

Mental Computation: An Overview on This Special Issue

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Mental computation involves engaging mental processes to compute an arithmetic result. However, there are different perspectives on the type of computation that can be done and *how* it can be done. Concerning the type of computation that can be done, some researchers argue that mental computation is the process of computing an exact arithmetic result (Reys et al., 1995), while others consider that mental computation can be performed using estimation (Heuvel-Panhuizen, 2001), as Andrews and colleagues present in their article. Regarding *how* it can be done, some argue that mental computation only uses mental processes without any external support (Reys et al., 1995), while others accept the use of paper-and-pencil as a support of students reasoning (Buys, 2001; Sowder, 1990). Actually, mental computation involves computing with numbers and rather than with digits (Buys, 2001) and it can be performed to reach an exact arithmetic result, or to estimate a result. Mental processes are the basis of computing mentally, even if paper-and-pencil is used to note some intermediate results. Further discussion needs to go deeper, so that we can understand why computing mentally is important to include in mathematics curriculum, and what it involves.

Mental computation is an important skill for daily life, and it involves high cognitive demands. Every time we compute mentally, our number and operation sense emerges in our strategies, as Magiera and Leigh stressed in their study. In order to compute mentally, students need to select strategies that include knowledge about numbers, operations and their properties (Alsawaie, 2012; Schneider & Thompson, 2000) as highlighted in Carvalho and Rodrigues article. These strategies reflect students' conceptual understandings (Serrazina & Rodrigues, 2017; 2021); and, when it involves creativity and flexibility in manipulating numbers and operations (McMullen et al., 2016) alternative and specific routes can appear, adapted to tasks and personal characteristics, as we can see in Proulx's study. A strong relationship between arithmetic and algebra can also be identified in mental computation processes where relational thinking occupies a central role, as Osana and Kindrat argue in their article.

This special issue about Mental Computation has five articles. Each of them presents several ideas that highlight the importance of conceptual understanding as a foundation to develop mental computation, from elementary school students to prospective teachers.

It begins with a literature review on computational estimation and its development and importance for mental computation. Subsequently, there are several empirical studies ranging from elementary school to prospective teachers' instruction, that discuss and come up with new questions about how mental computation can be developed and its implication in students learning and teachers' practices.

All of the contributions given to this special issue point in similar directions. Mental computation develops over time; and, daily routines using numbers and operations involving classroom discussions of students' strategies are important for students' conceptual understanding and mathematics learning. Mental computation needs to be addressed as a part of an interconnected web of related concepts such as estimation, relational thinking, number and operation sense and flexibility. All these concepts are adaptable to intervention; and therefore, in almost all of the articles some concerning issues arise about teachers practices and how their design and intentionality can influence the development of students' mental computation skills.

The first article of this thematic issue, entitled "Computational Estimation and Mathematics Education: A Narrative Literature Review, is a narrative literature review authored by Paul Andrews, Pernille Bødtker Sunde, Mona Nosrati, Jöran Petersson, Eva Rosenqvist, Judy Sayers, and Constantinos Xenofontos. The authors present a review of relevant empirical research studies on computational estimation and its development, an important aspect of mental computation, yet there is little research in the field of mathematics education. This review brings together the extensive research of the cognitive psychologists and the limited research of the mathematics educators to clarify the nature of this construct. In their review, the authors present a synthesis of the literature on three themes: strategies employed in computational estimation; development and manifestation of computational estimation in children; and computational estimation competence of adults. The major conclusions are drawn from the results of studies which show that computational estimation develops over time and is susceptible to interventions and draws on a wide range of strategies that are reciprocally dependent on a secure understanding of number and operations. The authors note that there seems to be a strong reciprocal relationship between computational estimation competence and flexibility in arithmetic.

In the article "Day Number Routine: "An Opportunity to Understand Students' Uses of Numbers and Operations", Carvalho and Rodrigues present a study where students that are 8–9-years old (grade 3) are challenged to

construct trains of calculations that need to end with the exact number of the day in which they are doing a number routine.

This exploratory study aims to identify the arithmetic knowledge that third graders acquire when they are free to use any numbers and operations to calculate mentally in a “Day Number” routine. The authors wanted to know what numbers and operations students would choose to construct their trains of calculations and if they rely exclusively on number facts or use mental calculation strategies. According to the findings, a wide range of knowledge was shown by the students. Students’ trains of calculations involved multi-digit numbers with the four basic operations, with a preference to start with multiplications or division and to end with additions or subtractions. Several mental calculation strategies emerged in the whole class discussion, such as decomposition in all the arithmetic operations, sequential strategies in addition/subtractions, and varying strategy in multiplication and division.

Osana and Kindrat developed a study reported in the article entitled “Examining the Impact of a Mental Computation Classroom Intervention on the Relational Thinking of Seventh-Grade Students.” This study, involving 66 seventh graders students from three classes, aims to investigate the impact of mental computation on the relational thinking and equivalence knowledge of students through a 12-week instructional intervention. Classroom discussions, in which students shared their strategies and the teacher provided conceptual explanations to frame and compare students’ strategies and approaches, were an important key aspect of this intervention. Findings show that all students improved relational thinking skills as well as their mental computation performance in two of the three classes that participated in the study. According to the authors, this finding highlights the benefit of engaging students in daily routines activities involving mental computation. They argued that a few minutes of mental computation, incorporated into a teacher’s practice, may represent a contribution for students’ mathematical success.

The Proulx’s article, entitled “Investigating the Specific and Alternative Nature of Mental Mathematics Strategies: The Case of Systems of Equations,” is primarily concerned with how mental mathematics strategies are specific to mental mathematics, aiming to study, not only the solving processes but also the nature of the mathematical activity in which solvers engage. The author provides a case study, grounded in *enactivist theory of cognition*, of an in-service group session where 10 secondary-level mathematics teachers were asked to solve a system of linear equations task. The analysis is conducted on strategies engaged by the participants in a mental mathematics environment. This analysis illustrates the ways in which the strategies represent specific and alternative solving routes that are different from algebraic methods used in paper-and-pencil work. The author argues that the local nature of the strategies points to how mental mathematics contributes to an extension and enrichment of the school mathematics landscape.

Finally, the article “Exploring Prospective 18 Teacher’s and Operation Sense in the Context of Fractions,” from Magiera and Leigh is an exploratory study of prospective teachers that aims to examine their number and operation sense as well as any possible connection to their number and operation skills in the context of fractions. Ten prospective teachers, who attended a teacher education program, participated in this study. They answered a series of 23 tasks aimed to elicit their thinking about fractions and to examine their use of number and operation sense skills while solving fraction-related problems. Seven skills for fraction-related number and operation sense were designed in order to analyze teachers’ problem-solving solutions. In this research, prospective teachers showed a strong argumentation based on recognizing the meaning of symbols and formal mathematical language in the context of fractions, but less stronger arguments on reasoning about different representations of fractions and operations, composition of numbers, and about the effects of operations on pairs of fractions. From the authors’ perspective, these findings suggest that these prospective teachers relied more on procedural than on conceptual understanding. Findings also indicate that reasoning about models of fractions and operations, recognizing the meaning of symbols, relationships among fractions, and reasoning about the effect of operations on pairs of fractions are directly connected.

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