

# Briefing on MM Education: A New Way of Mathematics Teaching

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*A new way of mathematics teaching was introduced applying the views of mathematical methodology that guides mathematics teaching. It is to put to use the laws of mathematical development and mathematical thinking and methods, using the laws of discovery, invention and innovation in mathematics, to design mathematics teaching.*

**Key words:** “MM” (mathematical methodology), teaching, discovery.

## Background

This paper introduces a new way of mathematics teaching, and this new way of teaching is designed for discovery, invention and innovation in mathematics and focuses on the mathematical development and mathematical logical thinking.

We began to use this approach at high school in 1989. The initial goal is to enlighten high school students’ creativeness and promote their discovery and invention in learning mathematics. The results from fifteen years of experiments show that the students’ achievement using MM approach is significant different comparing to their peers. Table 1 shows one of examples of theses results (Xu, 2003).

*Table 1*

### Student Achievement Difference at Four Schools from Wuxi City in 1993

	Mean of high school entry grade	Mean of high school exit exam grade	Mean of National exam for high school exit exam	Stand deviation high school exit exam grade	Stand deviation National exam for high school exit exam
Experimental group(N= 200)	107.20	81.77	80.35	14.76	13.15
Control group (N = 200)	107.97	72.05	71.29	16.86	15.87

From the experiment, we conclude that MM approach is an effective mathematical teaching approach for middle and high schools. To make effort of showing purity in mathematic, we have developed and constructed a series of mathematics teaching and

learning activities based on G Polya's mathematical approach which connected mathematics teaching and learning theories and classroom practices. In this article, we want to share some of our works.

### **Where is the Problem?**

New century with high technology has brought a new challenge to education, especially mathematics education. Engaging and motivating student to learn is a new task for all mathematics educators. We believe the motivation is depends on the mathematics teaching and learning method. So what is the most valid method? Providing lower quality of teaching materials and using it in mathematics teaching and learning is virtually equal to crossing off the elegance and perfection of mathematics step by step, resulting in thinner math textbooks. But the reduction of contents, until there is only the framework with no flesh and blood, will not increase the interest in the study on the part of the students. On the contrary, making unnecessary compromises and lowering the requirements of mathematics study may encourage a resistance to work on the subject, and a reluctance to deal with difficulty in mathematics and in general.

A question then arises: why, on the one hand, is mathematics education seen to badly need reform, but on the other, the reform is always greeted with setbacks and failure? Where and what is the problem? We can draw lessons from failures and gain experience from success. In our project we have attempted to apply mathematics methodology, such as that proposed by Polya, in mathematics education, and improve interest in and learning of mathematics, thus also enhancing students' character in an all-round way.

### **Will this Work?**

Modern mathematics education combines knowledge of pedagogy, psychology, physiology, perception science and even brain science, but sometimes neglects the thoughts and methods of mathematics itself. In fact, education can be seen from the standpoint of mathematical methodology and the developmental processes of mathematics itself. The Hungarian American mathematician G. Polya is a pioneer who tried to apply the mathematical methodology in the mathematics teaching. His mathematical works "How to solve it", "Mathematics and Plausible Reasoning", and "Mathematical Discovery" not only increased the American mathematics educational level after World War II but also had the important influences in international mathematics education, such as 'mathematical investigation' in the UK, 'problem-solving' in the US, and the project described in this paper in China.

From the standpoint of mathematics methodology, mathematics has a dual property: it is both systematic deductive science (when you see the final form of mathematics reached from facts) and inductive science (when you see it as a process of forming concepts) and its problem structure and mathematical modeling. For this reason, mathematics teaching should be in line with the dual property, both deductive and inductive, enabling the students to get an all-round mathematics education.

If we use mathematics methodology and psychology to analyze the kinds of thinking which historically and pedagogically lead to the development of mathematical knowledge, we can see that mathematical thinking also has a dual property: abstract thinking relates to

logical reasoning and image thinking relates to plausible reasoning, which employs observation, experiments, analogy, association and incomplete induction, etc. This dual property not only plays an important role in the process of mathematics discovery, but also has extensive application in social life. We, thus, should not treat mathematics as a pure scientific tool, but should treat it as cultural development, and try to improve people's general cultural accomplishment in mathematics. It is understood that we must take measures to ensure the maximal development of this mathematics cultural educational function. G. Polya himself gave a lot of examples to illustrate and reinforce his opinion that people find it hard to accept an abstract conclusion without being able to support it with plausible reasoning – logical reasoning on its own is not enough. Hence a logical argument about the irrationality of the square root of 2 does not necessarily convince even a learner who accepts the reasoning that it is, indeed, not rational!

This typical example shows that the difficulty of studying mathematics does not result solely from its abstract form. Rather, it is the result of being away from its background, from the absence of plausible reasoning to discover its process, from not analyzing the feedback information after setbacks, and from the lack of lively activities of creation and invention. Our duty as educators is to offer learners opportunities to decode the invisible codes and show that abstract ideas are seated in real experiences. Hsu (1983) put forth the suggestion that the principles of the mathematics methodology (MM) can be applied in mathematics education.

### **How to Operate?**

What is the MM method? How can this kind of method be organized appropriately in mathematics teaching and learning? In the process of mathematics teaching, mathematics should play two important roles: being an artifact and tool of science-technology and contributing to aspect of culture. These fundamental principles should be carried out in teaching and learning. That is: teaching through discovery and reasoning can be taken place at the same time, aiming at three targets: 1) to cultivate students' general quality of science, 2) to improve increasingly students' learning skills of mathematics culture by themselves, and 3) to form and develop the quality of students' mathematics.

Through experimental studies, we have identified 8 features which would then be important for mathematics teaching and learning:

- 1) To connect mathematics to real lives;
- 2) To view the beauty of mathematics and mathematics education;
- 3) To have an opportunity of mathematical discovery;
- 4) To have productive mathematics disposition;
- 5) To value the contribution of mathematics history;
- 6) To explore patterns of mathematics;
- 7) To have Adaptive reasoning;
- 8) To use problem-solve approach.

Teachers with professional development on MM approach have made significant changes on their teachings from a passive mode for students to an active mode, exploring

rules through both proof and guesswork, the whole teaching process melting into the main melody of invention and discovery.

### A Brief Example

In analytic geometry of plane, there is a well-known exercise: two end points of a segment AB moving along the x-axis and y-axis of coordinate planes. Find the locus equation of the middle point which on the segment.

This curve is a circle (although students often guess it is going to be something else at first). The MM method suggests looking for a general solution to the problem by placing the solution process and its consequence at the heart of the work, rather than the answer. By investigating, experimenting, tallying-up, monitoring and reflecting continuously students can go deeply into the mathematics knowledge system. We have observed Mr. Wang's class (a male teacher with 10 years teaching experience in secondary mathematics), and his teaching using MM not only motivate student learning but also help student learning with understanding. Mr. Wang uses a teaching-stick, and makes two end-points and moves on the interior right angled sides of the blackboard. In this way his students can observe that the star curve resulting from this action might be seen as the common envelope of both a family of straight lines and a family of the ellipses an implicit solution which allowed his students to discover the mathematics method through students' exploratory activity. Here we give the intuitional figure as follows:

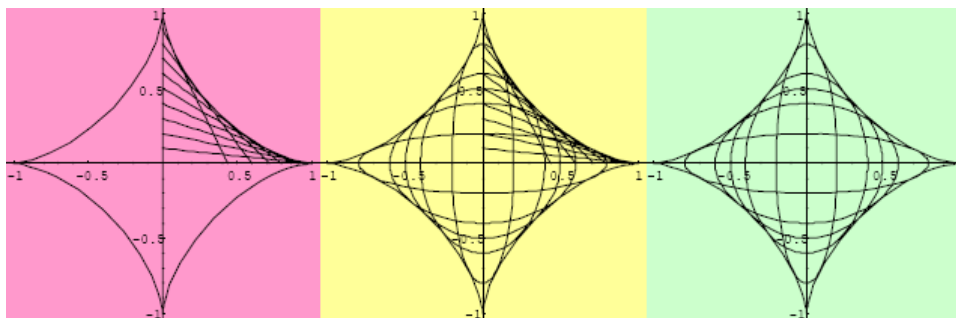


Figure 1. Intuitional figure

This example illustrates the eight features as variables which can occur in different amounts in different problems. In this lesson five were manifested as follows:

- The problem was simply stated;
- The problem was presented an aesthetic way;
- Student's imagination by mathematical discovery has developed and expended;
- Student disposition was productive, and they were acting like mathematicians;
- Students used reasoning, and they saw an ellipse they explored it.

By experience, we also realized that the lesson is no longer totally controlled by the teacher once students are active learning through discovery. For example: in a middle school algebra class, Miss, Li and her students together discussed a route of travel problem: an express train chases the local train. Through discussion, students brought up to class an apparently irrelevant question: How long after just 3:00, the hour hand with minute hand on the watch is going to be mutually perpendicular again? Miss. Li realized that it was a good chance to strengthen the students' mathematical consciousness and

increase their interest in learning. In the former case, the two trains move on the straight line, but in the latter case the two hands move on the circumference. The question extends easily to: “how long after any known point of time are the minute hand and the hour hand on the watch mutually perpendicular?” Miss. Li then encouraged this student to go on thinking after lesson, and also worked on the problem herself to show that the student’s question was valuable. We leave this as an exercise for the reader (solution on request). This example indicates that teachers can provide more opportunities to students’ mathematics discovery and imagination by changing teacher-centered teaching to student-centered teaching. The MM approach encourages teachers to provide more opportunities for students to explore, discuss, and discover their interesting mathematics problems.

### What about the Result?

The MM Experiment was carried out to cultivate “modern” mathematics teachers, who are not only good at teaching but also are able to associate research about new practices. The results of our experimental study show that students of the experiment class developed new ways to work with mathematics problems and improved their achievement. As has been noted, the MM method on the one hand thoroughly changes students’ situation of passive learning, providing them with an active thinking method. On the other hand, in methodology, middle school mathematics is no longer limited to the elementary mathematics, but connects the mathematics realm into one through application of mathematical methods of thinking and exploration. This develops a vast field for students to experience directly how to think within mathematics science and to participate in modern science cultivation by themselves.

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