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Case Study of Core Concepts of Statistics Teaching in Two High Schools in China

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Combined with APOS theory of conceptual teaching, this study examined a statistical conceptual teaching and analysis framework. The framework analysed mathematic lessons in two statistical classes at two key high schools in southern Jiangxi Province in China. Based on the interviews, the study analysed teachers' perspectives on the core concepts of high school statistics lessons. The results show that teachers still play a leading role in the statistics classroom. Students had few opportunities to lead the classroom teaching, mainly through answering questions from teachers or doing exercises. According to APOS theory, concept teaching can be divided into four stages: action, process, object and schema. From the results of the study, among the four stages of the conceptual concept teaching, the teachers' teaching mainly focused on the process stage and the object stage. Significant differences between the teachers' attention devoted to these two stages were also found.

Keywords**:** senior high school, statistics, core concept teaching, case study

With the continuous development of society and the advent of digital age of information, people increasingly use data in all aspects of their life. Central to the value of statistical education is the gradual development of habits for facts and analyzing problems through data (Shi, 2008). Statistics and probability are the core courses of high school mathematics in almost all countries at present (Xu, 2005). In April 2003, China's Ministry of Education published *Ordinary High School Mathematics Curriculum Standards (Experiment)* (hereinafter referred to as *High School Mathematics Curriculum Standards),* suggested to teachers that, the core concepts and basic ideas should be consistent throughout the teaching, in order to gradually deepen students' understanding. The recommendation by *High School Mathematics Curriculum Standards* (2003) for teachers was "to emphasize the basic concepts and the basic ideas of understanding and grasp of some of the core concepts and basic ideas (such as functions, spatial concepts, operations, vector, derivative, statistics, random ideas, algorithms, etc.) through thought process" (p.108). The standards explained, "The thought process includes visual perception, observed induction analogy, spatial imagination, abstraction, symbols, calculation solving, data processing, interpretation based reflection and construction" (p.3).

Though the core concepts were mentioned in the *High School Mathematics Curriculum Standards*, they were not specified. Moreover, Chinese Ministry of Education published the *Full-time Compulsory Education Mathematics Curriculum Standards* in January 2011. Its focus is “on developing students' number sense, symbol sense of space concepts, geometric intuition, the concept of data analysis, computing power, reasoning ability, the model of thinking , application and sense of innovation” (p.5). The core of statistics is data analysis; however, specifying what exactly is understood by its inner components and the logic of their relationships, had not been made very clear. That means that students entering the high school had knowledge of the concepts pertaining to data analysis; however, the logical and internal structure of the core concepts was missing from their understanding.

Therefore, the purpose of this study was to analyze the core concepts of high school statistics by investigating the following research questions:

1. What are the core concepts of high school statistics?
2. What is the “core of content” in high school statistics?

The statistical classroom teaching of two high school teachers were selected as a case to study how teachers conducted conceptual teaching of statistics, to analyze existing problems, explore reasons and seek countermeasures.

Theoretical Framework

The Core Concepts of Statistics

According to Hurd (1997), the concepts and principles that make up the science curricula present in contemporary disciplinary landscapes are the backbone of the discipline's structure and can be referred to as core concepts or representative ideas. Erickson (2003) also believed core concepts are the center of the discipline, along with key concepts, principles, and methods of lasting and migratory value beyond the classroom. From Erickson's point of view, "core concepts" not only had "concepts" but also principles and ways of thinking. Some concepts were more basic and more systematic. They not only served as a foundation, but also summarized "the core of the content" in a certain field of mathematics as well as "the core of the method of thinking." In the secondary stage, such concepts were limited, counted and operated (Zhang, 2011).

Core concepts can be incorporated to render the concept map (Zhang, 2011). Zhang believed that in order to understand the organization of the key concept, it is required to consider not only their generation, but also how these concepts form a network system. This belief leads to the notion of the Concept Maps (Novak & Gowin, 1984), which are the hierarchical collection of relevant concepts and the relationships between them. The main idea of the concept map is that nodes represent the concepts while the connecting lines represent the relationships between them (Novak & Gowin, 1984). Not all knowledge points can serve as the core concept; only the most general and inclusive knowledge point can serve as the core concept. Drawing the concept map can better clarify the connections and the levels between concepts (Zhang, Song, Wang, & Zhou, 2013). From the study of Zhang (2011), we can clearly understand that "the core of the content" of the statistics of high school in the United States is "data analysis", and there are four related core concepts of statistics: (a) put forward the problem of using data, (b) collect, organize and display data, (c) analysis data, and (d) reasoning, prediction. According to Shi, Zhang, and Zhao (2008), data analysis is reflected from three aspects. First, students should understand that there is data related d information. They should survey, collect and analyze data before they make a conclusion. Second, students should know there are many ways to deal with the same problem. They should learn how to choose the most relevant methods according to the environment. Third, students should know the randomness of data analysis (Shi, Zhang, & Zhao, 2008). Zhang (2010) indicated data analysis contains data collection, information extraction and problem solving. The four core concepts are based on the "process of statistical activity" conceptualized as the development context and internal logical connections.

In summary, the core concept of mathematics is based on a mathematical system of knowledge, summarized and extracted from the major concept and corresponding method of thinking. The core concept not only contains the key concepts, but also can contain mathematical principles and mathematical ideas and methods. The core concept is generally centered at the center of the concept map.

Framework of Statistical Core Concepts

Zhang (2014) argued that mathematical content is divided into two aspects: process and object. Which means the same mathematical concept is not only a process operation, but also an object structure. The concept of process is also linked to APOS theory (Tall, Gray, 2001; Tall, 2006). APOS is the acronym for action, process, object, and schema. The basic assumption of APOS theory is that mathematical knowledge is obtained by individuals during the process of solving the perceived mathematical problems. Students' learning of concepts was divided into the four stages of the APOS process. (Wen, 2015). APOS theory is a learning theory that analyzes how learners learn concepts from the perspective of learners. This theory emphasizes the process and object duality of mathematical concepts. The learning of the learner generally goes through three stages of the operation including the action, process, and object stages. The experience of completing these three stages of reflection leads to form schemata to reach and grasp the understanding of the concept within the problem situation, and beyond.

Yu (2009) believed that revealing the background of knowledge is carried out at the stage of concept introduction. There are two sources of mathematical concepts, one from the quantitative and spatial relationships in the objective world, and one from the logical construction of existing mathematical theories. Wen (2013) indicated that the teaching process model of the core concept of mathematics can integrate traditional concept teaching theory and APOS concept teaching theory. We suggested a mathematical core concept of teaching process mode shown in Table 1.

Table 1
Mathematical Core Concept of Teaching Mode Table

Stages	**Phenomenon**
Action Stage	Introduced from the concept background
	To introduce from actual needs
Process Stage	Abstracted from the instance
	Abstracted from the positive and negative examples to deepen understanding
	Compared to deepen understanding
Object Stage	Using formal symbols to express concepts
	Reveal characteristics
Schema Stage	Application of concepts

Classroom teaching is actually composed of the activities and teaching, the interaction between teachers and students, and the activities of students. These components can actually be divided into smaller pieces according to their theme or core features. Describing the theme or core features of these fragments, in Table 1, Wen (2013) called them “teaching phenomena.” These teaching phenomena also play a role in revealing teaching strategies. For example, in the “action stage” of concept teaching, the purpose is to let students experience the necessity of introducing concepts through specific activities or operations. Therefore, if the concept introduced at this stage is from the quantitative relationship and spatial relationship in the objective world, it is “introduced from the concept background.” In this study, it was coded as a1 if the concept is based on the existing mathematical theory logic, and then in other instances of this study when it was “introduced from actual needs," it was coded was a2.

Wen (2013) attributed the "enhanced understanding through positive and negative examples" to a teaching phenomenon in the "process stage", but in fact, the role of the positive and negative examples are different. Counterexamples can help students to compare, reflect, and correct from irrelevant attributes or misconceptions to deepen their understanding of concepts (Zhao, 2012). A positive example is a certain example that contains and reflects the essential attributes of the concept. A counterexample is a negative example. Examples that are not part of the concept category are counterexamples (Li, 2005). Regarding the number of positive and negative examples, some researchers believed that the number of positive examples should be appropriately greater than or equal to the number of counterexamples (Guo, Peng, & Yang, 2007). Since the positive and negative examples can be cited by the teacher or by the students, that is, the subject of the "teaching phenomenon" can be either a teacher or a student. Therefore, in this study, the two aspects of teaching phenomena, “enhanced understanding through positive examples” and “help understanding through counterexamples”, are all in the “process stage.” In addition, since teachers can obtain the essential attributes of the concept by “changing constraints” (Wu & Zhou, 2013), this study further refined and expanded the research framework of statistical concept teaching based on the research studies (see Table 2).

Table 2

High School Statistics Core Concept of Teaching: Four-Phase Observation Table

	Teaching Phenomenon	
A. Action Stage	a1	Introduced from the concept background
	a2	To introduce from actual needs
P. Process stage (concept object summary)	b1	Summarized abstraction through (rich) examples
	b2	Through positive examples to deepen understanding
	b3	Through Counterexamples examples to deepen understanding
	b4	Changed the constraints
	b5	By comprehending the concept of understanding
	b6	Given the concept name
	c1	Use formal symbols to express concepts
O. The object Stage (the consolidation and deepening of the concept, operating the concept as a standalone object of operation）	c2	Reveal the features
	c3	Methods of induction and summary
S. Schema stage (concept phase)	d1	Conceptual application
	d2	Solve problems

In addition to trying to figure out the core concepts of high school statistics, this study also selected the statistical teaching process of two high school teachers as a case study to examine how teachers taught statistical concepts, analyzed the existing problems, and to identify causes and solutions to problems.

The research data of this study came from transcribing the statistical classroom teaching videos into text and then analyzing the text. Based on the framework on Table 2, according to the different content of the activity, the whole classroom teaching was divided into several segments, and analyzed according to the kind of "Teaching phenomenon" each segment represented.

The concept of teaching of statistics was analyzed by the number of "teaching phenomena" implemented.

Research Methods

Research Site and Participants

The research sites were A and B high schools in G City, in the southern of Jiangxi province in China. A and B high schools were two of the five key high schools in G City. The students enrolled in these two high schools were excellent students in G City. The teachers who taught in these two key high schools were teachers with rich teaching experiences. The students in these two high schools were Han students, and their parents were citizens of G City.

The research participants were Teacher W and Teacher M. Teacher W was a math teacher of a high school with Bachelor's degree and taught for 33 years. Teacher M was a math teacher of the B high school and taught for 11 years. Teacher M was a master teacher of mathematics whose understanding of the theoretical knowledge of mathematics teaching, the ability to carry out classroom teaching and research made up for the lack of teaching experience in comparison to Teacher W.

Data Collection and Instrumentation

Data collection included two teaching videos of Teachers M and W and six interviews from other teacher participants. Two video lessons were statistics lessons. The contents taught by the two teachers was Section 8 Least Squares Estimation of the Compulsory 3 and Chapter 1 Statistics from a Beijing Normal University mathematics textbook.

In addition to the collection of video lessons from Teachers M and W, the interviews were conducted with six other teachers who taught at the same grade level as Teachers M and W. Three of the six teachers were colleagues of Teacher M and the other three were colleagues of Teacher W. Similar to Teachers W and M, the six teachers taught science classes at the same grade level. The decision to examine the classes of Teachers W and M was related to the differences in requirements for high school mathematics between students in liberal arts and science. Science students had higher mathematics requirements in the liberal arts students. Although high school statistics contained the same content and requirements for liberal arts in the *High School Mathematics Curriculum Standards* and the textbooks in terms of probability, science had more demanding curriculum than liberal arts. Although, in the case of higher probability content requirements, the statistical teaching requirements of science was not lower than that of liberal arts. Therefore, the statistical teaching of the science classes was chosen to observe as representative of courses with a higher mathematics requirement. Second, according to Tu (2011), it is best to collect case studies of the same topic teaching for a comparative study.

The interviews were audio recorded and conducted separately with each teacher's interview time approximately 15 minutes. Since the teachers taught at these two key high schools all had rich teaching experiences, their interviews were representative of other teachers with similar attributes at the schools. The purpose of the interviews was to explore the teacher's understanding of the core statistical concepts.

The interview questions were selected from the questionnaire by Li (2010) and were adapted with some changes to better suit the classroom conditions. The interview questions of this study were:

1. Talk about the "least squares estimation" teaching process (i.e. teaching design) and the reasons and ideas of it.
2. What do you think of the core concepts of high school mathematics statistics? Why do you think so?
3. Talk about the concept of teaching theory you know.
4. Have you learned statistical knowledge in college? Have you participated in the training of statistical knowledge since the new curriculum was introduced?
5. Why does Section 8 Least Squares Estimation in the textbook use the distance between sample points and the regression line? How do you understand this method of linear regression?

Data Analysis

Two video lessons of statistical classroom teaching were transcribed into text and then text was analyzed. According to the different content of the activity, the whole classroom teaching was divided into several segments, and the "teaching phenomenon" was coded, analyzed, and sorted into each segment of the APOS Four Stages. Next, the number of “teaching phenomena” were counted and analyzed.

The results of the codes and analysis of transcripts showed that Teachers W and M dominated the teaching phenomenon in all APOS stages. The data ratio of the four teaching stages of the two teachers was compared and shown in Table 3 below.

Counting the number of teaching phenomena codes by a1, a1, b1, b2 up to d1, d2, one can get the proportion of the occurrence of each code in the total teaching phenomenon, which were led by the two teachers respectively. In this way, the proportion of the specific teaching phenomenon was obtained from the teachers and represented on the four points on the histogram (see Figure 1).

Table 3
The Proportion Comparison Table of the APOS Four Stages for Teachers W & M

	A	P	O	S	A	P	O	S
	Teacher W				Teacher M			
# actions	1	7	9	0	1	8	19	3
% percent	5.00	35.00	45.00	0	3.13	25.00	59.38	9.38

Results

The Results of Statistics Teaching of High School

The results of the statistics teaching was obtained from the text transcriptions from the two teaching videos of Teachers M and W. Table 3 shows that the proportions of the two teachers in the teaching phase of "operation stage" were not different, but there was a big difference between the process stage and the object stage. The concrete manifestation was that in the process stage, Teacher W demonstrated teacher-led teaching phenomenon that took place 7 times, accounting for 35%, while Teacher M exhibited teacher-led teaching phenomena that occurred 8 times, but accounted for 25% , a smaller proportion than Teacher W. In the object stage, the difference between the two teachers was even more obvious. There were nine teaching phenomena times led by Teacher W, accounting for 45% of the total teaching, while Teacher M exhibited teacher-led teaching phenomenon 19 times, accounting for 59.38%. This result indicated that Teacher M tended to focus more on the object stage while teaching. In the schema stage, the two teachers were also quite different. Teacher W's teaching phenomenon appeared to be completed by the students, but Teacher M's teaching resulted in students not having opportunities to lead the classroom and were not allowed to practice solving math problems. Teacher-led teaching phenomenon appeared 3 times more in the teaching sequences and accounted for 9.38% of the teaching. In the four teaching stages, the "P process" and "O object stage" of Teacher W accounted for 80% of the two-stage teaching phenomenon, while Teacher M accounted for 84.38%. The average of the two teachers in these two stages was 82.19%.

The detailed results can be seen in Figure 1:

1. In the action stage, both teachers adopted the "introduction from concept background" with no obvious differences.
2. In the Process stage Teacher W's teaching resulted in b2 coding "deepen understanding through the positive example" and appeared a total of 4 times, accounting for 20% of all teaching phenomena, and for Teacher M only 2 times, accounting for 6.25%, and the difference was obvious. There was also a difference between the two teachers in encoding b4, "Change Constraints," with Teacher W, accounting for 5.00% and Teacher M's results accounting for 0%. In addition, there

was also a significant difference between the two teachers in implementing the teaching phenomenon b6 "giving the concept name," with Teacher W accounting for 5.00% and Teacher M accounting for 15.63%, indicating that Teacher M had mentioned the name of the concept several times.

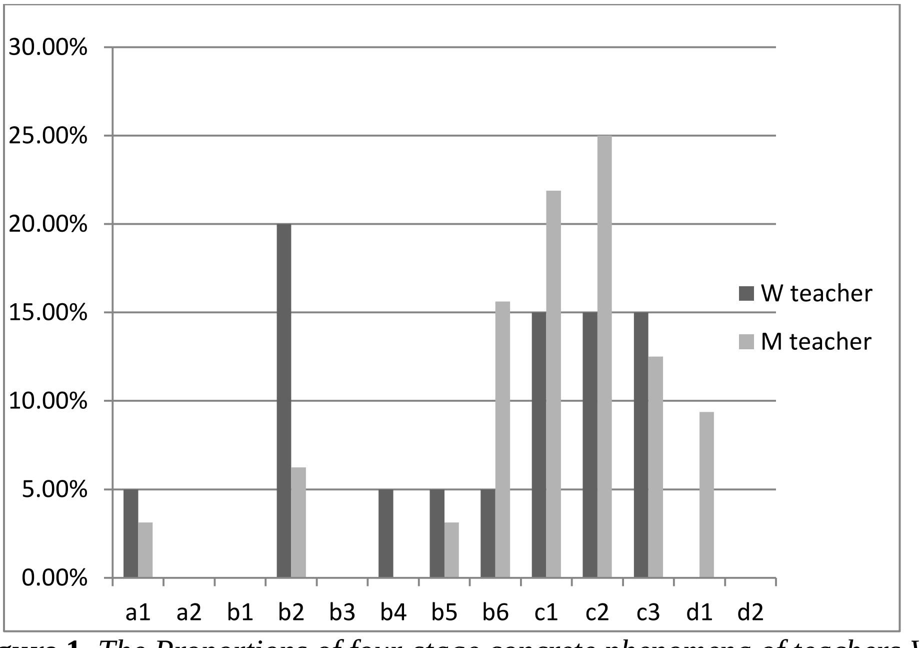

Figure 1. *The Proportions of four-stage concrete phenomena of teachers W & M.*

3. In the "Object stage", the two teachers differed in c1 "using formal notation to express concepts," with Teacher W accounting for 15% and Teacher M accounting for 21.88%. There was a significant difference in c2 "revealing features", where Teacher W accounted for 15% and Teacher M accounted for 25%, indicating that M teacher spent more time than Teacher W in repeatedly revealing the features of the concept. The two teachers were not very different in coding c3 that was "induction and summary of the method", Teacher W accounted for 15%, and Teacher M accounted for 12.50%.
4. In the "Schema stage," there was a big difference between the two teachers in the application of the concept of "d1" in the teaching phenomenon. The proportion of teacher-led teaching moments by Teacher W was 0%, indicating that at this stage Teacher W allowed students to practice by themselves, while Teacher M accounted for 9.38%, indicating that at this stage the teacher was still leading the class and did not provide practice time to the students.

The Results of Interviews

The analysis of the interviews with eight teachers, including interviews with Teachers W and M, showed the following results:

1. To answer the first question, “when teaching Least Squares Estimation,” only one teacher analyzed the formula. Two teachers taught two sections of "Section 7 Relevance" and "Section 8 Least Squares Estimate" together, the two other teachers considered that the linear regression equation could be directly calculated by students after they learned Relevance. The derivation process did not need to be analyzed with the students because the textbooks also did not make any demands. They just used this content as a "little information" for students to read and use. The remaining five teachers dealt the same way simply by focusing on letting students perform calculations. It can also be seen from the interviews that most teachers thought that the content of the least squares estimation had "nothing to say."
2. To answer the second question, six teachers did not consider the "core concept" important unless it was material needed for the college entrance exam. One teacher judged "inclusiveness" to be the standard of the core concepts; another teacher who used "logic" as the criterion of judgment, believed that the core concept of high school statistics was an internal logical connection based on students completing statistical activities.
3. The third question, “whether they understood the concept of teaching theory,” was answered negatively by six teachers; another two teachers, although they had heard the concept of teaching theory, did not know the specific content.
4. To answer the fourth question, “Whether the statistics had been learned in the university,” one teacher answered that he did not take a statistics course, and another teacher answered that he learned mathematical statistics in college, but high school content such as relevance, or independence tests had not been learned at the university. The other five teachers answered, "Learned but not quite remember" or "cannot remember." There was only one teacher who answered positively that he was knowledgeable about it. All teachers answered that they were not trained in any statistics course that was relevant for their teaching at present.
5. In response to the last question, only one teacher thought it was related to the distance from the point to the straight line. Four teachers considered "this was a regulation" and three teachers considered it as "easy to calculate."

Discussion

This study first identified the problem in the core concepts of senior high school statistics. It shows that the core concepts of statistics were not listed in the curriculum standards through research analysis of the *High School Mathematics Curriculum Standards* and its supporting textbooks including compulsory and elective content (Beijing Normal University Press, 2010, 2008). Next, this study described the status of the concept teaching of high school statistics and the teacher's understanding of statistical knowledge by analyzing two videos on teaching statistics concepts t and interviews with eight teachers.

The Core Concepts of High School Mathematics Statistics

This study identified data analysis as the core of the content of mathematical statistics at the senior high school level. The purpose of teaching high school statistics is to improve students' ability to deal with data analysis. There are five core concepts of high school statistics: random sampling, sample estimate overall, linear regression, independence test, and regression analysis. These core concepts were based on the basic statistical methods - the common method for the development context, including random sampling, sample estimate overall, and linear regression was the basic method of statistics in the high school compulsory modules. The independence test was a common method in elective modules. The system of knowledge that consists of these five core concepts and extended sub-concepts covered an important part of high school statistics. These five core concepts were highly generalized and inclusive. However, as presented in the textbook, they were not logically coherent and lacked the coherence of the statistical activities.

The teachers in this study judged the importance of the core concept in teaching solely based on whether the core concepts occurred on the national entry exam. In other words, those teachers taught to the test. Small number of teachers used “inclusiveness” or “logical coherence” as a criterion to choose the core concepts.

Conceptual Teaching in High School Statistics

The results of this study show that teachers still played a leading role in the statistics classroom. In the two teachers' classrooms in this study, opportunities for students to lead classes were obtained either by answering questions from teachers or by doing exercises. The teaching phenomenon led by the teachers accounted for 90.94% of the teaching phenomena that occurred in the entire class on average, and there were few teaching phenomena led by the students with an average level of 9.06%.

The results from analyzing the teaching of high school mathematics statistics classroom with APOS theory of concept teaching in this study show that, overall speaking, in the four teaching stages, teachers' teaching mainly consisted in the "process" and "object" stages. In these two stages, the average level of teacher-led teaching phenomenon was 82.19%, but there was a significant difference between the teachers in these two stages, which was manifested by the distribution of teachers-led teaching phenomena. A large part of the teaching phenomena was in the "object" stage, indicating that some teachers were weakening the "process" stage of the phenomenon.

From a concrete stage, in the "operation" stage, teachers generally introduced the statistics lessons from the background of the concept. The reason why it has not been introduced into practice was that the teaching of "Section 8 Least Squares Estimation" followed the "Section 7 Scatter Plot" and was more conducive to introduction from knowledge before and after, accounting for only 4.06% of the total. The teaching phenomena in "process" and "object" occupy 82.19% of all the teaching phenomena for the whole class on the average, indicating that teachers' teaching focused on these two stages. However, there was a difference in detail between the teachers in the two stages of "process" and "object." The concrete manifestation of that difference was that in the "process" stage, teachers helped the students deepen their understanding through positive examples, but there were differences in the numbers of examples, with significant differences in their proportions. In this study, some teachers repeatedly gave “names of concepts” to students instead of letting them explore the concepts.

Implications for Improving Instructional Practice

In the teaching process of statistics, teachers should maintain the integrity of the four stages of APOS concept teaching theory, namely the action stage, process stage, object stage and schema stage. In particular, teachers should pay attention to the teaching of the process stage. They cannot simply deal with the teaching of process stage and only give formulas, and not guide students to experience the production process of concepts and formulas. Having students experience a complete process of thinking and reasoning is helpful for students to truly understand the concepts and formulas of statistics and to grasp the thinking methods of statistics.

According to Yu (2009), the purpose of pursuing "process" is to "result", because result knowledge is the source of knowledge transfer, and a better teaching model should be "process + result." This study suggests that in the four stages of APOS, it is only after students have experienced the process stage that their experience and activity experience can be enriched, and only after they have experienced enough can they reflect on the mathematical activities and objects in the previous stage. Otherwise, even if the teacher gives a formula, the students cannot connect these formulas with other knowledge, and knowledge will only be "fragments" one by one. There is no way to establish a complete schema structure, let alone to truly understand and master the thinking method of statistics. This study also suggests that teachers should provide students with more opportunities to participate in statistical teaching instead of completing classroom exercises.

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