Engaging In-service Teachers in Mathematical Problem-solving Activities during Professional Development Programs

Asmamaw Yimer
Chicago State University, U.S.A.

Consistent with the broad research on teachers’ lack of deeper understanding of the mathematics they are teaching, in-service teachers who participated in the problem-based professional development have been observed lacking such qualities as conceptual base, problem-solving skills, and self-confidence about teaching mathematics. As a result of their participation, they have been observed developing positive attitudes about mathematics teaching and learning in general and in taking risks in problem solving in particular. Furthermore, in their reflections, participants revealed their confidence in ability to lead mathematical discourse, challenging their students and being challenged by their students, and developing mathematical autonomy in their students.

Key words: professional development, problem solving, in-service teachers, confidence.

Introduction

Although there are several factors that affect student achievement, educators believe that teachers play an important role in helping students improve their achievement in mathematics (Hill, Rowan, & Ball, 2005). This role of teachers in student achievement forced educators and school administrators to focus on several professional development programs that are geared towards equipping teachers with skills in specific aspects of teaching. These professional development programs focus on diverse issues. To mention a few, issues include developing teachers’ listening skills, developing and selecting appropriate tasks, how to lead discussion, how to lead and encourage problem-solving performances, and how to use manipulatives and other teaching aids (Mewborn, 2003; Skibba, 2008; Wu, 1999).

Instructional practices that seemed to contribute to student’s achievement include time for students to work on problem-solving contexts,
opportunities for students for discourse, adapting instruction to the level of students (Mewborn, 2003). Others viewed that children learn more mathematics if instruction is based on students’ thinking, engaging students with rich problem-solving contexts, and helping students see the relationship between various concepts and their corresponding symbolism (Gearhart et al., 1999). On the other hand, Wu (1999) argued that teachers who did not themselves problem solve would not be able to lead their students to a problem-solving environment. Wu further argued that, “you cannot teach what you don’t know, but what mathematics teachers do is teaching what they themselves had not a deeper understanding” (p. 535). In other words, teachers only teach the same way they have been taught. This, according to Wu, precludes “the possibility of good teaching since one cannot teach what one does not know” (p. 535).

Research reveals that teachers’ mathematical knowledge matters for student achievement (Hill & Ball, 2008; Ball & Bass, 2000; Hill, Rowan, & Ball, 2005). But what is not conclusive and seems to require more research is what subject matter knowledge is. Subject matter knowledge for teaching has often been considered to be the subject matter knowledge that students are to learn. In other words, many assume that what teachers need to know is what they teach. This assumption of what teachers’ subject matter knowledge is, according to Ball and Bass “has blocked the inquiry needed to bring together subject matter and practice in ways that would enable teacher education to be more effective” (p.95).

Several justifications have been given to the fact that teacher’s mathematical content knowledge matters. Hill, Rowan, & Ball (2005) argue that flexibility and ability to choose representations are qualities teachers need to possess. Others suggest many other qualities. For example, Ma (1999) used an interesting metaphor that teacher’s knowledge of mathematics for teaching being like an experienced taxi driver’s knowledge of a city who can get to significant places in a wide variety of ways, flexibly and adaptively.

The NCTM (2000) addresses these practices thereby suggesting professional development practices and directions. However, these visions need to focus on how these desired practices could happen. Otherwise, good intentions and visions remain in paper being just attractive. Cooney (1988) stated on such plans that “reform is not a matter of paper but a matter of people” (p. 355).

It is striking to read the comments from prospective teachers as they are asked to solve mathematical problems or as they engage in reflecting on
their teaching. In many cases, these teachers are fully aware that they lack a conceptual understanding of mathematics. For example, one student teacher noted, “I don’t just like saying ‘Well, this is pi. Remember it,’ … [but] where does pi come from? Well, I don’t know.” (Eisenhart et al., 1993, p. 18). Another preservice teacher noted, “I am really worried about teaching something to kids I may not know. Like long division – I can do it – but I don’t know if I could really teach it because I don’t know if I really know it or know how to word it” (Ball, 1990, p. 104).

**What Should Professional Development of In-service Teachers Be?**

There seems to be evidence that teachers, preservice or in-service, have a strong command of procedural knowledge, but lack a conceptual understanding of on what the procedures are based on (Mewborn, 2003). It is unfair and being naïve to expect teachers who do not have conceptual understanding themselves to help students build a conceptual understanding of mathematical ideas and principles. Nor do those teachers who have not been involved in inquiry and problem solving would lead students to be autonomous and thoughtful.

Professional development programs for teachers should focus on bridging the gap between the weak mathematical background of teachers and what is expected from them in order for their students be autonomous and problem solvers. However, recent professional development in mathematics, according to Wu (1999), is essentially a race against time. Such rushed programs or those taking place once in a while do not bring about the change we want teachers to have. Instead, according Wu, attention will be on “bread-and-butter” topics in school mathematics.

Consequently, professional development programs should be content-focused (Hill, Rowan, & Ball, 2005) and focus on exposing teachers to the process of doing mathematics (Wu, 1999). This is especially useful for teachers with weak subject matter knowledge as well as in-service teachers who have been teaching for a long time. This, according to Hill, Rowan, and Ball is an investment in quality of mathematical content knowledge of teachers.

In this paper we will present professional development programs that are geared to engaging in problem-solving activities enhanced the self-efficacy of in-service teachers. The tasks in-service teachers engaged in helped to develop confidence in taking risks and contributing to classroom discussions.
Their written reflections revealed that these teachers not only became reflective but also develop their self-efficacy in helping their students in a better way.

**Theoretical Perspectives**

Bandura (1986), in his social cognitive theory, provided a model for self-regulated learning where personal, contextual, and behavioral factors interact to give learners an opportunity to control their learning. Drawing on social cognitive theory, Pintrich (1999) describes self-regulated learning as an active, constructive process whereby learners set goals for their learning, plan actions, and monitor and regulate their cognition. These actions can mediate the relationships between individuals and the context and their overall achievement (Zimmerman, 2000).

The rationale behind engaging learners in a constructive process of learning is that we cannot impose ideas on learners and achieve understanding. Mewborn (2003) summarized the relevance of engaging teachers in professional development programs as “…one cannot expect teachers to change their teaching practices simply because they have been told to do so” (p. 49). On the other hand, Lappan and Briars (1995) characterized learning during professional development programs to be contextual; that learning occurs through dialogue, discussion, and interaction; that learners must be actively involved; that a variety of models be used; and that learners be engaged in reviewing, critiquing, and revising one another’s work. Mousoulides and Philippou (2005) described self regulation as students’ ability to setting goals planning activities, monitoring progress, controlling, and regulating their own cognitive activities. This description is what Garofalo and Lester (1985) viewed as metacognitive aspect of mathematical problem solving.

These perspectives guide this study from the design of the lessons, the strategies employed in classroom interactions, and in identifying the behavioral changes participants exhibited as the lessons progressed. In particular, the effort participants exhibited their willingness to take risk, their pride in discovering a sense-making representation, and their confidence in justifying their actions and critiquing others’ works are identified and situated to self-regulated learning.
Data Sources

During the summer of 2005, 42 teachers from two school districts in the upper Midwest who teach mathematics in grades 5-10 participated in a two-week intensive refresher course. They also participated in a 15-day, over two months duration, follow-up pedagogically-focused course. The grant was sponsored by Cooperative Educational Service Agency (CESA). CESA paid an incentive to each participant. In addition to the incentive, some of the teachers received graduate credits for the renewal of their certification process.

Participants were diverse in their educational background. Five of the participants have B. A. majoring in mathematics, seven have B. A. as mathematics minor, and 22 have B. A. majoring in Elementary Education.

A needs assessment has been done on participating teachers to identify their needs and priorities of mathematical areas. Participants were asked to list content areas in mathematics in which they have difficulties and would like to be addressed during the lessons. They were also asked to list areas of difficulties their students have. Nearly all teachers (with the exception of two who are teaching in grades 9 and 10) mentioned fractions, measurement, and geometry in order of their priorities.

Problem solving was the main focus not only because it was the goal of the grant, but also because of its potentially rich domain to study self-regulated learning (SRL) due to demands of cognitive and metacognitive skills (Panaoura & Philippou, 2003). Based on the needs assessment result, problems were designed in the five content strands (number and number operations, algebra, geometry, measurement, Data and probability). Major emphasis was given to fractions, measurement, and geometry.

For the grant purpose, all sessions have been videotaped. The cameraman moved around group discussions alternatively and recorded presentations in front of the classroom. There were two assistants who have known these participants and have worked with them. The data comes from observing classroom discussions, watching the videotapes, and participants’ written reflections.

Results and Discussion

After solving problems in addition, subtraction, and multiplication of fractions, the division of fractions received relatively more time. The measurement form of division both in whole numbers and fractions has been discussed extensively. Participants developed an interest not only in solving
problems, but also in identifying critical features of the problems. After participants showed confidence in handling measurement problems, the discussion moved to solving division problems that are partitive in nature.

In solving division problems that are partitive in nature, participants attempted to solve using the strategy that they used in solving division of measurement type. For example, in the following two problems, they used the measurement model of solving division problems.

**How many apples would each child get if three children share 12 apples equally?**

Earlier, a word problem that can be modeled by $12 \div 3$, but is measurement type has been discussed. The problem was “How many bags do I need to put 12 apples if I want to put three apples in a bag?”

The following shows discussions that took place. All names are pseudonyms.

[It was immediate that every one answered $12 \div 3 = 4$]

Teacher: Is this problem the same as the one we just solved?

[Most of the participants agreed that they are the same.]

Teacher: Can I use

1. $12 - 3 = 9$
2. $9 - 3 = 6$
3. $6 - 3 = 3$
4. $3 - 3 = 0$

Hence $12 \div 3 = 4$?

[Everyone agreed.]

This was the procedure that has been used in solving the measurement form of $12 \div 3$ earlier. Participants, knowing that the mathematical statement is $12 \div 3 = ?$, they imitated the strategy they saw before in the existing partitive problem.

Teacher: What are 12, 3, and 9 in $12 - 3 = 9$?

Chris: 12 apples, 3 children, and 9 apples.

Teacher: Does it make sense to you? What does it mean to subtract three children from 12 apples?

Teacher: How do children solve such problems?

Mary: They give out one apple to each child until they ran out.

Participants became aware of the difference between problems that are
measurement and partitive in nature. But they still rely on imitating the solution strategies they used for measurement forms in solving division of fractions that are partitive in nature.

Participants recognized that the following problem is partitive form. Participants were asked to solve the problem in a variety of ways.

I can walk 3/2 miles in 2/3 of an hour. How long can I walk in an hour? Do not convert in minutes or seconds.

David: I can set up a proportion: If 3/2 is to 2/3, then x is to 1.
Then cross multiplying, (2/3)X = 3/2. If we multiply both sides by 3/2, we get X = 3/2 x 3/2 = 9/4.
Bob: Shouldn’t we divide both sides by 2/3?
David: It is all the same. Dividing by 2/3 and multiplying by 3/2 have the same effect.
[Bob agreed with the equivalence of the two actions.]
Teacher: What will happen if we divided by 2/3?
David: We get X = 3/2 ÷ 2/3.
Teacher: Good! How do we divide 3/2 by 2/3?

Earlier, they solved a measurement form of division, 3/2 ÷ 2/3, by asking “how many two-thirds exist in three-halves?” They came up with variety of attempts to represent the solution. They have identified a pictorial representation of the solution that appeared to be reasonable to them.

Participants were asked to think about a pictorial solution within their groups for five minutes. Most groups imitated the measurement concept of division and drew a measurement model of division of fractions.

Teacher: It seems that everyone used the measurement model. Specifically, I saw everyone asking “how many 2/3 are there in 3/2?” Am I right?
Trish: I think that is right.
Teacher: Does it make sense to ask “how many 2/3 of an hour exists in 3/2 of a mile?”
[Participants were reminded that this is a partitive model of division and to work in groups to come up with a meaningful drawing that has a solution for 3/2 ÷ 2/3.]
Participants were able to come up with a solution by setting up proportions and by using strategy they used for measurement type problems. This is consistent with Wu’s (1999) argument that students rely on imitating solution methods when they do not know how to solve a problem different in nature.

**Sample Pictures for Solving the Problem**

One group shared the following drawing shown in Figure 1 to represent their solution.

![Figure 1. A group’s pictorial representation.](image)

A group member explained that the whole box represents one hour. If you see the two columns to the left, they represent the 3/2 miles walked in 2/3 of an hour which means that in 1/3 of an hour, you can walk 3/4 of a mile since there are four parts in one mile and 1/3 of an hour corresponds to 3/4 of a mile. As one can see, in one hour, you can walk 2 miles and 1/4 of a mile. That is 9/4 of a mile. The column to the right is the distance you can walk in 1/3 of an hour in addition to the 2/3 of an hour.

[The class applauded with appreciation.]

Teacher: Does it make sense?

Everyone nodded.
Participants were given time to talk about the pictorial solution presented and if they can come up with an alternative drawing. In addition, they were asked if the algorithm they know is visible in their drawing. This took a long time. They were convinced that the algorithm has been seen as an action on their drawings, but could not see this action in their drawing this time.

Trish: I drew two different wholes one for distance and one for time. I can see the solution in my picture but I cannot see where the invert and multiply existed in the picture. [She came forward and displayed her drawing as shown in Figure 2.]

![Figure 2. Trish’s pictorial representation.](image)

I can see you can walk 2 miles and small piece.
Teacher: How big is that small piece? Where do you get that small piece?
Trish: It is half of half of a mile. Four parts represent one mile and the piece I am talking about is half of half of a mile.
Teacher: What is half of half of a mile?
Trish: I got it! It is ¼ of a mile. You can walk 2 ¼ of a mile.
Teacher: What happened in her picture? Can someone describe her actions?
Susan: It seems that she divided the 3/2 into two equal parts. I think she wants to make a correspondence between each of these parts and
the shaded parts of the hour. Since she wants to know how long you
can walk in one full hour, she needs to add one more part of half of
the 3/2.
Teacher: Can you (or someone else) put these actions she described in
symbols?
No one seemed to understand.
Teacher: What did she do with 3/2?
Brad: She divided it into two equal parts
Teacher: Could it mean \( \frac{1}{2}(3/2) \)?
Everyone agreed.
Teacher: Then what did she do?
Kelly: She added one part of half of 3/2.
Teacher: All in all how many parts of \( \frac{1}{2} \) of 3/2 were in the answer?
Beth: I think three.
Teacher: Do you all agree?
It seems that every one saw there were three parts of \( \frac{1}{2} \) of 3/2.
Teacher: Can I write it as \( \frac{1}{2} \times (3/2) \)?
Bob: Why do you multiply by 3?
Teacher: Can anyone answer this? Why did I multiply by 3?
Trish: Since all the three parts are of the same size, adding three same
parts and multiplying one part by three are the same.
Teacher: Do you agree?
Everyone seemed to agree.
Teacher: Is \( \frac{1}{2}(3/2) \times 3/2 \) the same as \( 3/2 \times 3/2 \)?
Everyone nodded.
Teacher: Do you see inverting and multiplying as an action on the
drawing now?
[Everyone was talking within ones group, the class was louder than
ever. Everyone seemed to talk about the picture and some were
redrawing it and explaining to one another.]

Participants welcomed the challenges everyday. They were willing to
take risks and try drawing a picture. They shared how they thought about a
problem, their attempt in solving a problem, and whether their solutions make
sense or not. Sometimes, they shared their attempts even if they do not have a
solution. They were willing to take risks that were nonexistent during the first
two days.
They have been reflective of the relevance of the problem-based learning to understand the material conceptually on their part as well as what they will do differently for their students. Their reflections showed that participants have modified their beliefs about teaching and learning mathematics as the following reflections reveal.

It is still challenging. You (referring to the teacher) let me place myself in my student’s shoes. I cried a lot because of my misjudgment on my students. I can feel their pain now. I do not think I helped my students. I just like to make their days smooth. I can see the disservice I did to my students. I will not give up no matter how the coming days are difficult. (Chris).

I wish all teachers teaching math took part in this program. This is a different professional development program. We saw why formulas work. I haven’t seen anyone explaining why we invert and multiply. I was told that at grade school, I was practicing it at college, and I let my students repeat what I did. How could I explain this to my students when I, myself, do not see one? (Brad).

A teacher who had been very emotional during the first two days wrote the following in her reflection:

I have been very emotional the last two days because I could not see myself as a learner. I thought I was doing very well for my students. I thought I know what I teach until this moment. I failed to be open-minded to explore what is at the table. When I saw pictures and explanations about division of fractions, I saw the whole idea of the meaning behind each principle. I think it is time to say hallelujah! (Nancy).

The above reflection is consistent with the belief one has about teaching and learning as well as the view one has about his/her own success on problem solving tasks. According to Pintrich (1999), task value beliefs are correlated to performance.

The teachers were relaxed during the remaining days. They were willing to share their thoughts in public whether they arrived at a solution or not, whether they are right or wrong, or whether others in their group agreed or not. By doing so, they have developed their confidence that teaching and
Engaging In-service Teachers in Mathematical Problem-solving

learning is about sharing thoughts, discussing and modifying ideas, and justifying findings and solutions. This is consistent with what Graven (2004) called confidence as part of an individual teacher’s way of learning.

It has been observed that the belief teachers have about themselves and about problem solving has an impact on their performance. This is consistent to the nature of self-regulated learning described by Pintrich (1999) and Wolters and Rosenthal (2000). It is supported that high self-efficacy functions as an incentive for the pursuing of a goal; on the contrary, low self-efficacy functions as a barrier that urges to avoiding the goal (Hamilton & Ghatala, 1994; Seiferd, 2004).

The following reflections reveal participants’ development of positive dispositions about teaching and learning as well as their determination to change their classroom instructions.

I cannot wait to see my students with confidence and better power of mathematics more than ever. Now, I will not shy away from taking risks and letting my students explore mathematical ideas. One of my biggest fears was what if a child challenges me. Now I am equipped for such challenges. (Trish).

Another teacher wrote:

I do not want to apologize to my students out loud for the disservice I have done to them. But I did it with all my heart inside me. I made my promise to my self to serve my students better. I have now opened my eyes. Ready-made knowledge is not helping them at all. (Susan).

The above two reflections are consistent with what Hamilton and Ghatala (1994) called intrinsic goal orientation that is concerned with the degree to which a learner perceives himself to be participating in a task for reasons such as challenge, curiosity and mastery, using self-set standards and self improvement.

Conclusions

The literature and the data are in harmony in revealing that teachers do not have a conceptual understanding as well as the confidence to explain it to their students. What would these teachers do in their classrooms? It is
unrealistic to expect these teachers to develop the mathematical power we envision children to have for they (teachers) themselves do not have not one. These teachers will focus on providing to their students readily available rules and focus on skill manipulation. It happens that students will try to imitate the rules as demonstrated by their teacher. If a student gets confused about which one of the fractions to invert while dividing fractions, for example, such teachers will not help except, of course, repeating the rule over and over again until the student seemed to get it right as described in one of the reflections (Brad).

It has been very frustrating for teachers to consider themselves as learners. In fact, it is difficult to expect changes when these teachers were engaged outside their comfort zones. Teachers need to see a sense-making discussion to appreciate the newer approach, problem solving. As has been observed and described in their reflections, teachers moved from imitating strategies they have seen in similar problems to inventing their own and trying multiple representations. They were ready to challenge others’ ideas and their ideas be challenged by others. This was a shift from being defensive to being open for exploring new avenues, a shift from algorithm-motivated drawings to meaningful representations, and a shift from old beliefs about mathematics teaching as being teacher-centered to viewing students at the center of instruction.

The teachers gradually have been engaged in classroom discourse. They were willing to share their thoughts about specific problems even if they do not have readily available solutions to the problem. They were willing to try to show their solutions even if they knew that their solution is wrong. As a result, they developed the feeling that mathematical ideas are developed as a result of mathematical discourse within the community of learners. This risk-taking phenomenon was crucial for mathematics teachers. They will likely encourage their students to take risks and be open to face challenges.

**Reflections**

It is true that research as well as practice is growing with time. New ideas emerge to better handle classroom teaching. These new findings and practices are often times communicated with teachers in the form of professional development programs. This approach is analogous to painting a crack on a wall. Sure enough, the paint will cover the crack for a while. But when the paint is dry, the crack will be seen again. After a professional development program, some teachers practice what they have learned with
their students. They repeat what they did in their professional programs. But when they come across a different situation, they go back to what they used to do.

It is understandable and is appreciative that resources are invested on the professional development of teachers. It is through such an occasion that new ideas can be shared. It is refreshing for teachers to see alternative strategies. But these actions do not solve the real problem – sealing the crack. We cannot substitute the knowledge gap that exists with the sharing of few strategies. One should remember that these strategies that may be shared during professional development events may not work for every class.

Teacher preparation in colleges needs to change for the better. What good will new graduates (fresh teachers) do if they are not confident in what they do in their mathematics classes as students? Such teachers will not have the confidence to guide students to discovery when they themselves have not been engaged in one, these teachers will not engage their students in problem solving activities when they themselves have not done one, these teachers will not focus on conceptual development of mathematical ideas when they themselves have not seen one.

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Author:

*Asmamaw Yimer*

*Chicago State University, U.S.A.*

*Email: ayimer@csu.edu*