The Role of Translations between and within Representations on the Conceptual Understanding of Fraction Knowledge: A Trans-Cultural Study

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This research investigated the effects of using a representation-based intervention (RBI) on fifth grade students' abilities to translate within and between different representations when solving fraction related problems. The study employed a quasi-experimental design. The sample comprised 5 schools randomly selected from a population of 14 schools subsidized by a philanthropic organization. Eighteen fifth grade classrooms were randomly assigned to experimental (RBI) and control (TBI) groups. Results of the quantitative data showed that RBI students outperformed the TBI students on RBI and school assessment tests. Furthermore, RBI students employed problem solving schemas that reflected more translation within and between different modes of translations than did the TBI students.

Key words: representations, rational number concepts, conceptual understanding, mathematics for elementary students.

Introduction

Around the globe, political, social, and religious clashes have been inescapably notorious for inflicting insurmountable damage on the educational systems and the intellectual advancement of countries wedged in such conflicts. Lebanon is no exception. Thirty years of sporadic war and civil conflict had a devastating effect on the Lebanese educational system. The substantial out-migration of the skilled and educated drained the human reserve base. Schools' infrastructure and facilities suffered widespread damage, and the overall quality of public and private educational services weakened markedly (World Bank, 2000). However, and particularly after the

end of the civil war in 1991, there has been a gradual recovery of the Lebanese educational system, including reconstruction and rehabilitation of school facilities, which progressed rapidly. Despite this recovery, the Lebanese school system is still saddled with the self-defeating pessimism that accompanied decades of civil war (Chahine, Post, & delMas, under review).

Compared to other Arab countries in the Middle East, Lebanon has shown some advancement in the last decade. According to a report by the World Bank (2000), "… the adult literacy rate in Lebanon reached 85 percent in 1997 (from 65 percent in 1972); primary education became universal; net enrollments in secondary education have reached the 50 percent level; and enrollments at the post-secondary education level hovered around 25 percent" (p.3). Moreover, discrepancy by gender became low at all levels. In a survey conducted by the Consultation and Research Institute (CRI, 2005), enrollment of girls between the ages of 12 and 21 in public schools exceeded that of boys by far.

In 2003, Lebanon was one of the 60 countries worldwide participating in the Trends in International Mathematics and Science Study (TIMSS) (Mullis, Martin, & Foy, 2003). While TIMSS assesses student learning in mathematics and science at the 4th and 8th grade levels, Lebanon participated only in the 8th grade assessment. Findings of this study showed that students in Lebanon performed significantly below the international average in mathematics and science (international average score is 467 for mathematics and Lebanon's score is 433). Analyzing performance by the five domains tested, Lebanese students performed lower than the international average in all five domains. Compared to the eleven participating Arab countries and authorities, namely Bahrain, Egypt, Iran, Jordan, Lebanon, Morocco, Saudi Arabia, Syria, Tunisian, Palestine, and Yemen, Lebanon performed higher in mathematics. Nonetheless, almost no or very few students taking the examination in Lebanon got to the "advanced" or "high international" achievement benchmark in either mathematics or science.

In Lebanon, mathematics became an obstacle to many students, preventing them from pursuing higher educational levels (Chahine et al., under review). The gate-keeping power of mathematics as a determinant of college entrance has been reported in two successive studies by the Lebanese Educational Center for Research and Development (CERD) in collaboration with UNICEF and UNESCO. Both studies provided achievement data of students in the fourth elementary grades and intermediate classes for the year 1994-1995. In the fourth elementary grade, more than one fifth of the students

had a low average in mathematics (39.4% of students scored below 50% of the maximum score), specifically in problem solving (82.8% of students scored below 50% of the maximum score) (CERD, 1996). This decline in achievement becomes even more serious in middle school, for more than 50% of students failed in math (55.2% of students scored below 60% of the maximum score). Additionally, the percentage of students with low averages increased dramatically when attempting to use analysis and synthesis in problem solving (83.7% of students scored below 60% of the maximum score) (CERD, 1997). In 2007, the success rate in the official exams did not exceed 60% (Al-Mostaqbal, 2007). This decline could be due, in part, to the narrow description of what counts as mathematics and to the lack of updated pedagogical and technological support that foster meaningful instruction in the classroom. The heavy emphasis on the use of symbols and procedures prior to establishing conceptual understanding through the use of multisensory representations is a clear disadvantage for many students enrolled in most traditional school systems. In a recent study on the curriculum goals perpetuated in the school-leaving examinations of the Middle East and North Africa (MENA), Valverde (2005) asserted that more than 70% of core performance expectations in Lebanese exams are directed towards knowledge and recall of basic facts as well as performing routine procedures.

Theoretical Framework

The theoretical model adopted in this study is the Lesh's Translation Model (1983). Lesh's model highlights the role that "external" representations and translations within and across representations play in enhancing the acquisition and understanding of mathematical concepts as well as problem solving. Lesh's model emphasizes the use of five modes of representations: *spoken symbols*, *real world situation, manipulatives, pictorial representations, and written symbols*. Furthermore, Lesh's model capitalizes on translations within and across the five representation modes as necessary prerequisites for understanding and internalizing mathematical concepts.

Cramer (2003) argues that experiencing the benefits of using multiple representations as well as possible translations among these different modes of representations helps teachers become more aware of the weaknesses inherent in any curriculum and thus respond by incrementing it with outside resources.

One striking implication for using the Lesh Translation Model in analyzing students' performance on fraction related tasks is its heightened emphasis on the role of representation in the acquisition and development of concepts and skills. Lesh (1979) defined the ability to translate from one mode of representation to another as related to mental processes and abilities and differentiated between within-mode translation processes and across-modes translation processes.

Method

The deep concern about the widespread failure of students in mathematics in Lebanon gave impetus to the present study. This study investigates the effects of using a representation-based curriculum on fifth grade students' ability to solve fraction related problems by translating within and between different representation modes including spoken symbols, real world situation, manipulatives, pictorial representations, and written symbols.

Hypothesis

The study examined the hypothesis that students using a representation-based curriculum will demonstrate a greater ability to solve problems by translating within and between modes of representations in fraction related situations than students using the traditional Lebanese curriculum.

Sample

Five schools were randomly selected from a population of 14 semi-private schools. At the time of the study, all 13 teachers teaching grade five in the five participating suburban schools were selected and were randomly assigned to RBI (i.e., Research-Based Instruction) and TBI (i.e., Traditional-Based Instruction) groups. The RBI group included 285 students and the TBI group 223 students.

Data Collection

We employed two methods for collecting quantitative data: tests and classroom observations. The two instruments used to collect quantitative data were the RBI posttest and a school assessment test that covered similar

material as the RBI test to ensure the alignment of both assessments with respect to the desired content goals. We translated all instruments to Arabic, the language used in the study, and carefully attended to the cultural dynamics of the Lebanese classroom.

Description of Instruments

RBI test. This instrument included a total of 31 items. Some items required only a single answer. Others required an explanation of the student's problem solving protocol. We designed the test items for the RBI test to assess students' conceptual as well as procedural knowledge of fraction related concepts. The RBI test covered 4 strands which cover the major areas in the RBI curriculum namely (a) concept of unit, (b) fraction equivalence, (c) fraction order, and (d) fraction comparison. We administered and collected the RBI posttest from RBI and TBI groups at each of the five schools at the end of the curricular intervention.

School test. The school assessment was prepared by the schools and was based on the material taught during the period of curricular intervention using the traditional Lebanese curriculum. This test included 17 items, most of which required one answer and covered the same strands as the RBI posttest. RBI and TBI teachers administered and collected the test from their respective students at the end of the study.

Teachers' Professional Development

Training sessions were held independently for RBI teachers, i.e. teachers using RBI and TBI groups, i.e., those using Lebanese traditional curriculum. Three 2-hour training sessions were administered for RBI teachers and another three 2- hour seminars were given to TBI teachers during the course of the study. The first session was held the first week of intervention, the second was administered two weeks during the intervention, and the third was carried out two weeks before the end of the intervention and students' performance assessment. The sessions were given in Arabic and the topics discussed were related to the teaching and learning of fractions for elementary school students. RBI teachers were trained in the philosophy and implementation of the RBI curriculum and lesson plans. Training for TBI teachers involved administering a series of focus group sessions in which teachers discussed and

reflected on episodes of teaching situations involving problems with fraction concepts.

Data Analysis

The Lesh's Translation Model was employed as the theoretical framework for data analysis. Students' responses on the RBI test and the school test were analyzed using a mixed between-within subjects multivariate analysis of variance (MANOVA) technique.

The type of translation between/across five modes of representation (pictorial; real life; symbolic; manipulative; written) was identified for each question sub-item on the RBI and school assessment. Based on Lesh's (1979) representational model, each question in both the RBI test and the school assessment was assigned either a between-mode translation (i.e. across two different representations) or a within-mode translation (i.e. within the same "semi-autonomous subsystems" within the same representation i.e. from one picture to another) (Lesh, 1979, p.168).

For the RBI test, we extracted 9 translation processes, 6 between-mode translations and 3 within-mode translations. Similarly, for the school test, 5 translations surfaced after examination, 4 between-mode translations and only one within-mode translation. The translation processes by the type of representation and the number of items that measured such translation for both RBI and school tests are included in Table 2.

Table 1
Translation Processes by Type in RBI and School Tests

Type of translation	Direction of translation processes	Number of test items	
		RBI test	School Test
Between-Mode	Symbolic-Written	1	4
	Written-Symbolic	7	-
	Pictorial-Symbolic	9	-
	Graphical-Symbolic	1	1
	Real world-Written	4	4
	Manipulative-Symbolic	1	-

	Real world –pictorial	-	1
Within–Mode	Pictorial-Pictorial	2	-
	Symbolic-Symbolic	4	7
	Written -Written	2	-

To test whether there are differences between RBI and TBI students' abilities to translate within and across modes of representations for RBI test and school test, a mixed between-within-subjects MANOVA was employed. The rationale for using the mixed model MANOVA design was to evaluate whether the population means on the set of two dependent variables, (scores on within-mode translations and across- mode translations) vary for RBI and TBI groups.

Results

Quantitative Analysis

Results of MANOVA conducted to test the impact of RBI on students' performance produced a statistically significant main effect for the type of test (Wilk's Lambda =. 978, F (1, 501) = 11.35, p =.001). The multivariate η^2 = .022 showed a small effect size and indicated that around 2% of the multivariate variance in test scores was associated with the difference in scores on the school test and the RBI test. Overall, students' average percent correct score was higher on the RBI test (71.62) than on the school test (68.16). Means and standard deviations of scores on the two tests were broken down by curriculum in Table 3. Scores are percent correct of total possible points (Chahine, Post, & delMas, under review).

Table 2
Means and Standard Deviations of RBI and TBI Groups on RBI Test and School Test

Test	Group	Mean	Standard deviation
RBI	RBI	76.23	15.59
	TBI	65.75	16.67

School	RBI	70.86	23.54
	Control	64.71	24.90

Additionally, the interaction effect between curriculum and type of test was also statistically significant (Wilk's Lambda =. 990, F (1, 501) = 5.17, p =.023) with a multivariate η^2 = .010 indicating a small effect size and suggesting that about 1% of the multivariate variance in test scores was associated with the interaction between Group and Test.

To explore the nature of this interaction (see Figure 1), simple effects analyses were run in order to examine the mean differences among test scores on RBI and school tests. A separate one-way MANOVA with type of test as a within-subjects factor was run for each curriculum. After splitting the sample by group, the mean difference between scores on the RBI and school tests was sought first for the TBI group then for the RBI group.

Results of analysis of the simple effects produced a statistically significant difference between mean scores on the RBI and school tests for the RBI group (p < .001) but not for the TRAD group (p=. 378). While students who received the RBI curriculum scored higher on the RBI test than the school test, the TBI students had a similar mean on both tests (see Table 3).
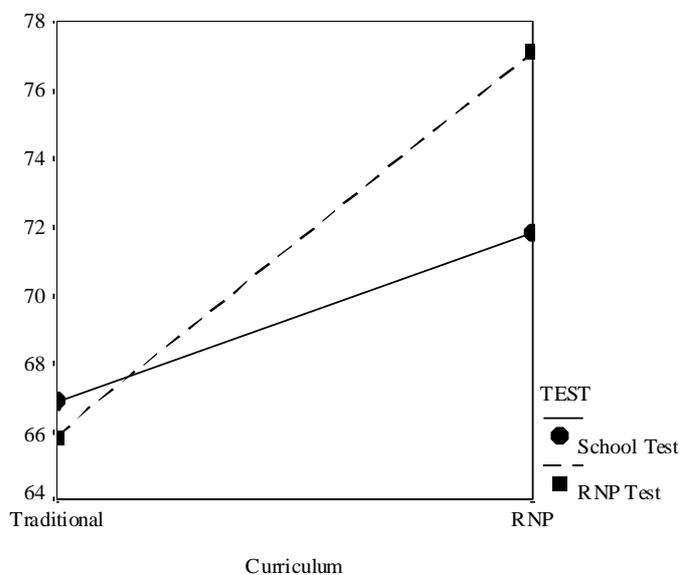

Figure 1. **SPSS output of a profile plot indicating the effect of interaction between Group and Test.**

Moreover, the means and standard deviations for RBI and TBI groups on each of the two translation modes in both RBI and school tests were calculated using SPSS and are given in Table 4. Results for the MANOVA conducted to test students' abilities to translate between and within representation modes produced a statistically significant main effect for TEST (Wilk's Lambda =.983, F (1, 501) = 8.883, p =.003). The main effect for TRANSLATE was not statistically significant (Wilk's Lambda =1, F (1, 501) = .093, p= .761). Moreover, the effect of the three-way interaction was statistically significant (Wilk's Lambda=.988, F (1, 501) = 6.012, p =. 015).

Table 3
Means and Standard Deviations of RBI and TBI Groups' Scores Per Translation on RBI and School Tests

Test	Translation mode	Group	Mean	Standard deviation
RBI	BETWEEN	RBI	77.04	16.47
		TBI	65.65	18.47
	WITHIN	RBI	74.14	18.72
		TBI	66.00	18.43
School	BETWEEN	RBI	69.22	24.50
		TBI	64.68	24.49
	WITHIN	RBI	72.50	27.82
		TBI	64.73	30.60

To explore the nature of the three-way interaction between type of curriculum, type of test, and type of translation, a separate 2X2 MANOVA was conducted for each curriculum group. A 2x2 within-subjects MANOVA for the traditional group did not produce a statistically significant interaction between type of test and type of translation (Wilk's Lambda = 1, F (1, 220) = 0.020, p = .888). The two-way interaction was statistically significant for the RBI group (Wilk's Lambda = .952, F (1, 281) = 14.289, $p < .001$).

Separate plots were created for RBI and TBI groups of mean scores for between and within translations for each test (see Figure 2). As can be seen in Figure 2a, students who received the traditional curriculum had about the same mean score for each type of translation on each test. Figure 2b shows that the mean score for within translations was about the same on both tests for

students who received the RBI curriculum. Relative to performance on the within translation items, the RBI students performed better on between translation items for the RBI test than on between translation items for the school test. Note that the mean scores for RBI students were higher than those of students in the traditional curriculum for all subsets of items.
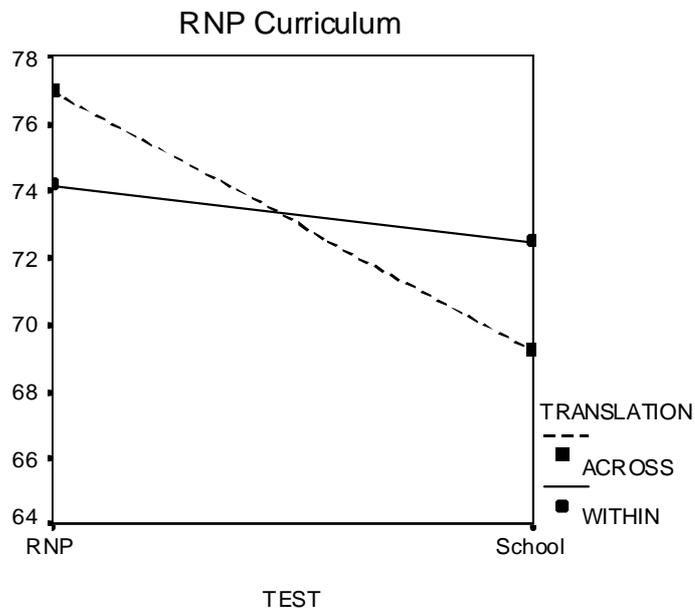
3a

Traditional Curriculum

RNP Curriculum
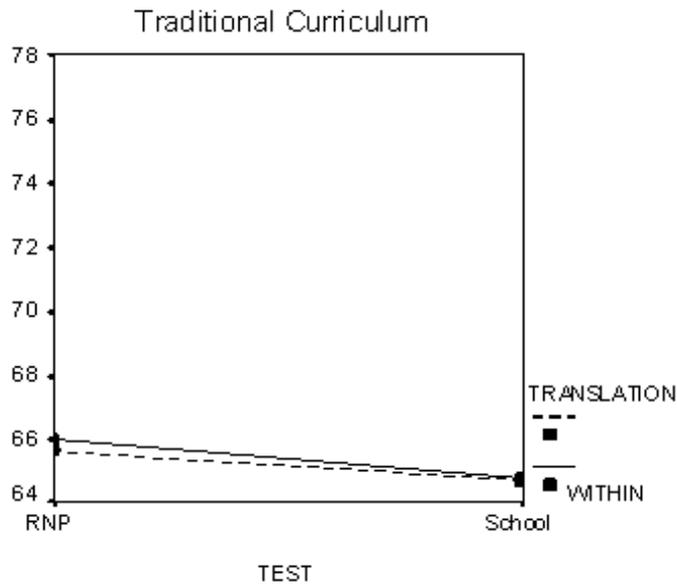
3b.

Figure 2. **Mean scores for BETWEEN and WITHIN translations on each test separated by group.**

Discussion

There was an implicit assumption in this study that using different representations when solving problems that involve fractions secures conceptual understanding of rational number concepts and consequently enhances students' performance. RBI students were afforded enriching opportunities to use different representations using the representation-based intervention tasks. This, in turn, significantly influenced their ability to develop flexible strategies to translate between and within multiple representations and thus successfully solve fraction related problems. Using multiple representations encouraged students to take initiative in developing their own representational models for understanding fraction related concepts. Such representations helped students shift from using strictly procedural strategies and routines to utilizing more cognitive means to reason about fraction related concepts and successfully solving problems. As compared to the traditional curriculum where the teacher or textbook serves as the decisive authority as to right and wrong solutions, using multiple embodiments or representations supported students' inquiry and empowered them to serve as their own authority, in control of their own learning. These shifts in both the problem-solving process and the allocation of authority can have implications on the performance of all students.

This study contributes to an extensive body of research that highlights the complexity of rational concepts and the difficulty that students confront when understanding fractional knowledge. Moreover, this study provides an outlook on students' performance with fraction concepts from an international perspective. Despite the tremendous efforts that have been made to reform fraction instruction, overemphasis of rote procedural repetition and mere recall of facts when learning remains a major cause of low performance in Lebanese classrooms. Nonetheless, findings of this study revealed that exposing students to opportunities for constructing their understanding of fractions using different representations increased students' success in learning rational number concepts. A direct implication of this work involves the need to sensitize teachers to research-based pedagogies that introduce mathematical content in meaningful ways that will enhance students' critical thinking and meaning making. Unless teachers are engaged in exploring novel instructional strategies through research-based interventions, rote drill instruction will most likely persist as the most prevalent teaching method in the mathematics classroom

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