learning where students are struggling	• Most struggled with creating their own word problem • Modeling is also difficult
Math Standards 5%	• How to focus on specific math standards for assessment purposes	• Creating appropriate leveled content standards based problems
Analyze Assessment 13%	• How to put data in SPSS and use SPSS • How to analyze assessment data • Better at compiling database	• Using a MSA rubrics • Grading from rubrics
Design Assessment 13%	• How to create meaningful MSA assignments • How to modify assessments for first grade • Designing assessment that fits my students	• I was able to better assess student academic level • This allows me to know how to assess my students
Knowledge of Students 32%	• How students solve the problem • See how my students model problem solving • Help students better to understand content • It can show the different methods students used to calculate • Got an idea at my students' skills early on • It allows me to help my students understand more of how word problems are constructed	• Student math knowledge • See how creative students are • Student language ability • What students retained from 6^{th} grade • What students retained from one MSA to another MSA Student ability to solve the problems given
Other 13%	• Communication with students • Share a better idea of the academic levels of my students • Writing improves student learning	• Student did really enjoy MSA work • My students like doing them

Improvement in Teacher Knowledge

The results of the paired t-tests that examined teachers' knowledge growth from course 1 to course two show that teachers' knowledge of student thinking, knowledge of student strengths and weaknesses, knowledge of assessment, knowledge of coding and analyzing student assessment were greatly improved (see Table 3). We attribute this to the learning that took place from doing the MSAW weekly and doing the MSA practice, and conducting the MSA assessment with their students. Among the areas of teacher learning, teachers' knowledge of student strengths and weaknesses (t (22) = -2.761, p = .011), knowledge of assessment (t (22) = -3.237, p = .004), knowledge of analyzing student assessment were significantly improved (t (22) = -2.646, p = .004) from course 1 to course 2 (see Table 4).

Table 3

Knowledge Improvement: Paired Samples Statistics

	N=23	Mean	SD	SE
Pair 1	Stu_Thinking1	4.109	.5632	.1174
	Stu_Thinking2	4.30	.703	.147
Pair 2	Stu_StrWk1	4.065	.5288	.1103
	Stu_StrWK2	4.48	.511	.106
Pair 3	Assess1	3.500	1.0766	.2245
	Assess2	4.35	.487	.102
Pair 4	Code1	4.1522	.41106	.08571
	Code2	4.17	.937	.195
Pair 5	Analysis1	4.065	.5288	.1103
	Analysis2	4.4783	.59311	.12367

Table 4

Knowledge Improvement: Paired Samples Test

N=23		Paired Differences					t	Sig. (2-tailed)
		Mean	SD	SE	Confidence Interval			
					Lower	Upper		
Pair 1	Stu_Thinking 1 Stu_Thinking 2	.1957	.7796	.1625	.5328	.1415	1.204	.242
Pair 2	Stu_StrWK 1 Stu_StrWK 2	.4130	.7175	.1496	.7233	.1028	2.761	.011
Pair 3	Assess 1 Assess 2	.8478	1.2562	.2619	1.3911	.3046	3.237	.004
Pair 4	Code 1 Code 2	.02174	.98256	.20488	.44663	.40315	.106	.916
Pair 5	Analysis 1 Analysis 2	.41304	.74852	.15608	.73673	.08936	2.646	.015

Differences in Teachers' Knowledge Growth

Differences in teacher knowledge growth in course 1. The GLM test results show that the elementary teachers' content knowledge and pedagogical content knowledge grew more than the secondary teachers due to doing the MSAW and MSA practice (see Table 5). This difference is significant, *F* (1, 21) = 4.626, *p* = 0.043, Partial Eta Squared = 0.181 (see Table 6). Figure 1 confirms the differences between the two groups.

Table 5
Descriptive Statistics

	Grade Level	Mean	SD	N
CK_MSAW	Elementary	4.231	.5991	13
	Secondary	3.800	.6325	10
PCK_MSAW	Elementary	4.231	.5991	13
	Secondary	4.000	.4714	10
PCK_MSAP	Elementary	4.38	.506	13
	Secondary	3.70	.675	10
PCK_MSABK	Elementary	4.23	.599	13
	Secondary	3.80	.422	10

Table 6
Tests of between Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	1481.244	1	1481.24	1535.67	.000	.987
Grade_Level	4.462	1	4.462	4.626	.043	.181
Error	20.256	21	.965			

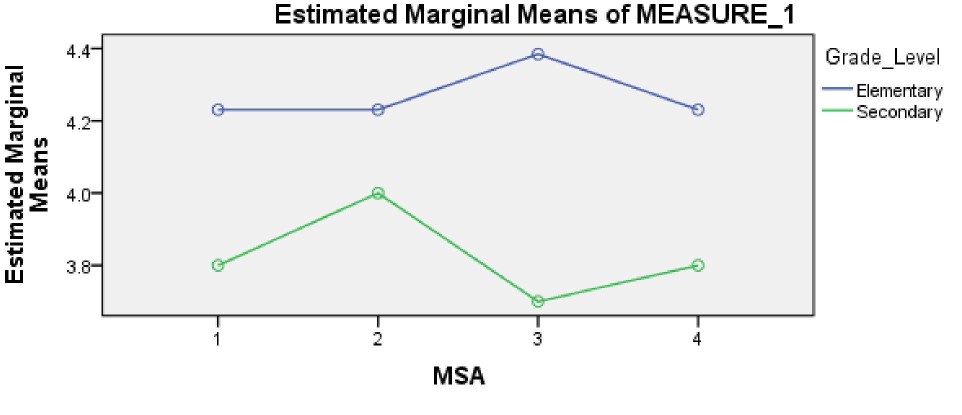


Figure 1. *Difference in learning between elementary and secondary levels in course 1.*

The differences in teacher knowledge can be reflected on the example of MSAW2 responded by a secondary teacher in Figure 2 and by an elementary teacher in Figure 3:

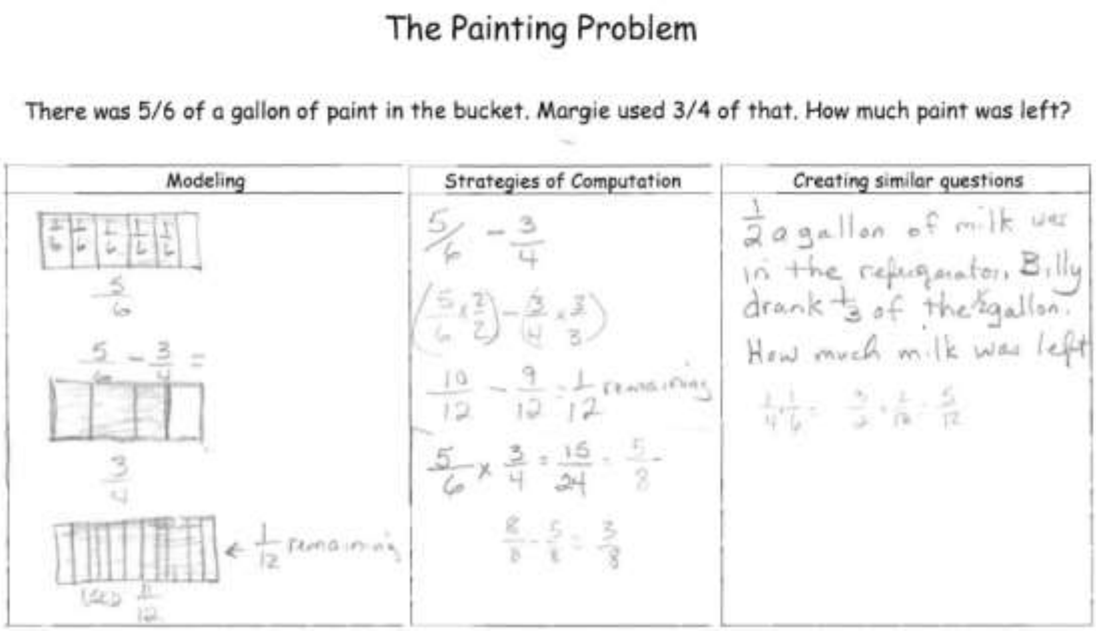


Figure 2. *The solution of MSAW2 by Barbara.*

Barbara is a 7th grade teacher. She seemed to be unfamiliar with the elementary level of mathematics teaching. In MSAW2 The Painting Problem, she first mistakenly just subtracted the two fractions $\frac{5}{6}-\frac{3}{4}=\frac{1}{12}$, ignoring the fact that fractions represent relative amounts. She did right in step 2 by multiplying $\frac{5}{6}\times\frac{3}{4}=\frac{15}{24}=\frac{5}{8}$, but she mistakenly used a whole $\frac{8}{8}$, not $\frac{5}{6}$ subtracted $\frac{5}{8}$ to get the wrong result of $\frac{3}{8}$. Barbara's visual models reflected her misconceptions and computation errors.

Ella is a 5th grade elementary teacher who solved MSAW2 differently. Ella found how much paint was used first by doing $\frac{3}{4}\times\frac{5}{6}=\frac{3}{8}$. After drawing a visual model by dividing 5/6 into 4 parts , she got "$\left(\frac{5}{6}\div 4\right)\times 3=\frac{5}{24}\times\frac{3}{1}=\frac{5}{8}$ gallon used." Ella then figured out "$\frac{5}{6}-\frac{5}{8}=\frac{20-15}{24}=\frac{5}{24}$ gallon left."

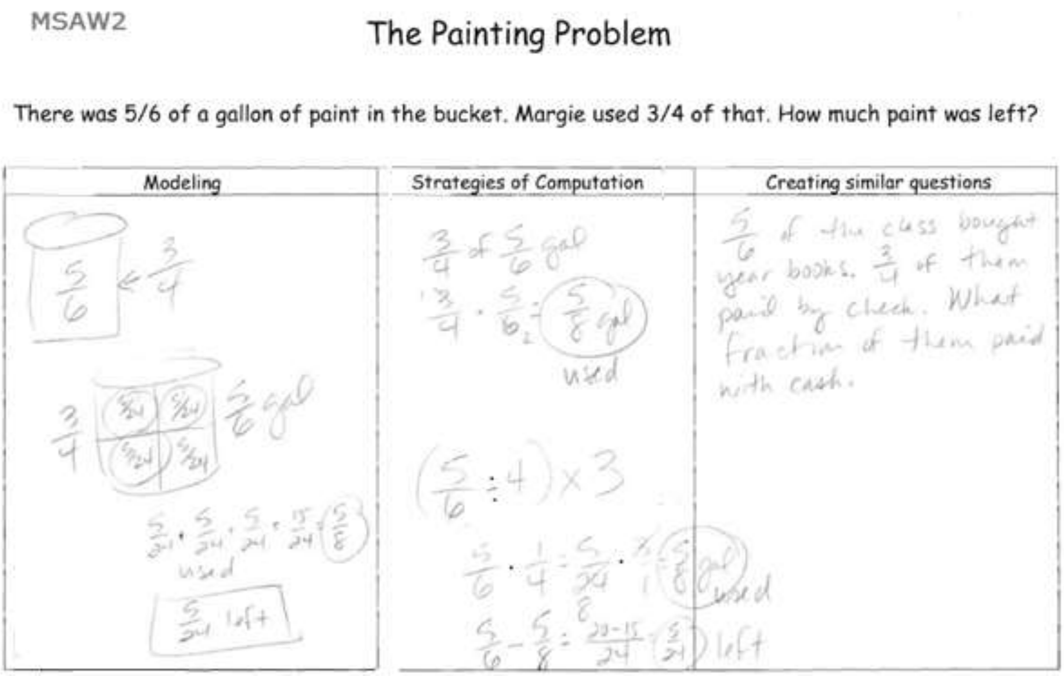


Figure 3. *The solution of MSAW2 by Ella.*

Differences in teacher knowledge growth in course **2.** Table 7 shows that in course 2, elementary teachers' growth of pedagogical content knowledge is higher than secondary teachers' pedagogical content knowledge (see Table 7). The highest difference is in the knowledge of student thinking. It shows that elementary teachers had higher growth of knowledge of student thinking, while the secondary group of teachers had higher growth of knowledge of student strengths and weaknesses. Figure 4 verifies these differences. However, Table 8 shows that these differences are not significant ($F(1, 20) = 4.032$, $p = 0.58 > 0.05$).

Table 7
Descriptive Statistics

	Grade_Level	Mean	SD	N
PCK_MSA5P	Elementary	4.00	.577	13
	Secondary	3.56	.527	9
Assess1	Elementary	3.692	.9473	13
	Secondary	3.500	1.0000	9
Stu_Thinking1	Elementary	4.308	.4804	13
	Secondary	3.833	.6124	9
Stu_StrWk1	Elementary	4.154	.5547	13
	Secondary	3.944	.5270	9
Teaching2	Elementary	4.154	.6887	13
	Secondary	3.722	1.1487	9

Table 8
Tests of between Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	1606.46	1	1606.46	1983.50	.000	.990
Grade_Level	3.265	1	3.265	4.032	.058	.168
Error	16.198	20	.810			

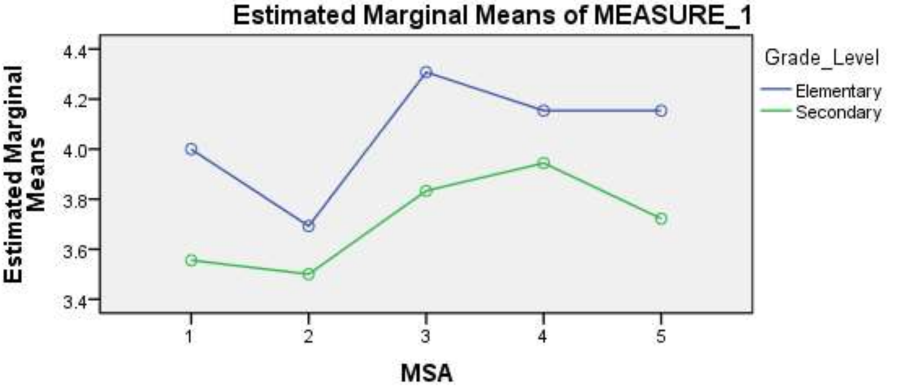
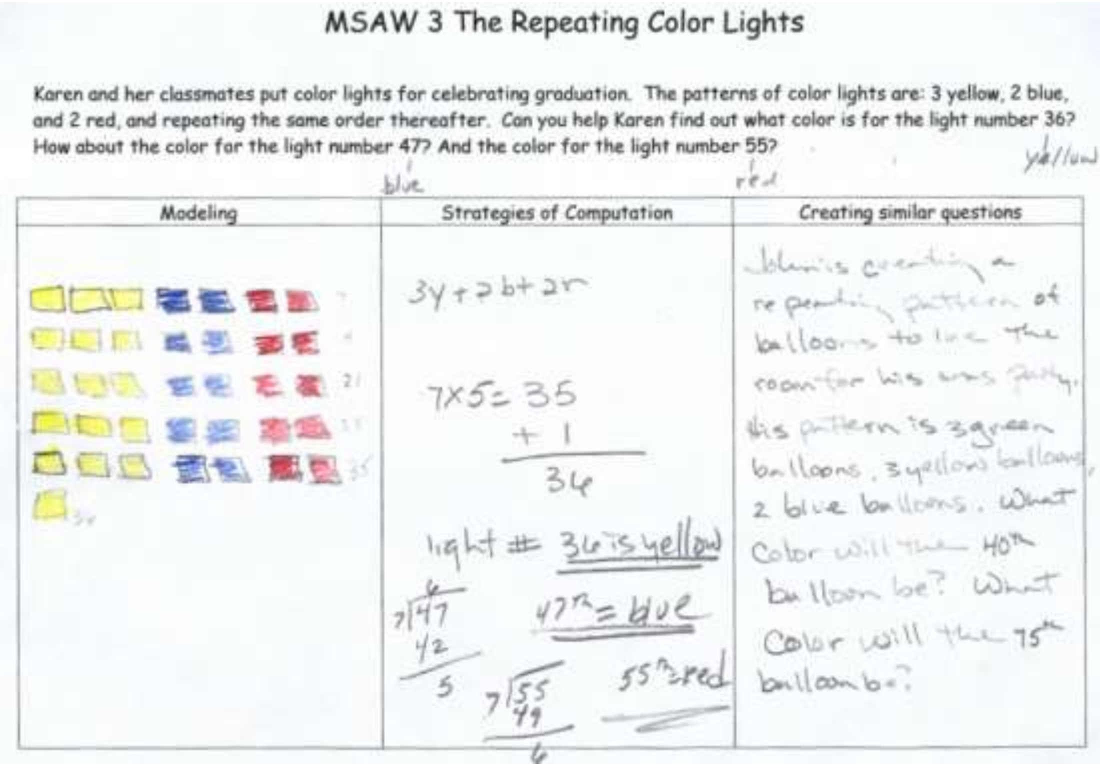


Figure 4. *Difference in teacher knowledge between elementary and secondary levels in course 2.*

The equal growth of teacher knowledge in both groups can be shown in the examples of MSAW3 The Repeating Color Lighter in course 2 in Figures 5 & 6. Sandra, an elementary teacher used colorful visual representations to show patterns of three color lights and used the patterns to find the solutions in her visual model. She verified the result using division with remainders. Helen, the secondary teacher used symbolic representations to show the patterns of three color lights, and use modulus as a strategy of computations to the same results.



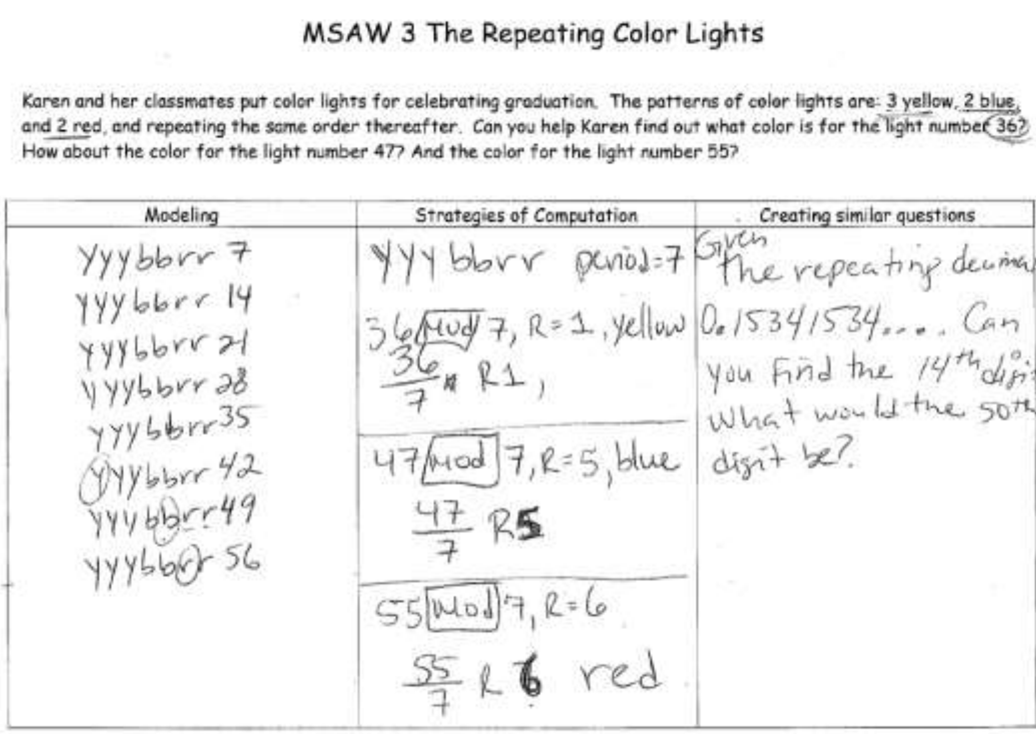
MSAW 3 The Repeating Color Lights

Karen and her classmates put color lights for celebrating graduation. The patterns of color lights are: 3 yellow, 2 blue, and 2 red, and repeating the same order thereafter. Can you help Karen find out what color is for the light number 36? How about the color for the light number 47? And the color for the light number 55?

blue red yellow

Modeling	Strategies of Computation	Creating similar questions
7 14 21 28 35 36	3y + 2b + 2r 7×5 = 35 + 1 36 light # 36 is yellow 7)47 6, 42, 5 47th = blue 7)55 49, 6 55th = red	John is creating a repeating pattern of balloons to line the room for his mom's party. His pattern is 3 green balloons, 3 yellow balloons, 2 blue balloons. What color will the 40th balloon be? What color will the 75th balloon be?

Figure 5. *The solution of MSAW2 by Sandra.*



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Modeling	Strategies of Computation	Creating similar questions
yyybbrr 7 yyybbrr 14 yyybbrr 21 yyybbrr 28 yyybbrr 35 yyybbrr 42 yyybbrr 49 yyybbrr 56	yyybbrr period=7 36 Mod 7, R=1, yellow 36/7 R1, 47 Mod 7, R=5, blue 47/7 R5 55 Mod 7, R=6 55/7 R6 red	Given the repeating decimal 0.1534153 4.... Can you find the 14th digit? What would the 50th digit be?

Figure 6. *The solution of MSAW2 by Helen.*

Differences in Teacher Knowledge Growth at the End of the Program

Table 9 shows that both groups had an almost equal growth in knowledge of assessment, knowledge of student thinking, and knowledge of student strengths and weaknesses at the end of the program. Table 10 confirms that there is no significant difference in teacher's knowledge in assessment, student thinking, and student strengths and weaknesses between the two groups (F = (1, 21) = .222, p = 0.642). Figure 7 confirms the same results.

Table 9
Descriptive Statistics

	Grade Level	Mean	Std. Deviation	N
Assess2	Elementary	4.31	.480	13
	Secondary	4.40	.516	10
Stu_Thinking2	Elementary	4.23	.832	13
	Secondary	4.40	.516	10
Stu_StrWK2	Elementary	4.46	.519	13
	Secondary	4.50	.527	10

Table 10
Tests of Between Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	1303.18	1	1303.18	1706.87	.000	.98
Grade_Level	.170	1	.170	.222	.642	.01
Error	16.03	21	.763			

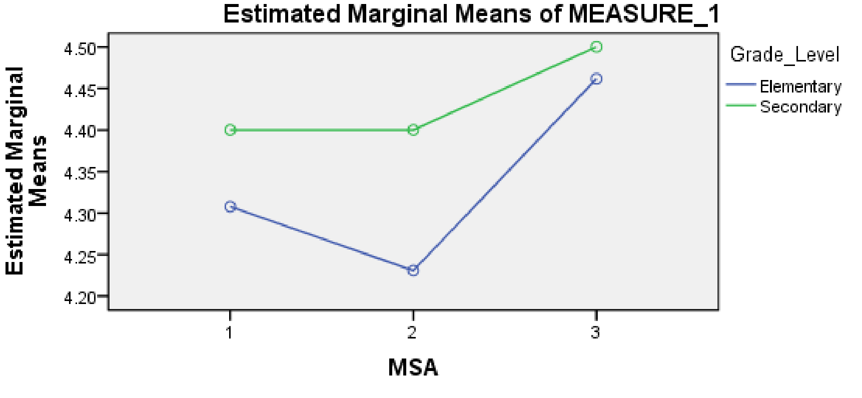

Figure 7. *Difference in teacher knowledge between elementary and secondary levels at the end of the program.*

Discussion and Conclusion

The results of this study show that the MSA approach in this study provided teachers a learning opportunity on how to improve their knowledge of student mathematics learning and knowledge of assessment in a structured and measurable way. The three components of the MSA approach are supported by the new Common Core Standards for Mathematical Practice Standard (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) that calls to model mathematics, look for and make use of structure **a**nd express regularity, and make sense of problems. The teachers in this study not only used the MSA assessment as a useful tool for assessing students' mathematical learning, but they also used it as an instructional model to address concepts, procedures, and problem solving. The teachers' responses to the survey questions provided evidence of their knowledge growth in student mathematics learning and assessment from implementing the MSA approach in the teacher education program.

One of the interesting findings in this study is that with implementing the MSA approach, teachers' knowledge was improved from course 1 to course 2 between two groups of teachers. In course 1, the elementary teachers had a higher growth in pedagogical content knowledge than the secondary teachers. This finding is consistent with the findings by An's study that more experience in teaching and more educational courses make teachers more knowledgeable in their efforts to support student learning (2009). Two reasons could affect this difference in course 1. First, in this study, course 1 focused on the elementary math content areas; second, in the U.S., elementary teachers usually take more educational courses than secondary teachers. In addition, Tasleema and Hamid (2012) found that elementary teacher educators have more teaching aptitude as compared to secondary teacher educators on total score of teaching aptitude test battery. However, with a continuous learning of the MSA approach from course 2, the secondary teachers' knowledge of assessment, knowledge of student thinking, and knowledge of student strengths and weaknesses improved equally as the elementary teachers in this study.

The results of the qualitative and quantitative analysis in this study shed light on the effective teacher education program. Although teacher learning appeared to be many approaches in teacher education programs, one or two separated courses alone were not enough for teachers to make substantial changes in their knowledge; teacher education programs need to provide a series of connected courses providing connected and structured methods for teachers to learn and analyze student mathematics learning in multiple aspects.

The structure of the MSA approach in this study provides a concrete, practical, structure, and measurable way for classroom teachers to create a set of rich assessment tasks and questions to assess student learning in diverse ways. Such assessment tasks and questions are supported by various studies and different domestic and international assessments (Kulm, 1990; Wu, 2008). It also provides a useful tool for teachers to know students' mathematical thinking and assess their strengths and weaknesses in mathematics learning. Importantly, it provides teachers feedback on the effectiveness of their instruction and supports them find appropriate ways to assist students who need help in a specific aspect of the MSA to improve their mathematics learning and achieve mathematics proficiency (NRC, 2001; RAND, 2003). In addition, it makes an insightful contribution to the current inquiry in the mathematics education field to answer the questions such as, "What knowledge is really needed for effective mathematics teaching?" "What is the effective way to assess students, and how can we operationalize such assessment to advance our efforts in both research and teaching practice?"

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Appendix A

Post Survey in Advanced Mathematics Methods Course 1 Spring 2011

Part I Please select one answer from each item:

	Never Agree	Strongly Disagree	Disagree	Agree	Strongly Agree
My math content knowledge has improved due to doing the weekly MSAW problems					
My PCK has improved due to doing the weekly MSAW problems					
My PCK has improved due to working on the MSA Practices					
My PCK has improved due to reading the MSA book chapters					

Post Survey Advanced Mathematics Methods 2 Fall 2011

Part 1. Please select one answer from each item:

	Never Agree	Strongly Disagree	Disagree	Agree	Strongly Agree
My PCK has improved due to designing five MSA problems for my students					
My knowledge of assessment has improved due to the MSA assessment					
My knowledge of student thinking on math has improved due to scoring student MSA assessment					
My knowledge of student strengths and weaknesses of math has improved due to scoring and analyzing student MSA assessments					
My knowledge of coding qualitative data has improved due to scoring the student MSA					
My knowledge of analyzing quantitative data has improved due to working on statistical analysis for my student data on the MSA					
My knowledge of teaching has improved due to working and discussing the MSAWs weekly					

Final Survey Spring 2012

Part 1. Please select one answer from each item:

	Never Agree	Strongly Disagree	Disagree	Agree	Strongly Agree
My knowledge of assessment has improved due to assessment projects					
My knowledge of student thinking on math has improved due to scoring and analyzing student MSA assessments					
My knowledge of student strengths and weaknesses in learning math has improved due to scoring and analyzing student work					
My knowledge of coding qualitative data has improved due to coding and scoring student MSA					
My knowledge of analyzing quantitative data has improved due to statistics tests and interpreting results for the MSA					

Appendix B

Example of MSAW

The Book Page Problem

Jessica is reading a story book. She reads 30 pages of the book per day. After three days of reading, 5/8 of the book are left. How many pages are in this book?

Modeling	Strategies of Computation	Creating and solving a similar word problem
Explain why and link models to underlying math ideas:	Explain why:	Explain why:

Appendix C

The MSA Rubrics

Level	Modeling	Strategies of Computation	Creating similar questions
Level 1	Either no model or model completely inappropriate	Either missing computation or many computational errors	Problem either missing or impossible to follow
Level 2	Appropriate model used, but either not fully demonstrated, or possibly based the operation only, did not show the process of conceptual developing	Only few computational errors, but followed rules and formulas on computations (routine way), or only by trial and error	Problem attempted, but difficult to understand
Level 3	Appropriate model used, and the process of modeling demonstrated	No computational errors, but solved problem by routine way or only by trial and error	Problem fairly clear, but not appropriate or not connected to real life application
Level 4	Model used highly efficient and meaningful, revealing comprehensive understanding	No computational errors and used a flexible or creative strategy in computation, revealing complete understanding of solving	Problem very clear, appropriate, and connected to real life application

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