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Taiwanese Elementary School Teachers Apply Web-based Virtual Manipulatives to Teach Mathematics

Yuan Yuan
Chung Yuan Christian University, Taiwan

The purpose of this study was to explore how elementary school teachers in Taiwan apply web-based virtual manipulatives to mathematics teaching and issues that arise in these applications. Four elementary school teachers in Taiwan were selected to participate in a case study. They selected applets from NLVM and designed learning activities to apply in class. Teachers and students indicated difficulties in using these applets in an English interface. Based on observations of the cases, it is better to have the activities pre-designed so teachers who wish to use these virtual manipulative can use them more effectively. In addition, thoughtful consideration must be given to the instructional design of the course and the appropriate use of Web-based mathematical manipulatives.

Key Words: elementary school teachers, virtual manipulatives, case study, mathematics teaching.

Physical manipulatives are useful tools in mathematics instruction. They can be used as a media to promote students' concrete thinking (Sowell, 1998) and to help students deepen their conceptual understanding (Schackow, 2007). Manipulatives offer alternative representations within which students can reconstruct concepts. Research has shown that students who used manipulative materials when learning mathematics performed better than those who did not (Raphael & Wahlstorm, 1989). However, implementing physical manipulatives remains to be a problem in real classrooms. These problems include lack of appropriate manipulatives, classroom management issues, and cleaning as well as saving problems.

Moyer, Bolyard and Spikell (2002) proposed the feasibility of generating virtual manipulatives, "an interactive, Web-based virtual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p.373). An appealing characteristic of the innovative technology is its interactivity and its capability to present multiple representations at the same time on the computer screen. It was suggested that "Work with virtual manipulatives …can allow young children to extend physical experience and to develop an initial understanding of sophisticated ideas" (NCTM, p.26-27). However, researchers argued that virtual manipulatives may hinder students from seeing the connection between their actions with the manipulatives and their actions with symbols (Kaput, 1989). Although a growing body of classroom research has begun to emerge on uses of virtual manipulatives as a representation for mathematics instruction, there are conflicting studies regarding the effectiveness of using Web-based virtual manipulatives over traditional approaches (Crawford & Brown, 2003). Research has shown that teachers' perspectives about effective computer-based pedagogy are related to the type of software they use with their students (Niederhauser & Stoddart, 2001; Pajares, 1992; Richardson, 1994). Although there are lots of virtual manipulatives on the Web now, few have researched how school teachers apply these applets to their mathematics teaching.

Theoretical Perspectives

Because of the development of information technology, virtual manipulatives recently became popular products for teaching mathematics. However, applications of virtual manipulatives in school are still at the initial stage. In order to understand the feasibility of using virtual manipulatives in teaching mathematics, theoretical foundation of the usage of virtual manipulatives in mathematics classroom will be discussed in this section.

Virtual Manipulatives as Modeling Tools

In classroom environments, there are two different approaches in using models to solve mathematics problems; "Learning *to* model" and "Learning *with* model" (Durmus & Karakirik, 2006). The learning to model approach focuses on learning how to model the reality. Learners are expected to create their own models when they are confronted with a problem situation. They also use these models to communicate and express their knowledge. On the other hand, the learning with model approach encourages learners to solve problems through the use of ready-made models. Usually, these models are created to address a specific question or situation; learners can solve related problems by changing the parameters in the model. With these models, learners can gain problem solving skills which are the major part of the mathematics curriculum.

Dienes (1961) proposed that manipulatives can serve as a concrete reference for a concept. Although manipulatives might be more abstract than a real situation, it is less abstract than the formal symbols (Post, 1981). Virtual manipulatives, as defined by Moyer, Niezgoda and Stanley (2005), are interactive, web-based virtual representations of dynamic objects which provide learners with visual images. Usually, they can be manipulated as physical manipulatives are and provide students opportunities for constructing mathematical knowledge. Therefore, virtual manipulatives can be used as a modeling tool for learning mathematics. Also, with its interactive features, using virtual manipulatives for teaching mathematics will not be limited to "Learning with model."

Virtual Manipulatives as Representations

In modeling a mathematics problem, representations are necessary for analyzing elements and their relationships in a problem. According to the National Council of Teachers of Mathematics (NCTM, 2000), representations have at least four functions. They are (1) to help learners understand mathematics concepts and relationships; (2) to communicate mathematics thoughts and arguments with other people; (3) to identify connections between similar mathematics concepts; and (4) to model real situations. These functions demonstrate the importance of representations.

However, evidence shows that students encounter difficulties in using multiple representations and in transforming between representations (Lesh, Post & Behr, 1987). Therefore, the ability to use multiple representations and transform between representations is an indicator of whether a student can create a model and understand mathematics concepts. To promote effective teaching, Moyer, Bolyard and Spikell (2002) argued that educators must choose different representations to help students learn mathematics meaningfully. Virtual manipulatives are exactly such tools. When using a virtual manipulative, connections among multiple representations can be shown simultaneously. They convey the various aspects of the same mathematical concepts, and thus can help students construct abstract mathematical concepts.

Purpose

One well developed site can be found in the U. S. is the National Library of Virtual Manipulatives (www.matti.usu.edu/nlvm). The National Library of Virtual Manipulatives (NLVM) is a NSF supported project that began in 1999 to develop a library of uniquely interactive, Web-based virtual manipulatives, mostly in the form of Java applets, for mathematics instruction (K-12 emphasis). Teachers apply these virtual manipulatives into their classroom teaching can help students visualize relationships and engage students learning actively. This study was conducted to understand how elementary school teachers in Taiwan apply virtual manipulatives from NLVM to mathematics teaching and address issues that arise in these applications.

Method

In order to understand the phenomenon mentioned above, a case study was conducted to investigate the research question: Based on the availability of Web-based virtual manipulatives from NLVM, what issues arise when elementary school teachers in Taiwan apply them to mathematics teaching? Four elementary school teachers took a graduate course where I was the instructor. I used two class periods (a total of 6 hours) to introduce ideas and trends of integrating technology into mathematics teaching. The teachers were introduced to the NLVM web site which contains many activities indexed by content strands and grade-level bands. Three applets were introduced thoroughly in Chinese. The introductions included the usage of these applets and designed activities suggested by NLVM. It was my hope that these explorations would help teachers understand how to apply the applets efficiently. The teachers showed interests in applying these applets to their classroom instructions. Sharing and presenting their experience of using these applets in class was one of the course assignments. The teachers selected applets from NLVM and designed learning activities that enrich classroom explorations. The selections of applets were based on their individual interests. In other words, these teachers decided what and how to use a specific applet for their students.

Four weeks later, they reported features of selected virtual manipulatives and discussed their experiences of using them in classroom. Each presentation took about two hours. Their presentations were video recorded and classroom discussions were transcribed to help an in depth analysis of data. Four teachers also provided me the documents that they have collected from their classrooms as further evidences. To protect the privacy of these teachers, pseudonyms are used throughout this report. The explorations included Mei-chi who used “Base Blocks” in grade 2; E-fan who implemented “Base Blocks Decimals” in grade 4; Fong-wei with “Diffy” in grade 5; and Kuo-jen with “Isometric Geoboard” in grade 5. In my graduate course, all of the teachers shared their experiences of using virtual manipulatives in school. The following results and discussions were based on these self-report observations from 4 teachers and classroom discussions.

Results

In order to acquire a better understanding of the teachers’ applications of virtual manipulative from NLVM, these four cases will be discussed individually first.

Case One: Mei-chi used “Base Blocks” in Grade 2

Mei-chi was a second-grader teacher. Two of her students, Yu-jen and Kuo-wha, often had difficulties in learning mathematics. According to the teacher, Yu-jen was not good at paper and pencil tests, and did not like to participate in learning. However, Mei-chi found that Yu-jen was especially interested in multimedia and paid attention to it. Kuo-wha had some developmental problems, and reading is a serious factor affecting his leaning. At the time of the development of NLVM, Mei-chi was teaching a unit called “numbers less than 1000.” She decided to use “Base Blocks” to help these two students. “Base Blocks” consist of individual units, longs (containing 10 units), flats (containing 10 longs), and blocks (containing 10 flats) which are used to show place value for numbers and to increase understanding of addition and subtraction algorithms (see Figure 1).

Before using this applet, these two students could only complete partial portions of the test questions with Yu-jen completing 75% and Kuo-wha at 25%. After using this applet in learning, their test completion rates increased to 100% and 75% respectively. Mei-chi described,

> When Yu-jen was using “Base Blocks”, I saw a smile on his face. Mathematics was not something he did not understand nor was it simply numbers and graphs. Instead, it was a game and he was interested in playing it. After these explorations, he completed his test quickly and was more

willing to do it.

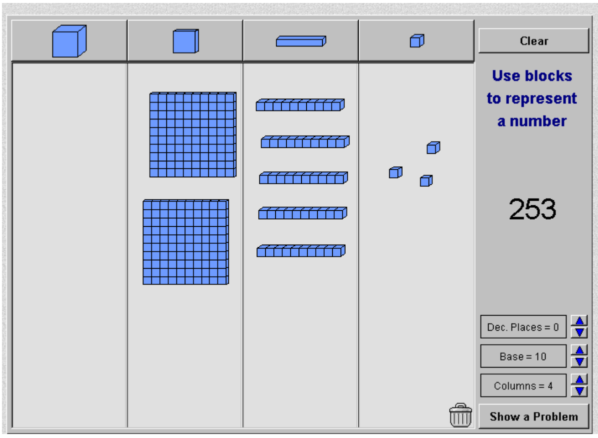

Figure 1. **"Base Blocks" shows place value for numbers.**

On the other hand, Kuo-wha did not display similar enthusiasm as did Yu-jen. This is possibly due of his reading problems. He needed more time to complete his tasks, but under his teacher's guidance, he completed most of his test problems himself. Mei-chi reported,

> I knew integrating technology into learning is important and is a new trend, but this is the first time I knew there is a wonderful resource like this. I used it with these two students, and I found it reduced their fear of mathematics. I am happy to find a new way to improve their leaning of mathematics. Through seeing graphs, visual stimulus and manipulations with this virtual manipulative, mathematics is much easier to understand for these lower achievers. They are great!

The only problem of using this applet in instruction was its English interface. Mei-chi indicated that when using these applets the teacher would need more time to explain their functions and students needed more time to learn about unfamiliar icons. She believed that if they were in a Chinese interface, they would definitely be more acceptable by school teachers and students.

Case Two: E-fan used "Base Blocks Decimals" in Grade 4

E-fan was a fourth-grader teacher. The students in her class already knew the relationship between 0.1 and 0.01. They also learned how to represent 0.13 on a 10x10 grid. Before using “Base Blocks Decimals,” the students in her class have explored some basic decimal concepts with 10x10 grids. These explorations included the comparisons of 2 decimal numbers, the relationships between 2 decimal numbers, and additions and subtractions of decimal numbers. After these initial explorations, E-fan found that her students had some misconceptions. For example, 10 of 35 students considered the number 4.68 was bigger than 4.8. They computed the sum of two decimals by lining up the right-hand digits, as they would with whole numbers. E-fan decided to use “Base Blocks Decimals” to help students clarify these concepts.

From E-fan’s point of view, using the applet to present place value concept is neat. In the previous classes, she remembered her students’ tables were full of 10x10 grids and students did not pay attention to her explanations. Students also reported that they liked to use this applet. They thought it was fun and convenient. They did not need to worry about losing paper grids. In addition to these, they had the chance to practice computational skills. However, problems occurred when using it in instruction. E-fan found students had problems choosing 10 units at a time, especially when there were units near the corner (see Figure 2.)

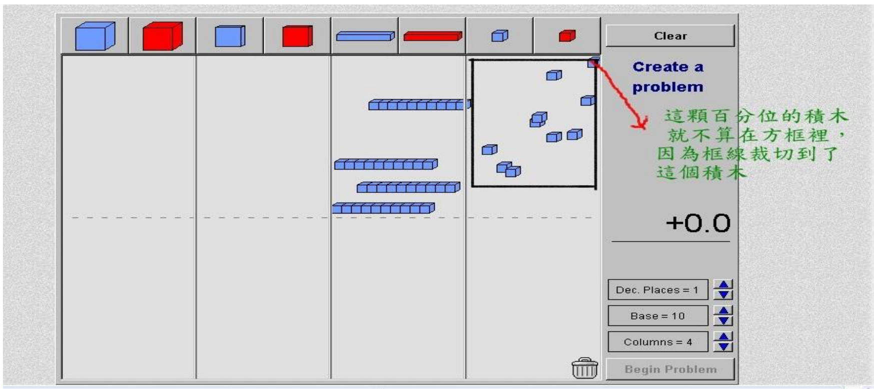

Figure 2. **Students had problems choosing a unit in the corner of screen.**

The other problem occurred when graphs were overlapped. Students often had problems separating and counting them correctly (see Figure 3).

E-fan also discovered new misconceptions students have after using this applet. For example, when asked how many 0.01 equals to 0.1? Students gave the correct answer, but the graph they drew on paper was a reflection of the representation that they saw from this applet. Figure 4 gives an example that students used a cube to represent .01. However, a cube is usually used to represent 1 when students were first introduced the blocks to represent numbers.

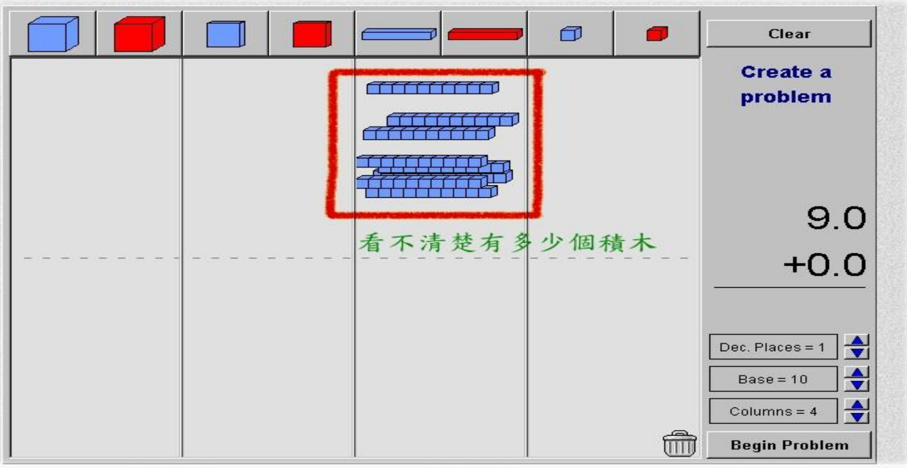

Figure 3. **Students had problems figuring out the number of flats on the monitor.**

E-fan also mentioned that the interface might affect students' learning. For example, the applet used 1/100's to represent the hundredths and students were not familiar with this representation. She also suggested the interface should be in Chinese, so students would find it easy to understand. Even with the difficulties of understanding the meaning of icons, almost all of her students (34 of 35) shared their love of learning with "Base Blocks Decimals".

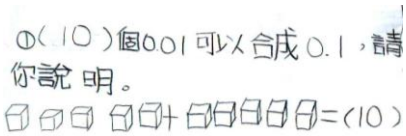

Figure 4. **Student used a cube to represent 0.01.**

Case three: Fong-wei used "Diffy" in grade 5

Fong-wei was a fifth-grader teacher. There were two gifted students in her class, so she decided to find a challenge activity from NLVM for them to explore. "Diffy" is a virtual manipulative selected for number and operations for 3rd-5th grade students. It is designed to give students a chance to solve an interesting puzzle involving the differences of given numbers (see Figure 5).

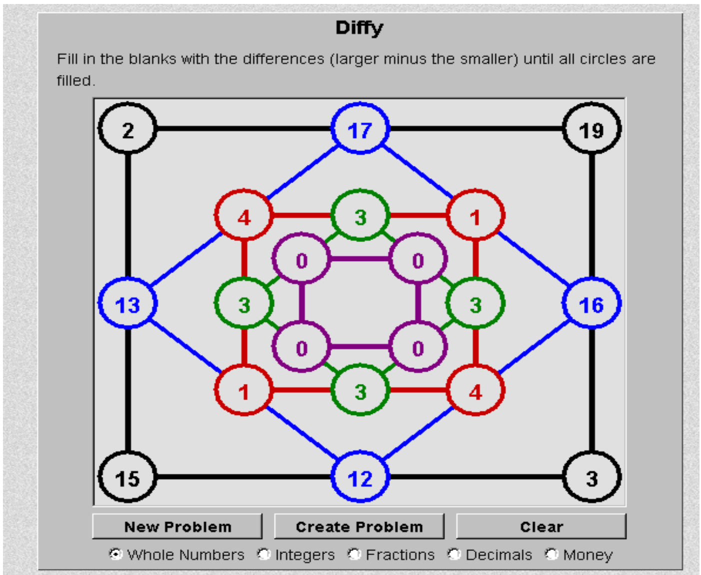

Figure 5. **"Diffy"- a puzzle for exploring patterns.**

Two students were introduced this applet and asked to find all possible arrangements of 4 numbers between 1 to 10 to make 7 squares where the inner square has four puzzle numbers that are all the same. Figure 6 gives an example of the result.

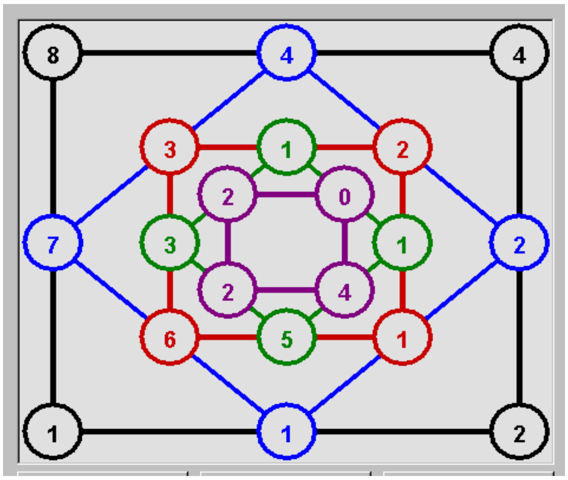

Figure 6. **An example that satisfies the requirement.**

Although figure 6 showed an example of this result, this graph did not complete the works. There should be another two squares with number 2,4,2,0 and 2,2,2,2. That is, the teacher asked the students to explore a problem that the applet cannot show completely. Fong-wei reported that these two students were not interested in using this applet. They preferred drawing on paper. These two students were interested in finding answers and they did have some amazing findings. For example, after a few explorations they found that they can always find a square with 4 numbers with the same. They spent a lot of time drawing squares and filling squares with numbers. However, there was a lack of pattern explorations.

While this activity did motivate students' experiences in exploring questions different from the textbook, the applet was not used appropriately. If the teacher could spend a little more time reading the instructions on the Web, she might design an appropriate activity for these two students. For example, there are two questions suggested for users: (1) What kinds of patterns can you see in the purple numbers when you complete Diffy? (2) Will you always get some zeros in the purple numbers, or a set of four purple numbers that are all the same, or two pairs of matching purple numbers? When a teacher proposes appropriate questions for students to explore, they may help students focus on what the teacher wants them to find.

Case four: Kuo-jen used "Isometric Geoboard" in grade 5

Kuo-jen was a fifth-grade teacher. He was teaching the concept of polyhedron. Before using **"**Isometric Geoboard**",** the teacher conducted a pretest to assess students' understanding of polyhedron. After pretest, the teacher asked students make different polyhedra with the **"I**sometric Geoboard**".** In class consecutive class periods, students went to the computer room and made polyhedra by themselves. The teacher designed tasks for them to explore and asked students to complete tasks without any lectures in class. Kuo-jen mentioned that he was so surprised that his students never worked hard like this before. After this activity, students were given a posttest. The test mean score increased from 79.54 to 89.49. Using pair sample t test to test the difference, there was a significance of difference between pretest and posttest ($t=-7.429$, $p<.001$, $N=35$). It showed that students' test score improved after the instruction. Thirty-three of 35 students responded that using this applet helped them learn this unit. Twenty-eight students described their love of using a computer to learn mathematics.

Kuo-jen reported his experiment in class and the results generated excited discussions among students in my course. Although students' test score were improved, the teachers in my course argued that this could be a learning effect. No matter what strategies the teacher used, students should learn from the instruction. So the results were not surprising. All teachers in my course gave ideas for using this applet. In our experiences, students liked to learn using computers. Whether they like using computers or learning mathematics from computers was not clear. In discussion of this application, we believed that "Isometric Geoboard" could be used more efficiently.

Discussion

Based on four teachers' application of the applets from the web resources, it seems that lower grade teachers were more likely to use skill-based applets than were teachers at the higher grade levels. This finding is consistent with the study results of Niederhauser and Stoddart (2001). However, when higher grade teachers selected more open-ended applets as their instructional tools, those applets were not appropriately integrated into instruction. They were more likely to be used as drill-and-practice software. Many of the open-ended applets are designed to help teachers design more student-centered activities, but this is not the case in this study.

In this study, E-fan was afraid that new misconceptions might occur when using these applets in real classroom. Kaput (1989) argued that virtual manipulatives may hinder students from seeing the connection between their actions with the manipulatives and their actions with symbols. This phenomenon is a special characteristic of most of the virtual manipulatives. Although educational technology is a tool that can be used to support a variety of approaches of instruction, possible drawbacks should be studied. This information is important for school teachers and it should be a continuous research focus in the future.

Four teachers were interested in exploring the use of virtual manipulatives to teach mathematics. The results from this study showed that virtual manipulatives could be used differently in mathematics instructions. Without any discussions in advance, the teachers in this study used virtual manipulatives differently. Applets on NLVM did provide teachers a great resource for teaching mathematics with different purposes. They used virtual manipulatives with students of with different abilities – lower achievers and high achievers. They also used virtual manipulatives to introduce new concepts and to enrich deeper understanding of a concept. Many studies indicated that teachers' belief systems exert a powerful influence on their curricular decision-making and on the instructional practices they use in their classrooms (Niederhauser & Stoddart, 2001). The application of instructional technology should be reflections of these teachers' belief systems. The teachers are happy to know the ideas of applying virtual manipulatives in teaching mathematics. This motivates their actions in trying to use virtual manipulatives in their classrooms. However, lacking a good "modeling" and training might decrease the effectiveness of the applications. In this study, Kuo-jen believes that students can learn more from manipulating those virtual manipulatives. Fong-wei considers "Diffy" a wonderful challenging activity for gifted students, and asks them to explore patterns from playing. In these two cases, virtual manipulatives are used only as tools to solve problems. It seems that the teachers need help to learn how to match computer use with their instructional goals. This also should be the focus of professional development of teachers.

When presenting their application results in class, it is evident that these teachers did not read the instructional suggestions provided by designers of virtual manipulatives. They seemed to spend a lot of time designing activities by themselves. On one hand, this is good because they did prepare for the lessons. On the other hand, this also showed that the teachers did not adopt good ideas to apply these applets to their classroom teaching. These teachers seemed to have difficulties with reading instructions in English, even though they are graduate students. Reading in a foreign language may be an additional endeavor for teachers and students. There is a need for translating these useful virtual manipulative Web sites into multiple languages. School teachers have neither the time nor the skills to produce their own educational materials. It would be more beneficial for teachers to have readymade activities, so teachers may use them effectively.

Conclusion

Numerous virtual manipulative Web sites are currently available in many countries. Many advantageous properties about virtual manipulative had been discussed. They are "the potential for alteration", "interactivity", "link symbolic and iconic notation", "unlimited supply", and "easy to clean" (Moyer, Bolyard, & Spikell, 2002). Using virtual manipulatives in instruction is a new trend of integrating technology into mathematics teaching and learning. It has been believed that instructional technology can be used to support student-centered inquiry-based learning (Cohen, 1990). However, these tools do not embody a student-centered instructional paradigm. In the implementation of the innovation, thoughtful consideration must be given to the instructional design of the course and the appropriate use of web-based mathematical manipulatives.

This paper pointed out issues that might arise when elementary school teachers in Taiwan apply Web-based virtual manipulatives to mathematics teaching. It was found that teachers need to be more aware of how these Web-based virtual manipulatives can be used to help their students meet a range of instructional objectives. Knowledge and beliefs of effective ways of using virtual manipulatives to meet a variety of instructional goals should be carefully integrated into teacher training and professional development. As an initial exploration we only selected four teachers participated in this study and results from this study were based on teacher self-reported data. Future research should be conducted to include more teachers to understand teachers' instructional perspectives on the use of virtual manipulatives in teaching mathematics. To get important and sufficient information, it is a need to have the researcher enter the classroom and observe students' work in class.

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Authors:

Yuan Yuan

Chung Yuan Christian University, Taiwan

Email: yuan@cycu.edu.tw