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Decimal Number System: Knowledge of Quebec Students Educated under the 2001 and 1981 Programs and Teaching Situations

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Decimal Number System (DNS) is fundamental in the teaching of arithmetic. Although several scholars have studied the system, it was mostly the studies carried out in the 80s-90s that brought out its complexity and also the difficulties students encounter with its learning. In some of them, the scholars saw the students' difficulties through the prism of the earlier program's behaviourist and objective foundation structure. However, the current curriculum in Quebec, based on social constructivism and structured according to competences is a departure from the past. In this context, this study, inspired by a doctoral dissertation, is intended to explore the strategies and knowledge in solving tasks on the DNS of 154 third grade students from six schools as well as teaching situations in two classes, more than 30 years after the studies were carried out in Quebec. The main data collected for the study included audio interviews with 18 students, a questionnaire (n=154), videotape of lessons and documentation. First, we outlined the strategies and knowledge of pupils on the DNS as well as teaching situations, and then we compared them with earlier studies as well as the current program's competence-based *structuring. We also discussed the scope of the task and class effects. We used the DNS frame of reference and the Theory of didactical situations for the analyses***.**

Keywords: Mathematics learning and teaching, student's understanding and knowledge, decimal number system, curriculum, elementary education.

The Decimal Number System (DNS) or base 10 positional number system is emblematic in the teaching and learning of arithmetic in elementary school. As such, it has been studied by scholars of mathematics education all over the world (Bednarz & Janvier, 1982; 1984a; 1984b; 1986; 1988; Brun, Giossi & Henriques, 1984; Deblois, 1993; Fuson, 1988; Kamii & Baker Housman, 2000; Koudogbo-Adihou, 2001; Koudogbo, 2013, 2015; Tempier, 2013), and more specifically in the two decades of 1980 to 2000. The studies carried out during the implementation of the program in terms of objectives (1980 to 2001), laid bare the complexity of the DNS, and identified students' difficulties and incorrect and false conceptions. Bednarz and Janvier (1982; 1984a; 1986) in particular, carried out diagnostic studies by presenting tasks to primary school students to identify their strategies, conceptions and difficulties, followed by longitudinal studies from 1980 to 1983, based on a constructivist frame of reference, with well-planned mathematical situations conducive to learning the concept (Bednarz & Janvier, 1984b; 1988). They aimed at helping students to build a meaningful and significant understanding of numeration and DNS. These authors, like other scholars (Deblois, 1993; Kamii & Baker Housman, 2000), interpreted these difficulties and conceptions as the effects of the objectives-based program that was in effect at that time. The efficiency of the teaching methods and even the structuring of the program itself, were called into question because they did not focus enough on the process of knowledge acquisition by the students, especially the necessary coordination between numbers-numeration-DNS, and working on numbers (Bednarz & Janvier, 1986).

In fact, the program introduced a sequential order numeration structure of numbers in terms of complexity. Thus, numbers less than 69 were studied in the 1st year, less than 99 in the 2nd year, and less than 199 in the 3rd year… However, it is while learning the higher numbers that the students' difficulties become apparent. Moreover, the analysis done by Bednarz and Janvier (1982; 1984a; 1986) of the proposed situations in the textbooks in Quebec, Canada, of this period shows a disconnect between numbers and operations.

In the early 2000s, in Quebec, as well as in many Western countries, there was educational reform. In place since 2001, the guidelines in the Quebec Education Program (Government of Quebec [GQ], 2001) is a major departure from the previous program. Whereas the 1981 program is based on a behaviorist learning theory and is structured in terms of overall and intermediate objectives, the 2001 program is based on social constructivism. The constructivism underlying this paradigm puts more "emphasis on skills development and output profiles rather than on the specification of numerous and fragmented objectives" (Legendre, 2004, p. 86). In this context, learning is a process favored by situations which represent a real challenge for the student and which involve a questioning of his knowledge and his representations (GQ, 2006). In addition, the organization of ''essential knowledges'' (pp. 150-152) shows a more integrated perspective of teaching and learning number system (and the DNS) compared to the previous program.

Thus, this program opts instead for a structuring in terms of competence and complex skills. This shift brought many changes. To enable the development of mathematical competencies, this program requires the implementation of mathematical situations of a certain complexity, i.e., situations that are different from exercises and which treatment calls for an organization and a coordination of new knowledge. For example, a structuring of the DNS teaching contents places this knowledge among the essential knowledge linked to the arithmetic of the "domain of mathematics, science and technology" (GQ, 2001, pp. 143-177). The preferred contents and their breakdown show a certain connection between numbers, counting, DNS, and number operations (GQ, 2001). There is also the updating of the teaching material (the student book, workbook, textbook, teacher's book and the teacher's guide…) in order to meet the objectives of the program. In accordance with the new guidelines, the teacher is supposed to target certain essential knowledge as prescribed in the program, to facilitate learning them. Within this framework, the DNS occupies a fundamental place, since it is at the heart of learning arithmetic in elementary school.

It is possible to assume that the implementation of the current program could bring about a new framework of essential knowledge and a renewal of teaching situations and their management by the teachers. It is also possible to assume that since its implementation, this program has played a structuring role in the teaching of DNS. It is also possible that this change could have some effect on students' knowledge in relation to earlier studies, particularly those carried out in Quebec (Bednarz & Janvier, 1982; 1984a; 1984b; 1988).

Thus, the research questions of this study are the following:

What are the portraits of knowledge of students who were educated under the competency program established in 2001 on the DNS and of teaching situations? Would the knowledge of students who were educated under the competency program established in 2001 on the DNS be similar to those implemented by the students of the 1981 program? Would the teaching situations be specific to the targeted knowledge and competences by the 2001 program?

In response to these research questions, three research objectives are formulated; the first two have to do with the learning component and the third with teaching:

1) Define the students' knowledge of the 2001 program in situations proposed by Bednarz and Janvier and compare them with those implemented by the students of the 1981 program.
2) Define the students' knowledge with respect to the DNS from tasks drawn up as part of this study.
3) Conduct an exploratory analysis of DNS-based lessons in order to identify the educational characteristics in terms of the specificity of the knowledge, and the skills approach promoted by the new program.

Reference Framework

The reference framework integrates the conceptual analysis of the DNS and two concepts covered by the Theory of Didactical Situations (Brousseau, 1998) and its developments (Conne, 1992).

Conceptual Analysis of the Decimal Number System

The conceptual analysis of the DNS helps to identify its fundamental principles, characteristics and functions. Among the principles, the decimal base, or regular grouping unit of the system (Guedj, 1996), enables counting by groups of ten using a set number of ten symbols (0-9). The exchange principle is reflected in the numbers operations and enables groupings: doing groupings (used in the addition), or undoing groupings, with the borrowing used in the subtraction. Another principle is the decimal place value, in which each digit in a number refers to a number less than the base. The position of the digit in the number gives its value, its weight. So a positional principle also characterizes this system (Tempier, 2010). But many studies revealed that the "positional" principle is mainly worked and taught to the detriment of its articulation with the "decimal" principle. For example, the results of the studies by Tempier (2010) in France reveal that only the "position" aspect is rather considered. In addition the resources offered to teachers are structured to work on the "position" aspect and don't promote a real understanding of the articulation between the principles of the DNS (Kamii, 1990; Tempier, 2010). Thus, if students easily learn to identify each position in a number, it is more difficult to grasp the value of a digit according to the position (Brun, Giossi, & Henriques, 1984), and mainly the value of a group of digits in a number (Kamii & Baker Housman, 2000; Perret, 1985; Tempier, 2010; 2013).

The DNS also takes on some of the roles of designation or representation of numbers (Ifrah, 1996), computation of collections/quantity of operations and recognition of certain properties of numbers. In addition, the DNS includes additive and multiplicative characteristics. Conceptually, the multiplicative structure has two interpretations: repeated addition, where a number of groupings can be repeated many times and an idea of exponentiation, i.e., the groupings of groupings. The coordination of these two interpretations is, for students, a very difficult task. These different elements account for the complexity of the concept and some interlocking multiple dimensions, as several studies have also shown empirically (Bednarz & Janvier, 1982; 1984a; 1986; Deblois, 1993; Fuson, 1988) in addition to the difficulties they raise for students. In fact, a lack of articulation of these multiple dimensions of DNS reinforced difficulties and false conceptions. It is important to present the studies of Bednarz and Janvier (1982; 1984a; 1984b; 1986; 1988) for two reasons: a) they pointed out these difficulties and misconceptions; b) they constitute the basis of this research. The authors carried out two studies during 1979-80 and during 1980-83. Their first diagnostic study on the treatment of collections and the meaning of actions on them was carried out with 40 students in grade 1 (6-7 years), 75 pupils (group A) in grade 3 (8-9 year) and 45 in grade 4 (9-10 years) and permitted to characterize students' inappropriate conceptions of the DNS. These conceptions could be explained by the structuring of the objectives based program which favors the fragmentation of teaching content and fragmented learning organized around reading, writing and representation of numbers. According to them, the rules of using material illustrate the conventional order of written symbolism. This favors a representation of the writing of the number as an alignment or juxtaposition of digits, not in terms of groupings and associated place values (Bednarz & Janvier, 1982; 1984a; 1986).

Their longitudinal study is carried out with the same group of students from grade 1 to grade 3 (39 students: grade 1; 22 students: grade 2 and 23 students: grade 3 or group B1). For Bednarz and Janvier (1988), "The operations are essential in our strategy because in addition to giving intentions for carrying out transformations on groupings, they inject meaning for those transformations (to make groupings, to “unmake” them…)" (p. 302). This study aims to test the effects of three years’ instruction prepared by the scholars Bednarz and Janvier based on a social constructivist epistemology and significant mathematical tasks. These tasks are based on problem-solving and promote understanding of the multiple dimensions of the targeted concept. The results revealed that the achievements of the 23 students (grade 3) of group B1 who benefited from Bednarz and Janvier’s didactical approach was compared to that of two other groups A (75 pupils) and B2 (26 students) and from the same school board as group B1 which received usual instruction related to the old program (objectives-based approach). The results show that the group B1 students have a better understanding of the rule of grouping and demonstrate a mastery of the DNS principles than those of the other groups that received regular education.

Distinction between Knowing and Knowledge

The question of the relationships between knowing (in French: connaissance) and knowledge (in French: savoir), which teachers aim to achieve through teaching situations, is relevant in light of this study. The theory of didactic situations (TDS) provides a modeling of these relationships by distinguishing between knowing and knowledge (Brousseau, 1997 & 1998; Salin, 2002). Knowing relates to the actions of a subject and the means of decision-making, the choice of an action, a formulation and a proof. In other words, knowing can be considered as individual cognitive constructs. It is applied/practical knowledge. Knowledge is, on the contrary, the "cultural and social means of identification, organization, validation and use of knowing" (Brousseau, 1997, p.10). This means that a knowing and knowledge have different cultural roles and status. However, more importantly is the triple distinction between a knowing, knowledge and instituted knowledge proposed by Conne (1992), in continuation of the TDS, but from a different educational perspective. For Conne, if knowledge consists of a recognized knowing useful for the control of a situation, then instituted knowledge refers to the concept of knowledge as defined by Brousseau in the TDS. For Conne, the recognition of the use of a knowing to get a grip on a situation and to control its transformation is the same thing as knowledge. The knowledge thus falls under situation control. Moreover, knowing that does not enable the control of a situation or results in its failure is ineffective knowing. However, although they are ineffective in the situation, what motivates the student in solving a task is nothing but knowing, according to Conne (1992).

The distinction proposed by Conne (1992) offers an advantage on two levels. First, students' difficulties and mistakes can be perceived as an otherwise valid knowing even if they pose problems in the mathematical situation. Second, it facilitates a more nuanced interpretation of students' strategies in distinguishing the knowledge that may enable or not the control of the situation. If a student knows the extent to which he manages to control the transformations of a mathematical situation, knowing—recognized as useful knowledge in this case—should be distinguished from Brousseau's knowledge --"cultural and social means of identification, organization, validation and use of knowledge" (p. 10). Thus, in the framework of this study, the situations in which students may or may not recognize the usefulness of certain knowing/knowledge should be identified. In the end, the distinction between knowing and knowledge makes it possible to identify the characteristics of students' knowledge, to interpret them in terms of useful knowing (knowledge) or knowing that does not recognize their usefulness for resolving situations (ineffective knowing).

Devolution and Institutionalization

To define the teaching situations in terms of their particularity with the envisioned knowledge as well as structuring in terms of the competence of the current program, we use the concepts of devolution and institutionalization. In the TDS, "Devolution does not consist only of presenting the student with the game the master wants him to play (instructions, rules, goal, final stage…), but also of acting in such a way that the student feels responsible rather than guilty for the knowledge of the result at which he has to aim'' (Brousseau, 1988, p. 89). The tools of the game that the student has refer to the knowing or knowledge (Conne, 1992) he uses to develop strategies and then reshuffles them to create new ones. Therefore, the teacher's action in the choice of situations whose traits would shape the relationship the student will have with is decisive. Additionally, devolution becomes meaningful in light of the situation and therefore of the didactic components concerning the challenges of knowledge.

Besides, it is the process of institutionalization that functions as an enabler of the passage from knowing to knowledge, conferring on some knowing the cultural status of knowledge. If knowing or knowledge is highly contextualized as means or implicit models of situation control, institutionalization enables its decontextualization so it could be identified by the culture and the society as knowledge, according to its use and importance (Brousseau, 1997). It is a process through which one reaches the level of symbolic knowledge. All in all, the concepts of devolution and institutionalization are useful in achieving the third objective of the study in relation with the analysis of teaching situations. Devolution makes it possible to study the didactic characteristics of teaching situations in terms of the specificity of the intended knowledge. Also, in the process of institutionalization there is the problem of understanding the way the teacher directs it in class in light of the challenges of the program of studies in place:

does the teacher decree it based on the skills underlying the situation or through knowing and knowledge?

Methods

Research Method Design, Site and Participants

A mixed method (Johnson & Onwuegbuzie, 2004) made it possible to respond to the three research objectives, combining qualitative and quantitative methods (frequencies and percentages, nonparametric tests). Data was collected through non-probabilistic and intentional sample (Patton, 2002) since it was carried out according to its quality to document the research question and help to the understanding of the research subject. In addition the tests used for the analyses are considered as a support for the interpretation of findings. Consequently, the findings of this research should not be generalized.

The process of participants' selection began with a study of Quebec Education Program in Mathematics and Progressing of Learning in Elementary School (GQ, 2001), followed by a study of approved textbooks and instructional materials and previous research findings on DNS (Bednarz & Janvier, 1982; 1984a; 1984b; 1988). Subsequently, the grade 3 was chosen and the schools located in Quebec were considered due to feasibility. After discussion with school boards and heads of schools, invitations to participate were sent to targeted teachers. We then discussed about the components of the study, negotiated with interested teachers (N = 14) and chose those who were truly interested in contributing (N = 8). Finally, we planned and collected data during the 2009-2010 school year.

More particularly, participants of the study (see Table 1) came from three regions and school boards in Quebec. They are composed of 154 students from third grade (aged 8-9) and from eight classes (group D). They were recruited in order to obtain a varied portrait of students' strategies and knowledge. The inclusion criteria are the grade 3 to which the subjects in preceding studies (Bednarz & Janvier, 1982; 1984b; 1988) belonged to and the essential knowledge aimed at in the 3rd year as per the DNS. In addition, two classes (RH and RJ) among the eight were involved in the videotape of their teaching situations. Besides, as the studies of Bednarz and Janvier constitute the reference of the current study (in response to objective 1) the different groups of students (grade 3) who participated to their researches will be also considered (groups A, B1 and B3).

Data Collection and Data Analysis

Three complimentary data collection methods were used. First, individual audio interviews of duration of 15 minutes were done with a limited number of 18 students (identified by their teacher) belonging to three different schools of the same school board (group C, derived from group D).

The interviews aimed to describe their strategies and knowing or knowledge in solving the two tasks taken from the studies of Bednarz and Janvier (1982; 1984b; 1988) in order to compare them against their results (objective 1). These results came from the strategies of three students’ groups: group B1 received three years’ instruction prepared by the scholars Bednarz and Janvier (1984b; 1986; 1988) based on a social constructivist approach and significant mathematical tasks; groups A and B2 received usual instruction related to the old program (objectives-based approach).

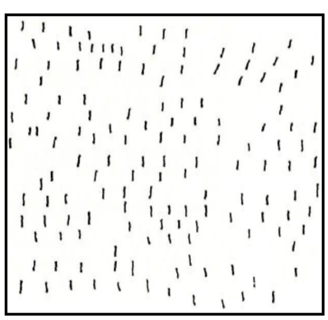
Table 1
Participants (Groups C and D)

Classes	LA	LB	BF	SD	RK	**RH**	RI	**RJ**	Total
Groups	N %	N %	N %	N %	N %	N %	N %	N %	N %
Group C: Audio Interviews						5	5	8	18 11,7
Group D Questionnaire	25 16,2	16 10,4	21 13,6	24 15,6	23 14,9	21 13,6	10 6,5	14 9,1	154 100
Lessons Observations									

The first task had to do with the enumeration of 144 rods drawn on a sheet of paper where the student was asked to quickly say how many rods there are (see Figure 1). Then, if the child failed to respond, the following instruction was formulated for him: “I am going to do the same thing later with a friend who will be here after you. Could you do something (we gave him the sheet) so that, when I shall show him the sheet, he will be able to tell me very quickly how many rods there are?’’

Description of the item.

‘’Present to the child a sheet of paper where a large number of rods are drawn […] The sheet is left before the child a short time and presented in a manner that he cannot touch it. Ask him: ‘’Can you tell me quickly how many rods there are drawn there?’’



Then, when the child has failed to respond:

‘’What did you do?’’ ‘’I am going to do the same thing later with a friend who will be here after you. Could you do something (we give him the sheet) so that, when I shall show him the sheet, he will be able to tell me very quickly how many rods there are?’’

When the child has finished: ‘’What did you do?’’...

Figure 1. *Task 1 – Enumeration of 144 rods (Bednarz & Janvier, 1982; 1984b; 1988).*

The second task had to do with the solving of a contextualized mathematical problem where a mother who was celebrating her child's birthday buys 234 peppermints and distributes 178 of those to the children at the party (see Figure 2). The student was asked to work on one of the collections arranged to discover the difference between the initial collection of peppermints (2 bags of peppermints, 3 rolls of peppermints and 4 units of peppermints) and the collection of peppermints distributed to children (1 bag of peppermints, 7 rolls of peppermints and 8 units of peppermints). The goals of such a task was to verify how the child operates with different orders of groupings, and especially the meaning given to borrowing in subtraction.

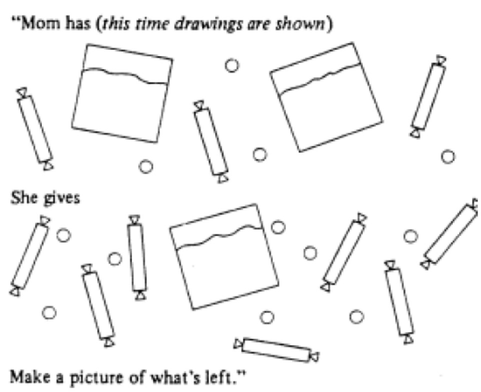
« Description of the item:

"A 'mom' buys peppermints for a birthday party. She wraps them in rolls like this one (showing sample of a roll on the desk) and put the rolls in bags like this one (showing sample of a bag on the desk) in order to give some to the children. She has some peppermints left over".

The child has before his eyes a bag, one or two rolls, and three or four peppermints.

Important: No mention is made of the number of peppermints in a roll or of the number of rolls in a bag. This number was not visible, but was accessible by examining the sample or by asking the question to the interviewer.

"The 'mom' prepared all this (showing the top drawing). During the party, she gave all this to the friends (showing the drawing below). Make a picture of what's left"



Then, when the child has finished: " Can you explain to me what you have done?"

Figure 2. *Task 2 – Problem solving (Bednarz & Janvier, 1982;1984b; 1988).*

Next, a written questionnaire (see Figure 3) was given to the students belonging to group D (*N* = 154) in a period of 50 minutes. The questionnaire is comprised of different tasks. The 31 items on the questionnaire, designed according to the principles, features and functions of the DNS, were divided into three categories: writing numbers, decimal place value of a digit or group of digits in the number and the number operations.

Finally, concerning lessons observations in classes RH and RJ, they were carried out throughout the school year, but in the end, two lessons taken as representatives were considered for analysis. In both classes, teachers used tasks from textbooks. For example, in Class RJ, the teacher used a contextualized task in which the students are asked to explain to a foreigner (another student) our number system. In class RH, students are asked to respond to exercises dealing with the DNS positional principle.

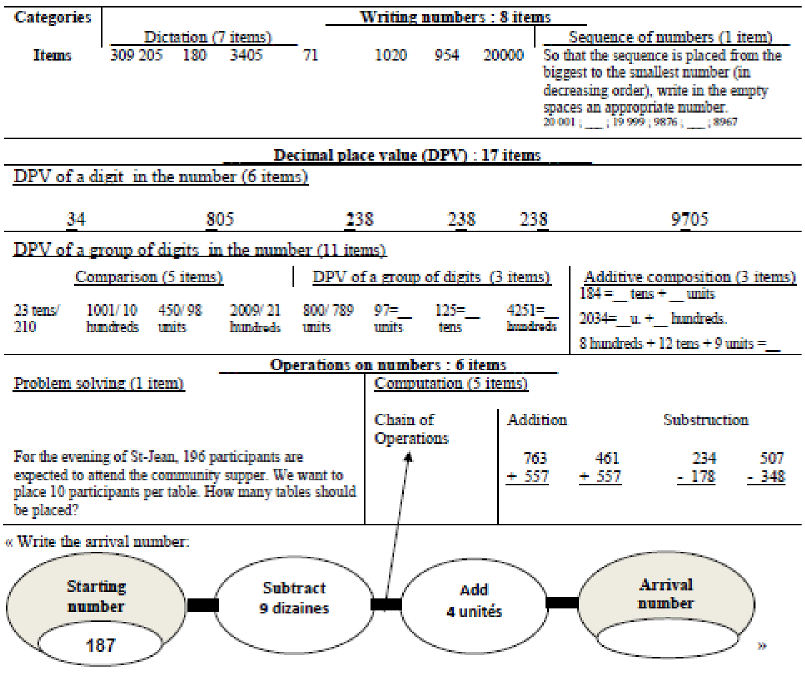


Figure 3. *Written questionnaire.*

The process of analyzing the interview data helps to bring out the underlying reasoning to the students' strategies and the knowledge they are based on. The analyses were made from the strategies of students previously identified in the studies of Bednarz and Janvier (1982;1984b;1988). For the task 1 (enumeration of 144 rods), there are two categories of strategies: 1) the coding of collections, with the use of groupings in order to write a code linked with them or the use of grouping of groupings. 2) The coding of collection of items, which is the grouping in order to count, the counting one by one and the estimation. For task 2 (solving problem), there are also two categories of strategies: 1) the use of the rule of grouping with the collection strategy (association of a number to each grouping and computation), the groupings strategy (proceeding from the picture and mental computation by unmaking grouping) and whatever the strategy (a confusion on groupings/impossibility to operate on them). 2) No use of the rule of grouping with the impossibility to solve the problem or partially/locally. Descriptive statistical analysis (frequencies and percentages) helped to establish a comparison between the strategies used by students in our study and Bednarz and Janvier's studies (1982;1984b;1988).

The analysis of the data from the questionnaire was carried out from the conceptual analysis of the DNS as well as the conceptual tools of the TDS, especially the distinction between knowing and knowledge (Conne, 1992). To facilitate the data analysis, the responses to the questionnaire submitted by group C were taken into consideration, followed by those of the other students of group D. The identification of particularly discriminatory tasks made it possible to classify the performance of the students according to certain coherence. In the end, five performance profiles were created; and this enabled the characterization of the knowledge of the group D about the DNS (objective 2), in addition to other elements that had an effect on the student's performance at a task. At this point, two variable categories were considered: the class the student belongs to and his performance on a task.

Moreover, since the data does not make it possible to fulfill the condition for the application of an analysis like ANOVA, two non-parametric tests were carried out. The statistics software SPSS, used for descriptive statistics, made it possible to ascertain whether a class difference exists with the help of Ficher's exact probability test (1954), in order be able to estimate the magnitude of the observed divergence, if possible, through Somers' D test (1962). The materiality threshold is 0.05. The Fisher's exact test (non-parametric) is used to satisfy some restrictions on eligibility requirements. The statistical power of the test is then somewhat lower. The Somers' D helps to measure the extent of differences in performance between groups to see how significant these differences are. The tests are useful complements to qualitative analyzes in order to avoid a student-centered interpretation as an individual for an interpretation of a pupil's performance as a subject of a class. If the Fisher test allows at least to identify the items for which there are significant differences according to the performance rates between classes, it is difficult to identify the classes between which these differences exist. Interpretations of the results are carried out with caution when considering which classes have the greatest discrepancy between students' performance when there are significant differences in the statistical analysis.

Finally, the qualitative analysis of lessons based on the TDS of Brousseau (1997; 1998) made it possible to define the teaching situations of the DNS in two third-grade classes in terms of the knowledge involved and the guidelines of the curriculum (GQ, 2001).

Results

Knowledge of Students of the 2001 Program in Solving the Two Tasks of Bednarz and Janvier (1982; 1984b; 1988)

Regarding the first objective of defining the students' knowledge of the 2001 program in two situations proposed by Bednarz and Janvier, and then comparing them with those of the students of the 1981 program, convergences and divergences appear in the work of the groups linked to the DNS. Concerning the first task of enumeration (see Figure 1), the majority of group A students (74%) and B2 (73%), as well as students of group C (55,5%), apply the coding of a collection of items (see Table 2), especially counting one by one. But those who received three years' instruction prepared by the scholars based on a social constructivist approach (group B1) mostly have recourse to the coding of a classified collection (92%), like the grouping of groupings, of which they are almost the sole users.

Table 2

Task 1—Results of group C versus groups A, B1 and B2

Categories of Strategies		Koudogbo *2001 Program* -> **C** (*N* =18)	%	Bednarz & Janvier *1983(a)* -> **B1** (*N* =23)	%	*1983(b)* -> **B2** (*N* =26)	%	*1980* -> **A** (*N* =75)	%
Collection of items	Counting one by one and estimation	4	22,2%	1	4%	10	38,5%	31	41%
	Groupings in order to count	6	33,3%	1	4%	9	34,5%	25	33%
Collections of groupings	Groupings in order to write a code	7	39%	14	62,5%	7	27%	19	26%
	Grouping of groupings	1	5,5%	7	29,5%	0	0%	0	0%

In resolving the second task - problem of peppermints (see Figure 2), group C (78%) as well as groups A (70%) and B2 (46%) have recourse to non-grouping strategies or confuse it with the instructions, contrary to group B1 (8%), which instead exercises control over the groupings, dismantling them so they could work (see Table 3).

Table 3

Task 2—Results of group C versus groups A, B1 and B2

Use Rule of grouping	Categories of Strategies	Koudogbo *2001 Program* -> **C** (*N* =18)	%	Bednarz & Janvier *1983(a)* -> **B1** (*N* =23)	%	*1983(b)* -> **B2** (*N* =26)	%	*1980* -> **A** (*N* =75)	%
No	Impossibility to solve	8	44,5%	1	4%	8	30,5%	45	60%
Yes	Collection strategy	4	22,2%	0	0%	4	15%	13	17%
	Groupings strategy	0	0%	21	92%	10	39%	10	13%
	Whatever the strategy	6	33,3%	1	4%	4	15,5%	7	10%

Knowledge of Students of the 2001 Program in Solving the Tasks of Koudogbo questionnaire (2013)

Regarding the second objective which is to define students' knowledge concerning the DNS from the responses to the questionnaire (see Figure 3), it was helpful to constitute five performance profiles of all the people and to subsequently distinguish types of useful knowing (knowledge), or the inefficiency mobilized by the students, especially the degrees of inherent mastery of different performances.

Around 47% of the people studied generally have enough knowledge to succeed at the different tasks, aligning respectively on profiles 1 (13%) and 2 (33, 8%), which is a testimony to the excellence of the students' knowledge (profile 1) or a very good mastery (profile 2). On the contrary, the results equally reveal how a little fraction of the people (13%) has a profile 1 performance where one remarks a coordination of *intra and inter-task* knowledge. The useful knowing that allowed these students to control the mathematical requirements of each task thus became knowledge (Conne, 1992). Likewise, a third of the people have profile 3 performance (30,5%) with some mastery of the DNS that is characterized by useful knowing, even if they are not fully integrated. Less than a quarter of the people (22,8%) are spread out on profiles 4 and 5 and for whom knowledge (knowing) is rather inefficient, and consequently marred by difficulties. The knowledge lacks inter and intra tasks coordination leads to difficulties and errors.

Emergence of Phenomena: Class Effect and Task Effect

Some phenomena like *class effect* and *task effect* emerge from the analyses. Certain tasks (or items) have success rates that vary considerably from one class to another (for example: problem solving, chain of operation and place value of a group of digits in the number). These are considered discriminatory in terms of classes and refer to a *class effect*—the most remarkable effect to emerge from the analyses. 20 out of 31 items (64,5%) are affected in all categories of tasks. This effect reflects the fact that belonging to a class has a significant impact on the performance of students on a mathematical task. A *class effect* is identified when a significant difference is observed among the performance obtained in each class. This effect is predominant with these tasks: problem solving, chain of operations and place value of a group of digits in a number (9 out of 11 items). This last category seems the most difficult for students to succeed. For example, the statistical analyzes reveal significant differences between classes' performances to the items of a task, with the materiality threshold to Somers' D and Fisher's exact test. In addition, the analysis of the procedures used by students provides indications, which help, especially during problem resolution, to distinguish mathematical practices peculiar to the classes from which the students come. For example students' procedures and tools (design, mathematical writing) are elements that distinguish the conduct of students according to the class they belong to. Another phenomenon is the *task effect*, which makes it possible to observe that the task failed massively or was passed by all the students in all the grades. Therefore, the performance of all the students at the same task makes little difference, because the task involved does not discriminate against students in terms of their class membership.

Analysis of DNS-Based Lessons in Class RH and Class RJ

The qualitative didactic analysis of teaching lessons in the two classes, as part of attaining the third objective, has made it possible on the one hand to define the proposed situations in light of the specificity of targeted knowledge as well as the approach of the current program, and on the other hand, to formulate explanatory hypotheses on the relationship between teaching situations and students' performance. Indeed, the teaching situations proposed by the two teachers come from textbooks and are not peculiar to the targeted knowledge. And this makes the process of devolution (Brousseau, 1998) rather hypothetical, as the situations didn't sufficiently take into account the interrelated knowledge of the DNS. For example, in Class RH, the teacher used tasks that are rather applied exercises on the positional principle of DNS than problems solving. In addition, in Class RJ, the manner of pedagogical organizations is favored by the teacher at the expense of didactic challenges specific to the knowledge targeted by the situations. Also, the development of students' skills recommended by the program does not appear clearly in practice *in situ*. The analysis of sessions equally made it possible to understand students' results from classes where lessons were observed, thus making sense of the identifiable difficulties and errors in their response, linking them to the type of instruction received, as was made clear much earlier.

Discussion

The purpose of this research was to provide the portraits of knowledge of students who were educated under the competency program established in 2001 on the DNS and of teaching situations. In response, three research objectives are formulated and have to do with the learning and teaching components. In response to the first objective, from what we know now about the knowledge of the students educated under the current program, compared to the students of the objectives-based program, there are divergences. Thus, the knowledge, the ineffective knowing and the difficulties concerning the DNS of 18 students (group C) are comparable to those of the students educated under the old objectives-based program (groups A et B2). Only the students (group B1) who were taught by Bednarz and Janvier (1982; 1984b; 1988) have the knowledge (or useful knowing) most in line with the DNS. These results reveal that the situations experienced by these authors presented several mathematical and didactical characteristics to address the teaching and learning issues. Effectively, these authors are educational practitioners and thus, teaching situations they have developed in their studies are based on the analysis of student conceptions, the curriculum and the usual teaching activities in textbooks, but also on a real conceptual analysis, taking into account the articulation of knowledge on numeration-DNS and operations as well as the scope of intermediate representations making it possible to give meaning to the number system. All these considerations seem not to be the case for the teachers involved in our study.

Again, in light of the results from the questionnaire of 154 students (objective 2), it is possible to see that the knowledge of the people studied seems satisfactory. But although the tasks we proposed to the group C considered the multiple dimensions of DNS, they are mostly structured in terms of questions/answers rather than in terms of solving complex problems. So these results apparently contradict those obtained from the interview of the 18 students on the resolution of problem (task 2) formally used in the studies of Bednarz and Janvier (1982; 1984b; 1988). In fact solving this problem was very difficult for them and therefore because of *the task effect* was. It is possible to suggest the current curriculum did not, therefore, modify the students' knowledge.

On the other hand, analyzing the performance profile of students made it possible to highlight, according to a given profile, what the student knows, i.e. his knowledge, or what he knows more or less, or does not know, i.e. his *inefficient knowledge* (Koudogbo, 2013; 2015) as well as the existing (or non-existing) links in the knowledge/knowing he mobilizes. For example, if the student recognizes the use of knowledge, he could invoke that in controlling the situation and succeed at a task in an intra-task manner or at the whole task, i.e. in an inter-task manner. The opposite is also true.

The emergence of phenomena such as the *class effect* and the *task effect* has a certain scope. Indeed, although statistical analyses do not permit the location of the classes for which the difference is significant, it shows, on the contrary the students' performance in relation to the class to which they belong. The data analyzed come mostly from the responses to the closed questions questionnaire. There is therefore little information that would allow an interpretation of the *class effect*. However, the analysis of the procedures used by the 18 students who also participated in the audio interviews (group C) provides indications, which help, especially during problem solving, to distinguish mathematical practices peculiar to the classes from which the students come, and it is due to the observations that took place in their own classes. The students' problem-solving procedures, reasoning, strategies, the tools used (design, mathematical writing), and the students' verbalization are elements that distinguish the conduct of students according to the class they belong to. It was possible to formulate the hypothesis according to which the *class effects* stem from the mathematical culture of the class.

From the above, the hypothesis that several factors interact to determine student performance in a task was formulated. Among those factors, there is the student's performance profile, mathematical culture developed in the class he belongs to (*class effect*), i.e., the students' way of doing mathematics (Conne, 1999), and the level of the mathematical difficulty of the task (the task effect) with regard to the educational level and knowledge of the student. In light of this, the level of difficulty of a mathematical task depends not only on the cognitive development of a student but also on the mathematical culture of the class to which he belongs, in addition to what is offered to him by the educational institution (GQ, 2001), according to his grade.

As for the analyses of the teaching lessons (objective 3) the findings allowed to consider two elements. First, the analysis made it possible to understand students' results from classes where lessons were observed, thus making sense of the identifiable difficulties and errors in their response, linking them to the type of instruction received (*class effect*), as was made clear much earlier. In addition, the analyses equally made it possible to consider the process of devolution (Brousseau, 1998) rather hypothetical as the situations (textbook exercises) didn't sufficiently take into account the interrelated knowledge of the DNS (Class RH). In fact they are exercises that focused not on the interrelated dimensions of DNS but on the positional principle as pointed out in the studies of Tempier (2010). They don't call for an organization and a coordination of knowledge as they don't represent a real challenge for the student. They don't involve a questioning of student's knowledge and representations (GQ, 2006).

So, what is the ''scope'' of the essential knowledge described in the program in current textbooks? How are they transposed? It is therefore important to consider training situations which make it necessary to question the teaching and learning issues specific to the knowledge on the basis of studies in mathematical education. In addition, the manner of pedagogical organizations is favored by the teacher at the expense of didactic challenges specific to the knowledge (Brousseau, 1998) targeted by the situations (Class RJ). Thus, the development of students' skills recommended by the program (GQ, 2001) does not appear clearly in practice *in situ*, considering the targeted classes in the study.

Consequently, it is possible to assume that a shift in curriculum orientation does not seem to be enough to change practice in classrooms (Cardin, Falardeau, & Bidjang, 2012), at least, the two teachers' practice we studied in the current research. Provide students with learning situations that have the didactical qualities required to build appropriate and efficient knowledge raised the problem of teacher training. Finally, even though the findings of this research should not be generalized, nonetheless, they would help to support the knowledge/knowing of students and teaching situations approached respectively in the study. The probative results relative to the three research objectives seem to be important for the mathematical education and should be considered in future research, in particular, how to design such learnings situations.

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