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A Study of Influential Factors in Mathematical Modeling of Academic Achievement of High School Students

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Applying path analysis method, the study explores the relationships among the influential factors in mathematical modeling academic achievement of 208 high school students. The results showed that all seven factors—academic achievement (MMAA) mathematical modeling self-monitoring ability (MMS-MA), creativity level (CL), the cognitive structure of mathematics and science subjects (CSMSS), mathematical modeling emotion (MME), creative inclination (CI), cognitive style (CS) and mathematical modeling belief (MMB) have significant positive correlation with mathematical modeling.

Key words: mathematical modeling, academic achievement, influential factor, path analysis method, high school students.

Introduction

Due to the powerful education value of mathematical modeling, many countries include this content in their mathematics curriculum (Werner, Galbraith, & Mogen, 2006). Although in China mathematical modeling is also included in *the Standard for Senior High School Mathematics Curriculum (experimental version)*, educators are not satisfied with the curriculum implementation effects (Li & Yu, 2008). More research needs to be done on high school students' mathematical modeling cognition patterns, based on what mathematics educators can design and teach of mathematical modeling curriculum to make more effective. The current study was to apply path analysis method analyzing the influential factors, path and function extent on mathematical modeling academic achievement (abbreviated as MMAA) of high school students. Based on related studies on cognitive process and the difference of mathematical modeling, we proposed a theoretical assumption on influential factors and the path of high school students' MMAA.

Theoretical Assumption on Influential Factors

Possible Influential Factors

Self-monitoring level of mathematical modeling. The self-monitoring level of mathematical modeling refers to modeling learner ability to achieve success which includes carrying on the plan, inspecting, evaluating, providing feedback, making adjustment and controlling the whole mathematical modeling activity. Self-monitoring is the key element of meta-cognition, and its level may significantly influence the possibility and efficiency of success in problem solving (Li, 1997). Researchers (Li, 2009; Li, Pang, & Yu, 2009) indicated that self-monitoring is the main operational variable of cognition. The cognitive activities, such as adjusting the understanding and presumptions, searching and selecting the strategies, constructing and solving the modeling, testing and discussing the modeling for questions according to feedback information may impact the results and efficiency of mathematical modeling. Li (2007) showed that undergraduates' mathematical modeling self-monitoring level influenced their cognitive and operational process through influencing their mathematical modeling self-monitoring behaviors. Li, Pang and Yu (2009) maintained that there was a significant difference between experts and novices in using mathematical modeling self-monitoring strategies, and the mathematical modeling self-monitoring level influences the level of mathematical modeling through selecting and using the strategies of mathematical modeling.

Creativity level. Creativity refers to both intellectual ability and characteristic intelligence demonstrated in innovation activities. Mathematical modeling is an activity greatly related with creativity. Usually no existing and pre-knowledge method is available for learners to apply mathematical modeling method to solve real situation problems. It usually requires the modeler to use creative thinking to link different knowledge, and then using creative methods builds a relation and rules between related factors. Werner, Galbraith and Mogen (2006) pointed out that creativity was a required ability for a modeler. There was a close inner connection between the mathematical modeling level and creativity level of engineering undergraduates - the higher level of creativity the students had, the stronger the competences of mathematical modeling they would have (Dan, 2007). According to the data of contests and mathematical modeling teaching, high achieving students who have high levels of creativity usually have flexible thinking and use novel methods to solve problems.

The cognitive structure of mathematics and science subjects (CSMSS). The cognitive structure of mathematics and science is a cognitive structure developed by learning mathematics and science subjects. Mathematical modeling often involves broad, cross-disciplinary knowledge, particularly, the knowledge of mathematics and science subjects. As a result, the cognitive structure of mathematics and science subjects is the foundation for mathematical modeling and the pool of background knowledge. CSMSS may help the modeler in selecting related information and problem schema, constructing appropriate representation for a question, and finding appropriate strategies for mathematical modeling. Li (2007, 2009) pointed out that many students had enough knowledge needed to apply mathematical modeling method to solve a realistic problem, but they could not make the model by themselves without getting a hint. One of the reasons was the unformed knowledge structure in the students' mind, which resulted in the difficulties for a precise selecting, and successful extracting of appropriate knowledge, which led to the wrong representation and strategy for a question. Li (2007) showed that the undergraduates' cognitive structure of mathematics and science subject had a significant effect on their MMAA.

Mathematical modeling emotion. Mathematical modeling emotion is a kind of relatively stable feeling experience and psychological sense, consisting of four factors, which include interest, motivation, attitude and anxiety. The mathematical modeling interest is a psychological disposition experienced by individuals in exploring and solving the real mathematical modeling problem. The mathematical modeling motivation is also a psychological disposition, which arouses and maintains an individuals' mathematical modeling learning activity and directs the activity to a given objective. The mathematical modeling attitude is a kind of reflective preparation state formed in the process of the mathematical modeling learning activity, pointing and influencing the selection to the activity. Also the mathematical modeling anxiety means a reflective inclination is produced by individuals for feeling or foreseeing a threat to their self-respect in the mathematical modeling learning activity. There was a closely intrinsic relationship between undergraduates' achievement motivation and their MMAA (Li, 2007). Engineering college students' interest and attitude in mathematical modeling influenced their achievements in mathematical modeling (Dan, 2007).

Creative inclination. Creative inclination is an active psychological disposition of individuals which cause them to create activities, which including four dimensions: the characteristics of adventure, curiosity, imagination and challenge. Creative inclination falls into the scope of creative personality, offering mental state and background for individuals' applying creativity through arousing, facilitating, adjusting and monitoring. Researchers (He, Zha, & Xie, 1998; You, Zhang, & Liu et al., 2006) stated that the characteristics of creative inclination (excepting for curiosity) and the total score had significant correlation with students' academic achievement, and the latter reflected largely the state of students' cognitive structure of mathematics and science subject. The meaning (four dimensions) and characteristics of creative inclination tell us all these including "guessing boldly" (the representation of adventure), "getting to the bottom of the matter" (the representation of curiosity), "inferring by intuitively" (the representation of imagination) and "willing to explore complex problems" (the representation of challenge) are the necessary psychological background and personality characteristics for learning mathematics and science courses, solving complex mathematical problems (e.g. mathematical modeling, and implementing self-monitoring).

Cognitive style. Cognitive style is the difference in personality inclination and characteristic performed by individuals in the cognitive process. Cognitive style is an important factor which impacted students' cognitive structure (You, Zhang, & Liu, 2006). Flexibility of thinking was considered as the basis of learning mathematics and science courses. Students with stronger field-independence tended to have flexible thinking (Li, 1994). Undergraduate students' cognitive style impacted their MMAA via their cognitive structure of mathematics and science subjects (Li, Pang, & Yu, 2009). The students, with stronger field-independence, often had stronger original, curious and critical characteristics, which reflect individuals' creativity level (Li, 1994; Lin & Xin, 1996).

Belief in mathematical modeling. Belief in mathematical modeling refers to students' overall cognition and perspectives on questions related to mathematical modeling learning. Belief in mathematical modeling has three elements: the cognition belief, the learning belief and the efficiency belief. The cognition belief in mathematical modeling is students' basic views on mathematical modeling knowledge and the knowing process. Belief in mathematical modeling also includes three aspects, the belief in knowledge structure, stability and validity of mathematical modeling. The efficiency belief in mathematical modeling is the modelers' understanding of their mathematical modeling ability. Students' beliefs deeply impacted their learning emotion, motivation inputting, behavior engagement and cognition processes, then, produced a far-reaching effect on the learning results (Tang & Yu, 2008). Students' beliefs on mathematical modeling impacted fairly their mathematical modeling learning and achievement (Li, 2009).

Theoretical Assumption of the Model

Based on the above analysis, we propose a theoretical assumption of influential factors and a path of high school students' MMAA: (1) factors which impact MMAA directly: mathematical modeling self-monitoring level, creativity level, cognitive structure of mathematics and science subjects and mathematical modeling emotion; (2) cognitive style, creative inclination and belief in mathematical modeling directly impact mathematical modeling self-monitoring level; (3) creative inclination and cognitive style directly impact creativity level; (4) creative inclination and cognitive style impact the cognitive structure of mathematics and science subjects; and (5) cognitive style and belief in mathematical modeling impact mathematical modeling emotion.

Methodology

Participants

The participants in the current study are 218 science students from four high schools (in each school one class is selected in the 12th grade) in

Guangdong and Jiangsu province, however, only 208 participants' data are valid enough to analyze for the current study.

Instruments

(1) Creative Inclination Test. This test was constructed by F. E. Williams and revised by Lin Xingtai. This test is as 50 topics and its reliability is between 0.49 and 0.81.

(2) Group Embedded Figures Test. This test was revised by Beijing Normal University Psychology Department. This test' retest reliability is 0.90 while the validity is 0.49.

(3) Torrance Tests of Creative Thinking. This test is one of the most authoritative scales for testing creativity level.

(4) Mathematical Modeling Belief Questionnaire. This test was initially developed as the Mathematical Modeling Belief Questionnaire, and consists of 82 items. In response to pre-test and experts in education and the mathematical modeling field, we revised the draft repeatedly and the questionnaire was reduced to 60 items (Li, 2009). This test' overall reliability is 0.81.

(5) Mathematical Modeling Emotion Questionnaire. This test was constructed with the same process as the Mathematical Modeling Belief Questionnaire, which involves 24 topics. This test' overall reliability is 0.78.

(6) Mathematical Modeling Self-Monitoring Ability Questionnaire. This test was also constructed as the same process as Mathematical modeling Belief Questionnaire, which involves 34 items. This test' overall reliability is 0.83.

(7) The Test of Mathematical modeling (Groups). This test was constructed based on the demands of *The Standard for Senior High School Mathematics Curriculum* and the mathematical modeling contents in high school mathematics teaching materials. This test consists of 6 problems, the first three questions are essay questions belonging to basic knowledge of mathematical modeling, the last three questions are mathematical modeling problems belonging to the skill test problems of mathematical modeling and are progressively difficult. Participants' scores in this test were used as index of their MMAA.

(8) The Average Score of Mathematics and Science Subjects. This variable focused on all science students in participants' schools (12^{th} grade). This score transformed the original scores scored by participants' in mathematics, physics, chemistry and biology into standardized scores, and used the average score as an index of their cognitive structure of mathematics

and science subjects.

Data Collection and Data Analysis

We provided the questionnaires and scale, and the following tests for the participants at the same time: Mathematical Modeling Emotion Questionnaire, Mathematical modeling Emotion Questionnaire and Creative Inclination Test. The Test Paper of Mathematical Modeling (Groups), Mathematical Modeling Self-Monitoring Ability Questionnaire, Torrance Tests of Creative Thinking and Embedded Figures Test were conducted independently. The original scores of participants' mathematics, physics, chemistry, and biology subjects were translated into standardized scores, and we then computed the average scores of the four subjects.

Results

Correlation Analysis among Factors

Table 1 (see below) shows the correlation coefficients among participants' mathematical modeling belief (MMB), cognitive style (CS), creative inclination (CI), mathematical modeling emotion (MME), mathematical modeling self-monitoring ability (MMS-MA), creativity level (CL) and the mathematical modeling academic achievement (MMAA).

Table 1

Correlation Matrix among Factors

	1	2	3	4	5	6	7	8
MMAA	—							
MMS-MA	0.526**	—						
CSMSS	0.509**	0.221	—					
CL	0.518**	0.191	0.156	—				
MME	0.492**	0.232	0.163	0.142	—			
MMB	0.421**	0.435**	0.222	0.131	0.569**	—		
CS	0.430**	0.438**	0.451**	0.449**	0.457**	0.164	—	
CI	0.438**	0.456**	0.413**	0.586**	0.139	0.151	0.216	—

Table 1 showed that (1) there is a statistically significant correlation among cognitive structure of mathematics and science subjects (CSMSS), mathematical modeling self-monitoring ability (MMS-MA), creativity level (CL), mathematical modeling emotion (MME) and mathematical modeling academic achievement (MMAA); (2) there is significant correlation among mathematical modeling belief (MMB) and mathematical modeling emotion (MME), mathematical modeling self-monitoring ability (MMS-MA), and mathematical modeling academic achievement (MMAA); (3) there is significant correlation among cognitive style (CS), cognitive structure of mathematics and science subjects (CSMSS), mathematical modeling self-monitoring ability (MMS-MA), creativity level (CL), mathematical modeling emotion (MME) and mathematical modeling academic achievement (MMAA); and (4) there is a significant correlation among creative inclination (CI) and creativity level (CL), mathematical modeling self-monitoring ability (MMS-MA), cognitive structure of mathematics and science subjects (CSMSS), and mathematical modeling academic achievement (MMAA).

Duplicate Regression Analysis among Factors

The results of five duplicate regression analysis are shown in table 2: mathematical modeling emotion (MME), cognitive structure of mathematics and science subjects (CSMSS), creativity level (CL), mathematical modeling self-monitoring ability (MMS-MA) and mathematical modeling academic achievement (MMAA) are looked upon as the forecast goals respectively,.

Table 2

Duplicate Regression Analysis among Factors

	MME		
	β	*t*	*p*
MMB	0.429	4.838	0.001
CS	0.396	4.361	0.001
	CSMSS		
	β	*t*	*p*
CS	0.466	5.112	0.001
CI	0.421	4.716	0.001
	CL		
	β	*t*	*p*
CS	0.414	4.561	0.001
CI	1.468	5.112	0.001

	MMS-MA		
	β	*t*	*p*
MMB	0.402	4.607	0.001
CS	0.411	4.622	0.001
CI	0.429	4.838	0.001
	MMAA		
	β	*t*	*p*
MMS-MA	0.415	4.643	0.001
CSMSS	0.375	4.476	0.001
MME	0.337	4.239	0.001
CL	0.393	4.615	0.001

From table 2 we see that: (1) through mathematical modeling belief (MMB), cognitive style forecasting mathematical modeling emotion (MME), the predictabilities are preferable, and the mathematical modeling belief (MMB) shows better predetermination; (2) through cognitive style (CS) and creative inclination (CI) forecasting cognitive structure of mathematics and science subjects (CSMSS), the predictabilities are preferable, and cognitive style (CS) shows better predetermination; (3) through cognitive style (CS) and creative inclination (CI) forecasting creativity level (CL), the predictabilities are preferable, and creative inclination (CI) shows better predetermination; (4) through mathematical modeling belief (MMB), cognitive style (CS), creative inclination (CI) forecasting mathematical modeling self-monitoring ability (MMS-MA), the predictabilities are preferable, and from high to low the predetermination sequence is creative inclination (CI), cognitive style (CS), mathematical modeling belief (MMB); and (5) through mathematical modeling self-monitoring ability (MMS-MA), cognitive structure of mathematics and science subjects (CSMSS), mathematical modeling belief (MMB) and creativity level (CL) forecasting mathematical modeling academic achievement (MMAA), the predictabilities are preferable, and from high to low the predetermination sequence is mathematical modeling self-monitoring ability (MMS-MA), creativity level (CL), cognitive structure of mathematics and science subjects (CSMSS), mathematical modeling belief (MMB).

Path Analysis Model of Influential Factors in MMAA

According to the path coefficient β of forecast variable (FV) to effect sign variable (ESV) and its decision coefficient R^2 ($R^2 = \gamma \times \beta$, and γ is correlation coefficient for FV and ESV), a path analysis model is established (see figure 1).

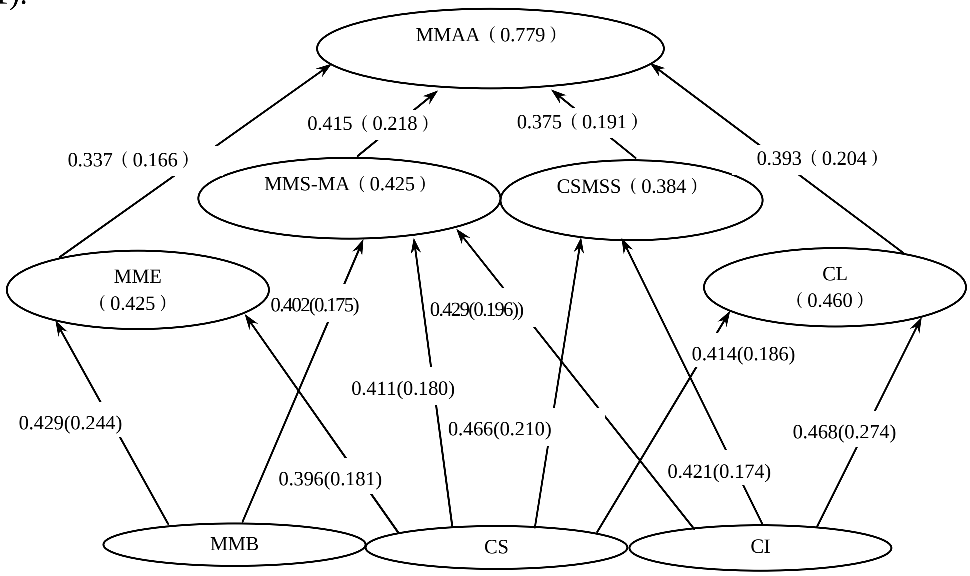

Figure 1. **Path analysis model of influential factors in MMAA.**

Note: Data in bracket expresses the decision coefficient, outside data expresses correlation coefficient.

Figure 1 shows that: (1) mathematical modeling self-monitoring ability (MMS-MA), creativity level (CL), cognitive structure of mathematics and science subjects (CSMSS) and mathematical modeling emotion (MME) directly influence mathematical modeling academic achievement; the variations of mathematical modeling academic achievement that they may explain are 21.8%、20.4%、19.1%、16.6% respectively, the total variations 77.9%; (2) creative inclination (CI), mathematical modeling belief (MMB) and cognitive style (CS) influence mathematical modeling self-monitoring ability (MMS-MA), and the three factors may explain 55.1% of the variations; (3) creative inclination (CI) and cognitive style (CS) influence creativity level (CL), and the two factors may explain 46.0% of the variations; (4) cognitive style (CS) and creative inclination (CI) influence cognitive structure of mathematics and science subjects (CSMSS), and the two factors may explain 38.4% of the variations; (5) mathematical modeling belief (MMB) and cognitive style (CS) influence mathematical modeling emotion (MME), and the two factors may explain 42.5% of the variations.

Discussion

Measurement Methods for Cognitive Structure of CSMSS

Although various methods (e.g. concept map, card arrangement, word association and ordinal branch technology) have been used to measure cognitive structure, all of them have limitations. For example, these methods usually have a premise that connection among different knowledge point is symmetrical. The cognitive structure of mathematics and science subjects implies a broaden subject knowledge structure; it is difficult to apply these methods scientifically, effectively and simply measure the cognitive structure. Our research sample the total science students as a whole in the participants' school (12th grade) respectively, and took their standardized average scores in mathematics, physics, chemistry and biology as an index of the cognitive structure of mathematics and science subjects. Students achieve test scores through learning with their teachers guidance and students organizing and internalizing curriculum content and ideas in their minds throughout the learning process. The test scores of mathematics and science subjects can reflect considerably student knowledge structure of mathematics and science subjects and is suitable for using as an index to measure students' cognitive structure of mathematics and science subjects.

Self-Developed Questionnaires

Besides authoritative questionnaires or scales that have been used generally, this research applied several self-developed questionnaires for measuring relevant elements as well. Although these questionnaires are developed based on other related tools and revision and reliability test, and they can be accepted as tools for measuring relevant elements in the scope of research condition and level at present. As few existing related studies on mathematical modeling cognition and non-cognition are still in initial exploring stage and provide little selectable information, especially measuring tools, our self-developed questionnaires need to be tested and improved through follow-up research.

Measure Methods for MMAA

Because there exist various methods, results and forms for mathematical modeling, the measurement of MMAA should involve multiple, broaden and gradient characteristics. This research used self-developed questionnaires, The Test of Mathematical modeling (Groups), as measuring tools testing participants' MMAA, and the test scores as an index to measure participants' MMAA. The first three questions are essay questions for measuring the knowing and mastering level of participants' basic knowledge and methods. The last three questions are mathematical application and mathematical modeling problems and their difficulty progresses successively. Though they are open ended questions, they are not "authentic" mathematical modeling problems in the strictest sense. Furthermore, the test form is closed and time-limited. The test of mathematical modeling problems in this research reflects the MMAA demanded by the mathematical modeling contents included in *The Standard for Senior High School Mathematics Curriculum (experimental version)* of China. It is not reliable for predicting participants' MMAA. As a result, optimal tools and methods for measuring high school students' MMAA should be developed.

Influential Factors of MMAA

Factors such as school, curriculum, teacher and assessment may influence students' MMAA in the macroscopic view (Dan, 2007). Considering the implementation and evaluative state of mathematical modeling curriculum, it still lacks manipulated tools for applying a quantitative method which explores the influence of these factors on the MMAA of high school students. The current research only explores the cognitive and non-cognitive factors of high school students' MMAA. Although our theoretical assumption of influential factors and path initially proposed has been supported by the path analysis results, and the causal relationship and active mechanism have been discussed in–depth, research is still necessary. Because of the complexity of data collection and analysis, some underlying cognitive or non-cognitive factors have not been taken into consideration (e.g. characters, types of temperament, and characters thinking), and these are possible factors influencing students' mathematical MMAA to a certain degree. This requires more study.

Conclusion

Within the scope and condition of the research, this paper draws the following conclusions:

1.All seven factors—mathematical modeling self-monitoring ability (MMS-MA), creativity level (CL), the cognitive structure of mathematics and science subjects (CSMSS), mathematical modeling emotion (MME), creative inclination (CI), cognitive style (CS) and mathematical modeling belief

(MMB)—have significant positive correlation with mathematical modeling academic achievement (MMAA).

2. All four factors—mathematical modeling self-monitoring ability (MMS-MA), creativity level (CL), cognitive structure of mathematics and science subjects (CSMSS) and mathematical modeling emotion (MME)—can predict mathematical modeling academic achievement (MMAA), and they may explain its variations to 42.5%.

3. The creative inclination (CI), cognitive style (CS) and mathematical modeling belief (MMB) can predict mathematical modeling self-monitoring ability (MMS-MA), and they may explain its variation to 55.1%.

4. The creative inclination (CI) and cognitive style (CS) can predict creativity level (CL), and they may explain its variations to 46.0%.

5. The cognitive style (CS) and creative inclination (CI) can predict cognitive structure of mathematics and science subjects (CSMSS), and they may explain its variations to 38.4%.

6. The mathematical modeling belief (MMB) and cognitive style (CS) can predict mathematical modeling emotion (MME); the two factors may explain its variations to 42.5%.

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