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Using the MSA Model to Assess Chinese Sixth Graders' Mathematics Proficiency

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This study assessed 491 Chinese sixth graders' mathematics proficiency reflected in conceptual understanding, procedural fluency, and competence in word problem applications using the model – strategy - application (MSA) approach. The One-Way Within-Subjects (Repeated Measures) ANOVA and Pearson Correlations were used in analyzing student proficiency in fractions and decimals. The results showed that Chinese students' procedural fluency was at a higher level compared to their conceptual understanding and word problem in real-world applications. The results revealed that a higher level of computation did not lead Chinese students to a deep understanding of fractions and decimals. The study suggested achieving a balanced way of teaching and learning mathematics through the MSA approach.

Key words: mathematics proficiency, conceptual understanding, procedural fluency, real-world application, visual models, representations, word problems, computations.

Introduction

In recent years, various comparative studies have revealed notable differences in student mathematics learning between the U.S. and Chinese students. Most of these studies show that the mathematical performance of Chinese students is higher than that of their U.S. counterparts (Cai, 2000, 2001; Lapointe, Mead, & Askew, 1992; Stevenson, Lee, Chen, Lummis, Stigler, Fan, & Ge, 1990; Zhou & Peverly, 2004; Zhou, Peverly, Boehm, & Lin, 2000). However, the question of whether Chinese students, though fluent in computations, have a better understanding of mathematics remains discernibly unanswered in current research.

Learning mathematics with understanding is an important issue discussed in a variety of studies that have advocated a vital goal in teaching mathematics: To sharpen students' mathematics proficiency, students must learn mathematics with understanding (Carpenter & Lehrer, 1999; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997; National Research Council [NRC], 2001; Shafer & Romberg, 1999). NRC (2001) and RAND Mathematics Study Panel ([RAND], 2003), for example, outlined student mathematics proficiency consisting of the following five aspects: 1) conceptual understanding; 2) procedural fluency; 3) problem solving in applications; 4) adaptive reasoning; and 5) productive disposition. These five aspects are connected with and build on each other because when students acquire knowledge with understanding, they must apply that knowledge to learn new topics and solve new and unfamiliar problems (Carpenter & Lehrer, 1999).

Considering conceptual understanding, procedural fluency, and problem solving in applications as core components in mathematics proficiency (NRC, 2001; RAND, 2003), this study integrated these three components in a connected knowledge base that was used as a fundamental framework to assess

Chinese student mathematics proficiency with a focus on assessing their understanding and its relationship to fluency in computation and application in word problems.

Theoretical Framework

Learning Mathematics with Understanding

In order to learn knowledge and skills, retain them, and be able to apply them to solve problems, students must learn with understanding (Carpenter, Fennema, Fuson, Hiebert, Human, & Wearne, 1999). According to Carpenter & Lehrer (1999), understanding is the process of mental activity consisting of five parts: 1) constructing a relationship; 2) extending and applying mathematical knowledge; 3) reflecting on experiences; 4) articulating what one knows; and 5) making mathematical knowledge one's own. These five components are consistent with the NRC and RAND's two main components in mathematical proficiency - developing conceptual understanding by constructing a relationship (1), and building competence in applying knowledge to make one's own problems (2 & 5), which are included as two of the main criteria used to measure Chinese students' mathematics proficiency in this study.

Various researchers have conducted studies using one or more of the five parts of mathematics proficiency to help children learn mathematics with understanding. Conceptually Based Instruction (CBI), for example, directed by Hiebert and Wearne (1993, 1996), focused on building relationships using different representations, which are used as tools for solving problems, demonstrating and recording children's strategies for solving problems, and communicating their strategies; Cognitively Guided Instruction (CGI) developed by Carpenter, Fennema and Franke (1996) focused on teachers' understanding of children's mathematical thinking; Supporting Ten-Structured Thinking (STST) project conducted by Fuson (Fuson & Smith, 1995) provided explicit guidance and support for making connections between representations; The Problem-Centered Mathematics Project (PCMP) focused on computational procedures that build directly on children's number concepts and their knowledge of properties of number operations (Murray, Olivier, & Human, 1992). The findings from these projects show that "When students learn skills in relation to developing an understanding, not only does understanding develop, but mastery of skills is also facilitated" (Carpenter & Lehrer, 1999, p. 31). Although each of these projects has helped children understand mathematics in different ways (Carpenter et al., 1999), "in reality they are not isolated but integrated" (Fennema, Sowder, & Carpenter, 1999, p.198). However, more studies are needed to tie conceptual understanding, procedural fluency, and competency in word problem applications into a connected network that builds student mathematics proficiency on a solid knowledge base.

Assessing Mathematics Proficiency through the MSA Model

According to NCTM (2000), "Assessment should support the learning of important mathematics and furnish useful information to both teachers and students" (p.22). However, studies reveal a problem in assessment: "Less attention appears to have been paid to how teachers' assessments might help improve mathematics learning" (NRC, 2001, p.40). "The integration of assessment and learning as an interacting system has been too little explored" (Glaser & Silver, 1994, p. 403). A good assessment would be "The tasks they frame create a strategic space for students' work and for gaining insight into students' thinking" (NRC, 2001, p. 349). Nevertheless, researchers recognized that assessing mathematical proficiency, especially assessing understanding, is not an easy task, and it requires combining knowledge from a number of sources (Fennema, Sowder, & Carpenter, 1999).

To construct a valid and practical assessment for measuring proficiency, RAND (2003) provides five indicators of student mathematics proficiency. Among them, conceptual understanding, procedural fluency, and problem solving in application are the essential components of the proficiency strands. These three components are also addressed in the guiding principles of the *California Mathematics Framework* (California State Dept. of Education, 2006): To achieve balance within mathematics – basic computational and procedural skills, conceptual understanding, and problem solving.

Based on the core strands of proficiency from NRC (2001) and RAND (2003), and the guiding principles of the *California Mathematics Framework* (2006), Wu and An (2006 & 2007) developed a unique approach in the **model–strategy-application** (MSA) to build a knowledge base for measuring pre-service teachers' knowledge of mathematics teaching and for improving their mathematics learning. Later, this model was adapted in a teaching ability study (Wu & An, 2008) as a measurement tool to gauge teacher changes. Figure 1 shows a structure of the knowledge base, indicating the conditions that must be met in order to achieve effective teaching: a teacher must focus on creating a variety of visual ***models*** to aid in addressing and proving mathematical concepts, building ***strategies*** for procedural and computational fluency, and ***apply***ing the strategies to solve real-life word problems.

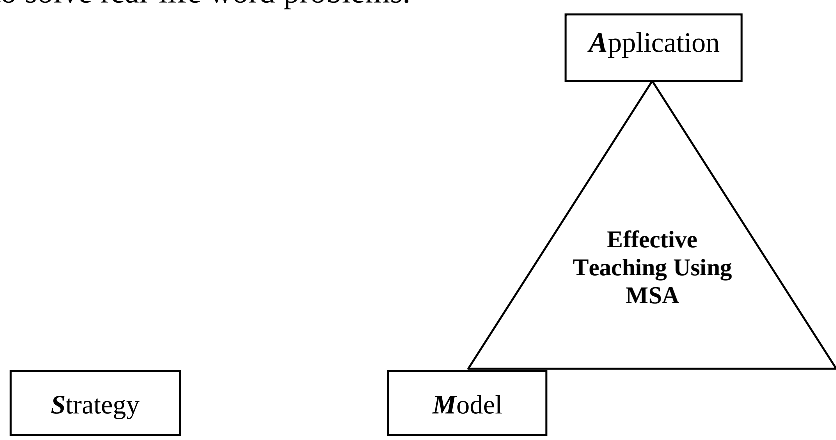


Figure 1. **The MSA approach.**

These three components are interrelated in a triangular network in which the model and strategy components form a foundation for application. The three components of the MSA model build upon each other; thus, ignoring one or another will result in ineffective teaching. Procedural fluency without conceptual understanding will yield non-meaningful and inappropriate strategies for solving applications; conceptual understanding without procedural fluency will yield inefficient strategic applications (Wu & An, 2007, 2008).

The learning process of the MSA provides teachers with the opportunity to integrate assessment and learning as a dynamic and interdependent system (Glaser & Silver, 1994) in assessing their students' proficiency in mathematics. As they evaluate student learning using the MSA approach, the students engage in an inquiry learning process: they demonstrate their understanding using visual representations, construct strategies of computations, and use their understanding and strategies to solve real-world word problems.

Although recent research has begun addressing the issue of building mathematics proficiency, there exists no study that focuses on the integration of the three components of the MSA as an interdependent system to assess K-8th student mathematics proficiency. The goal of this study was to investigate Chinese sixth graders' mathematics proficiency as reflected in their mathematics understanding, proficiency, and application in real world problems in an integrative system. Furthermore, this study examined the correlation among these three components in order to find out how each component related to the others.

The research questions asked in this study were: 1) What are the differences in mathematics proficiency as reflected in conceptual understanding, procedural fluency, and word problems in real-world applications among Chinese students at the sixth grade level? 2) What are the relationships between conceptual understanding, procedural fluency, and word problems in real-world applications in Chinese sixth graders' learning?

Methodology

Subjects and Procedures

The participants included 491 sixth grade students from six schools in two cities in Southeastern China. To assess the average student mathematics proficiency, each city included three schools at three different achievement levels – high, middle, and low. All Chinese students in this study used traditional textbooks that focused on rigorous computations in 2004-2005. The reasons for including sixth graders were to examine their mathematics proficiency focusing on understanding and to test their readiness and preparation for higher and more challenging mathematics courses at the middle school level. In addition, the data collected in this study were intended to be used in a comparative study that investigates the differences in mathematics proficiency between using traditional textbooks and using new textbooks two years later.

During the 2004-2005 school year, in collaboration with Chinese colleagues, the author developed the assessment instrument and translated it into Chinese. Chinese colleagues from China randomly selected three schools at three different levels in each city and provided the assessment to 491 sixth graders in China.

Data Collection and Instruments

The assessment focused on student mathematics proficiency in fractions and decimals. The assessment questions were designed using a unique approach **model–strategy-application** (MSA) to assess Chinese students' mathematics proficiency. The questions included 1) using various visual **models** to convey conceptual understanding; 2) using **strategies** to demonstrate procedural fluency; and 3) being able to **apply** knowledge in developing word problems. The MSA model integrated the three parts in an interdependent approach that gauged student learning in three aspects. There were seven problems, three related to fraction addition, subtraction, and multiplication, and four related to decimal addition, subtraction, multiplication, and division. The instrument was designed according to the MSA model (Wu & An, 2006) and was consistent with the proficiency components (Stein, Smith, Henningsen & Silver, 2000; Swoder, Philipp, Armstrong & Schappelle, 1998). The students were asked to solve each problem in three ways: calculate it, construct a visual representation to demonstrate understanding of it, and create a word problem and then solve it (see Table 1). This model is also a key approach to engaging students in an inquiry-based learning process.

Table 1

Using the MSA Model to Assess Student Mathematics Proficiency

Problems	Solve the problem by showing procedures, steps, or strategies.	Use a representation (picture, chart, or other manipulatives) to show your understanding and solving of the problem on the left.	Creating a real-world situation for the given problem on the left.
1. $\frac{11}{12}+\frac{5}{7}$			
2. $3-\frac{1}{2}-\frac{2}{3}$			
3. $\frac{4}{5}\times\frac{3}{7}$			
4. 1.25 + 9.89 + 1.62			
5. 62.12 – 24.3			
6. 263.6 $\times$ 0.465			
7. 24.275 $\div$ 1.25			

Data Analysis

The study used both quantitative and qualitative methods for data analysis. A qualitative data analysis was used to analyze students' construction on representations and their own word problems in applications. Students' responses were coded, categorized, and compared (Lincoln & Guba, 1985). The criteria in coding were based on the three parts of the MSA model that measure mathematics proficiency:

1. If a student can solve a problem using the correct procedures and get a correct answer, the student is considered to have computational fluency.
2. If a student can create his/her own visual model and use it to convey their understanding and get the right answer, the student is considered to have conceptual understanding.
3. If a student can apply his/her understanding to develop a word problem that connects and makes sense to the real-world applications, the student is considered to have competence in application in word problems.

When a student met these criteria, the student was considered to have mathematics proficiency with a strong understanding of mathematics.

For quantitative methods of data analysis, a One-Way Within-Subjects (Repeated Measures) ANOVA was used to find statistically significant differences in mean scores between conceptual understanding, procedural fluency, and application in word problems. Pearson Correlations were calculated to identify the relationships between conceptual understanding, procedural fluency, and competence in application in word problems.

Results

Differences in Mathematics Proficiency in Three Aspects in the MSA Model

Statistical Results

Table 2 shows the percentage of correct answers in each of three areas. The results show that Chinese students had a higher level of computations than use of models as well as word problems. The use of visual models was the weakest area among the three components.

Table 2
Results of Chinese Student Scores in the MSA Model (*N*=491)

Problems	Visual models		Procedures		Word problems	
	Correct	%	Correct	%	Correct	%
$\frac{11}{12}+\frac{5}{7}$	197	40%	469	96%	262	53%
$3-\frac{1}{2}-\frac{2}{3}$	238	48%	449	91%	279	57%
$\frac{4}{5}\times\frac{3}{7}$	178	36%	480	98%	245	50%
1.25 + 9.89 + 1.62	219	45%	449	91%	268	55%
62.12 – 24.3	223	45%	422	86%	264	54%
263.6 × 0.465	122	25%	309	63%	202	41%
24.275 ÷1.25	128	26%	303	62%	138	38%

A repeated measure ANOVA, with Greenhouse-Geisser correction, was conducted to assess whether there were differences between the average scores of the three areas of the MSA. Results indicated that Chinese students did score significant differences in three areas, $F(1.60, 784.54)=376.317$, $p < 0.001$, $R^2=0.434$, $eta^2=.434$.

The means and standard deviations for the model, computation, and application abilities are presented in Table 3. Examination of these means suggests that Chinese students had a lower level in modeling than in computation and application. Polynomial contrasts indicated, in support of this, that there was a significant linear trend, $F(1, 490)=87.694$, $p<0.001$, $eta^2=.15$. However, this finding was qualified by the significant quadratic trend, $F(1, 490)=477.681$, $p<0.001$, $eta^2=.49$. reflecting the higher score for computations than application.

Table 3
Means and Standard Deviations for the MSA (*N* = 491)

	Mean	Std. Deviation	*N*
Model	2.66	2.744	491
Computation	5.87	1.228	491
Application	3.48	3.059	491

Using Rule or Procedure to Demonstrate Computations

The results from Table 2 show that the Chinese students had high fluency in fraction computation skills. More than 91% of students calculated each problem correctly in fractions. In fraction multiplication, 98% of Chinese students showed correct computations, whereas only 91% of the students got correct answers in the fraction subtraction problem that had a whole number and two fractions with uncommon denominators, though they were good at fraction addition with uncommon denominators (96%).

Although 91% of Chinese students knew how to do decimal addition, only 86% of them did the decimal subtraction problem correctly. In addition, they seemed to be weak in decimal multiplication and division because these two required more procedures. The difficulty in these two parts was reflected in their scores - they had scores a little over 60% in these two parts. For students who got correct answers in the decimal addition 1.25 + 9.89 + 1.62, they added digits by lining up with decimals with two digits each after the decimal points, whereas with unequal digits after the decimal points in the subtraction problem 62.12– 24.3, some students had difficulties in lining up or borrowing. For the multiplication problem 263.6×0.465, the students made some careless errors because of the greater number of digits. Most students used the long division methods for the division problem $24.275 \div 1.25$, but some students had difficulties due to the large number of digits. It is worth noting that many students got wrong answers for the division problem such as 194.2, 0.1942, and 1.942, which showed their inadequate knowledge of number sense with decimals.

Constructing Visual Models to Show Understanding

The results of Table 2 show that the Chinese students seemed to be weak in constructing visual representations. In fractions, for example, they only scored less than 50% in each part. The Chinese students had more difficulties in decimal multiplication and division as shown in their scores showing less than 30% of the students doing the problems correctly, though they had scores of more than 45% in decimal addition and subtraction.

Fraction Addition and Subtraction

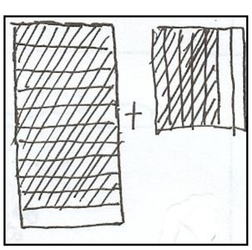
For constructing fraction addition or subtraction with uncommon denominators, two important concepts must be noted: 1) The whole unit 1 should be the same shape and same size when adding or subtracting fractions with unlike unit factions, and 2) To add fractions with unlike unit fractions, the unlike unit fractions should be divided into the same unit fractions first before adding or subtracting.

The main misconceptions were found from the Chinese students' models in this study as follows:

1. Using different sizes for the whole unit 1 when adding or subtracting fractions with unlike unit fractions.

   For example, Figure 2(a) shows that $\frac{11}{12}+\frac{5}{7}$ has two different sizes of rectangles – two different whole units - though the two fractions were correctly drawn. The model for the problem $3-\frac{1}{2}-\frac{2}{3}$ had the same error with the whole unit 1 as in Figure 2(b) showing the three number lines with different lengths.

(a) $\frac{11}{12}+\frac{5}{7}$



(b) $3-\frac{1}{2}-\frac{2}{3}$

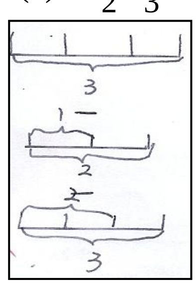


Figure 2. **Errors in adding and subtracting fractions with unlike unit fractions.**

2. Having the same sizes of whole unit 1s, but not changing fractions with the like unit fraction.
   For example, in the visual representation of $\frac{11}{12}+\frac{5}{7}$, two number lines had the same length, but the like unit was not shown in the number lines (see Figure 3 (a)).

(a) $\frac{11}{12}+\frac{5}{7}$ (b) $3-\frac{1}{2}-\frac{2}{3}$

Figure 3. **Not changing fractions to like unit fraction.**

With the subtraction problem $3-\frac{1}{2}-\frac{2}{3}$, although all whole unit 1s are the same in size, the process of subtraction by dividing the same wholes into the same unit fractions were not demonstrated (see Figure 3 (b)).

3. Having different shapes of the whole unit 1 when adding or subtracting fractions.
   Figure 4 (a) and (b) show the misconceptions in using different visual models to represent two fractions in the same problem. In Figure 4(a), $\frac{11}{12}$ was represented by a rectangle, while $\frac{5}{7}$was demonstrated using a number line, for the problem$\frac{11}{12}+\frac{5}{7}$. A similar error occurred in Figure 4(b): 3 and $\frac{1}{2}$was a denoted using circle, yet $\frac{2}{3}$was shown using a rectangle, for the problem $3-\frac{1}{2}-\frac{2}{3}$.

(a) $\frac{11}{12}+\frac{5}{7}$ (b) $3-\frac{1}{2}-\frac{2}{3}$

Figure 4. **Use of different visual models to represent fractions.**

4. Adding fractions on number lines without understanding the relationships.
   Figure 5(a) displays errors in the model for $\frac{11}{12}+\frac{5}{7}$. Two fractions were added as whole numbers without understanding the different sizes between the two. It mistakenly showed $\frac{11}{12}$ as about twice the size of $\frac{5}{7}$. The same error was found with the problem $3-\frac{1}{2}-\frac{2}{3}$.

Figure 5(b) shows an almost equal amount of lengths for three numbers: 3, $\frac{1}{2}$ and $\frac{2}{3}$. It also shows the student tried to use the

(a) $\frac{11}{12}+\frac{5}{7}$ (b) $3-\frac{1}{2}-\frac{2}{3}$

Figure 5. **Adding and subtracting fractions without understanding.**

computation result to fit into a representation rather than directly constructing a visual model with understanding.

Obviously, incorrect visual models were originated from the misconceptions in fractions, which led to incorrect applications in word problems. It is noted that most Chinese students' models were limited in number lines with few variations. Very few students used squares, rectangles, and circles to build visual models. Some of them made mistakes in not converting unlike unit fractions or not using the same whole unit with visual models, thereby they could not get correct answers. It is interesting to note that many students constructed a larger size of a whole unit for $\frac{11}{12}$ then for $\frac{5}{7}$, for the problem $\frac{11}{12}+\frac{5}{7}$, which displayed their unawareness of the same whole requirement.

Fraction multiplication

Results from Table 2 show that only 36% of Chinese students were able to draw visual representations for fraction multiplication, basing on their understanding of fraction addition and subtraction. However, more than 60% of Chinese students had difficulties in creating correct visual representations due to their incomplete understanding of fraction multiplication.

The major errors in developing visual representations for fraction multiplication from the Chinese students were the following:

1. Drawing two fractions separately without using the same whole unit and without dividing the unlike unit fractions into like unit fractions.

$\frac{4}{5}\times\frac{3}{7}$

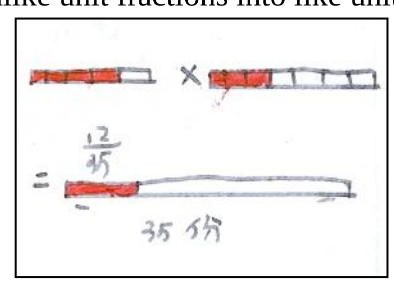


Figure 6. **Drawing fractions without the same whole unit 1.**

In Figure 6, two fractions $\frac{4}{5}\times$ and $\frac{3}{7}$ were demonstrated, as parts of two number lines and the product was shown. However, the two number lines did not have the same lengths, i.e. the same whole unit 1. In addition, the process of dividing the unlike unit fractions of $\frac{1}{5}$ and $\frac{1}{7}$ into the like unit fraction of $\frac{1}{35}$ was missing. Obviously, the student used the computation result to fit into the visual model.

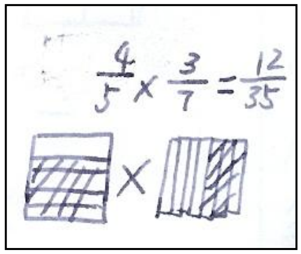


Figure 7. **Drawing fractions without showing a process.**

2. Using the same whole unit 1, but missing the process and product.

   In Figure 7, two fractions $\frac{4}{5}$ and $\frac{3}{7}$ were expressed, as parts of two rectangles, but the product of the process and result did not show in the drawing.

3. Using area representation, but the model does not reflect on the concept.

$\frac{4}{5} \times \frac{3}{7}$

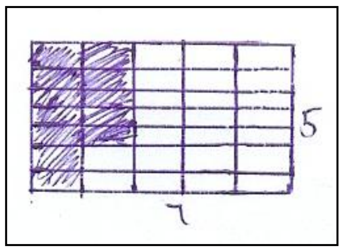


Figure 8: **Using area representation without understanding the process.**

Figure 8 shows the area representation with dimensions 7 by 5, but it does not demonstrate how the 12 shaded units came to be. The result seemed to be from the computation directly, not deducted from the visual representation.

The above mistakes in visual representations show the Chinese students' misconceptions in fraction multiplication. First, when developing visual models for fraction multiplication, it is very important to understand that the whole unit 1 should be the same in shape and size and the unlike unit fractions should be changed to a like unit fraction. Second, the multiplier is one of the key concepts. For a whole number as a

multiplier, such as $\frac{4}{5} \times 2$, it is easier to understand by using the repeated addition

$\frac{4}{5}+\frac{4}{5}=\frac{4}{5} \times 2$. However, when a multiplier is a fraction, such as $\frac{3}{7}$, students showed

confusion with it and had difficulties to visualize it and make sense of it. In terms of a variety of visual models, as in addition and subtraction of fractions, most Chinese students used number lines to draw visual representations for fraction multiplication.

Decimal Addition and Subtraction

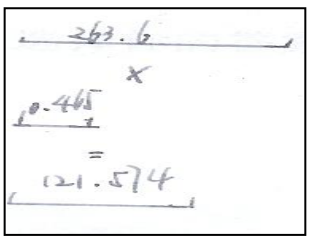
Table 2 shows that 45% of the Chinese students did the decimal addition and subtraction problems correctly. Most students were able to use money to build their visual representations for the decimal addition and subtraction problems. The money model is very helpful in decimal problems that have two digits after the decimal points, but still around 55% of the Chinese students could not demonstrate these two problems with correct visual models.

Decimal Multiplication

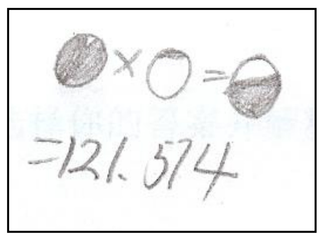
Table 2 shows that only about 25% of the Chinese students drew visual models for decimal multiplication and division correctly. The major misconception was not having a clear understanding of place value and expanded form for decimal multiplication. For example, if students understand that 263.6 can be expressed as 200 + 60 + 3 +.6 or know how to use Tenth, Hundredth, and Thousandth grids to represent visual models, they would draw models correctly. The following shows some misconceptions in visual models for decimal multiplication and division.

1. Not showing the process from the visual representation.

Figure 9 (a) shows that 263.6 and 0.465 were represented by two number lines respectively. It also indicates the answer 121.574 using another number line. However, the student did not demonstrate how to get answer 121.574 from the number lines. It is noted that the student also had unproportional number lines for 263.6 and 0.465. Figure 9 (b) shows that the student used circles to only demonstrate the problem and answer.



263.6 × 0.465



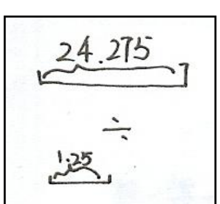
(a) (b)

Figure 9. **Not showing the process of decimal multiplication.**

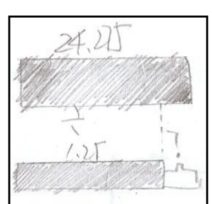
Decimal Division

Table 2 indicates that only 26% of the Chinese students were able to create visual models for the decimal division problem. The major misconception was that students do not understand the meaning of decimal division. If they knew that 24.275 ÷ 1.25 means to find how many 1.25s are in 24.275, the students would know how to draw a correct model. The notable errors in the decimal division are shown below:

1. Not understanding the meaning of decimal division, merely copying the problem.
   Two figures (a) and (b) in Figure 10 demonstrate only the problem of decimal division 24.275 ÷ 1.25. The line segment representation and regional representation did not show the process of the problem solving nor the correct answer.



24.275 ÷ 1.25

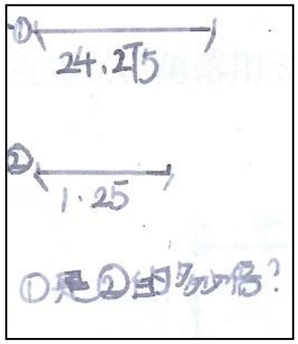


(a) (b)

Figure 10. **Not showing the process of decimal division.**

2. Understanding the meaning of decimal division, but not knowing how to represent it.
   Figure 11 shows the student understanding of decimal division by using the line segments to ask how many 1.25s are in 24.275. However, students did not know how to process it using visual models.

24.275 ÷1.25



(How many times is (1) longer than (2)?)

Figure 11. **Not showing the process of representation.**

3. Not understanding the meaning of decimal division as repeated subtraction.
   Figure 12 shows that the student knew: "The total is 24.275; subtract $1.25 from it, how much is left?" However, the student only subtracted 1.25 once, and did not repeat the subtraction. Although this can be a careless error, the misconception worth mentioning.

24.275 ÷1.25

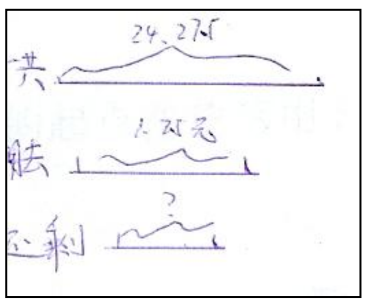


Figure 12. **Showing decimal division as subtraction.**

In summary, the above examples only show some Chinese students' common errors in building their own visual models. Although some problems are typical, these misconceptions imply that some Chinese students relied heavily on procedure development and ignored learning mathematics with understanding.

Creating Word Problems in Real-world applications

The results from this study show that the Chinese students seemed to have a better performance in creating word problems for applications than developing visual models. The Chinese students did much better with fractions as well as decimal addition and subtraction, as their scores ranged from 50% to 57% in these areas, while with decimal multiplication and division, only around 40% could create correct word problems.

The major errors displayed in word problem applications were not knowing how to apply their knowledge in applications, not understanding of the meanings of fractions and decimals, and problems unconnected with real-world applications. The following summarizes the Chinese student misconceptions in developing real-world application problems:

Word Problems in Fractions

Four types of misconceptions were shown in fractions:

1. Adding fractions with different units in real applications. $\frac{11}{12}+\frac{5}{7}$
   - Mei wrote a word problem for fraction addition: " There are $\frac{11}{12}$ of apples and $\frac{5}{7}$ of pears. How many apples and pears are all together?
   - Bing's example: My mother used $\frac{11}{12}$ of turnips and $\frac{5}{7}$ of cabbages when she cooked. How many did she use?

   Mei and Bing did not realize that unlike units could not be added and a total is missing in each problem.
2. Disconnected with real-world applications in fraction addition.
   - Xia's example: There was a basket of fruits. $\frac{11}{12}$ of the fruits were taken the first time, and then $\frac{5}{7}$ of fruits were taken the second time. How many fruits were taken in total?
     Obviously, Xia was not aware that one basket is not enough when putting $\frac{11}{12}$ and $\frac{5}{7}$ together.
   - Xiaoqi's example: I have three stamps. I lent $\frac{1}{2}$ of my stamps to someone and lent $\frac{2}{3}$ again. How many were left?
     Xiaoqi's misconception lied in a disconnection between mathematics and a real world application. How can he divide a stamp into $\frac{1}{2}$ or $\frac{2}{3}$? Similar examples were also seen as $\frac{4}{5}$ of a machine or a car, etc.
3. Only using symbolic representations for fraction multiplication.
   In fraction multiplication, some students could not provide an example related to a real-world application, and only used mathematics language in their examples.
   - Ping's example: A number is $\frac{4}{5}$, what is $\frac{3}{7}$ of it?
   - Hua's example: A basket of apples is $\frac{4}{5}$ kg, and another basket of apples is $\frac{3}{7}$ kg. How many kg of apples are there if multiplying the two baskets' apples?
4. Not making sense of real-world applications in fraction multiplication.

Some students' examples did not make sense with a real-world application. For example, Bingbing had the following example:

- Hong went shopping to buy a ruler. It costs $\$\frac{4}{5}$ for a ruler. If Hong bought $\frac{3}{7}$of a ruler, how much does it cost?

  In mathematics, it may be right, but in the real word, it is very difficult to buy $\frac{3}{7}$ of a ruler.

Word Problems in Decimals

Around 55% of students did well on decimal addition and subtraction due to their experience in shopping, which reflected in their word problems. Only few students' word problems did not make sense as shown in "1.62 birds and 21.25 white rabbets. Three types of misconceptions in word problems were shown with decimals:

1. Not having a deep understanding of the meaning of decimal multiplication and division.
   a. Qian's example for 263.6 x 0.465: There are 263.6 kg of apples in a fruit market. There are 0.465 kg more Pears than apples. How many kg of pears are there?
   Qian did not realize that this problem is not a multiplication problem.
   b. Ming's example for 24.275 $\div$ 1.25: There are 24.275 kg of seeds for making 1.25 kg of oil. How many kg of oil can you get from 1 ton of seeds?
   Ming's word problem is obviously not a division problem.

2. Only using symbolic representations for decimal multiplication and division.
   Many students used only mathematics language to write their word problems. For example, some students wrote, "Find 0.465 of 263.6" and "How many times is 24.275 larger then 1.25?" as their word problems, without connecting real word application.

It is worth noting that besides all the above misconceptions, many students could not write word problems connecting with real-world applications, which is one of the reasons for lower scores in decimal multiplication and division in this study.

Correlation between Conceptual Understanding, Procedural Fluency, and Word Problems

To identify the relationships between conceptual understanding, procedural fluency, and competence in word problem applications, Pearson Correlations were calculated between the scores on the three MSA areas.

Table 3
Correlations between Model and Application

		Model	Computation	Application
Model	Pearson Correlation	1	.133**	.781**
	Sig. (2-tailed)		.003	.000
	N	491	491	491
Computation	Pearson Correlation	.133**	1	.135**
	Sig. (2-tailed)	.003		.003
	N	491	491	491
Application	Pearson Correlation	.781**	.135**	1

	Sig. (2-tailed)	.000	.003	
	N	491	491	491

** Correlation is significant at the 0.01 level (2-tailed).

Strong Correlation between Model and Application

A strong, positive correlation was found to be 0.781 between modeling and application, which is significant at $p<0.1$.(see Table 3). Since a strong linear correlation was found, it can be concluded that scores on the modeling correlated significantly with scores on the application, and indicated that a student's scores on conceptual understanding strongly predicated the student's level of developing word problems – the higher scores in conceptual understanding, the greater the competency in word problem applications.

The results in Table 3 also show that a weak correlation was found to be .133 between computation and modeling as well as between computation and application, though it is significant at $p<0.1$. Since a weak linear correlation was found, it can be concluded that scores on the computation weakly correlated with scores on the modeling and application, and indicated that the higher scorers in computation may not have had a better understanding and competency in word problem application.

The correlations in Table 3 are supported by the line graphs in Figure 12 (a)～(d) below:

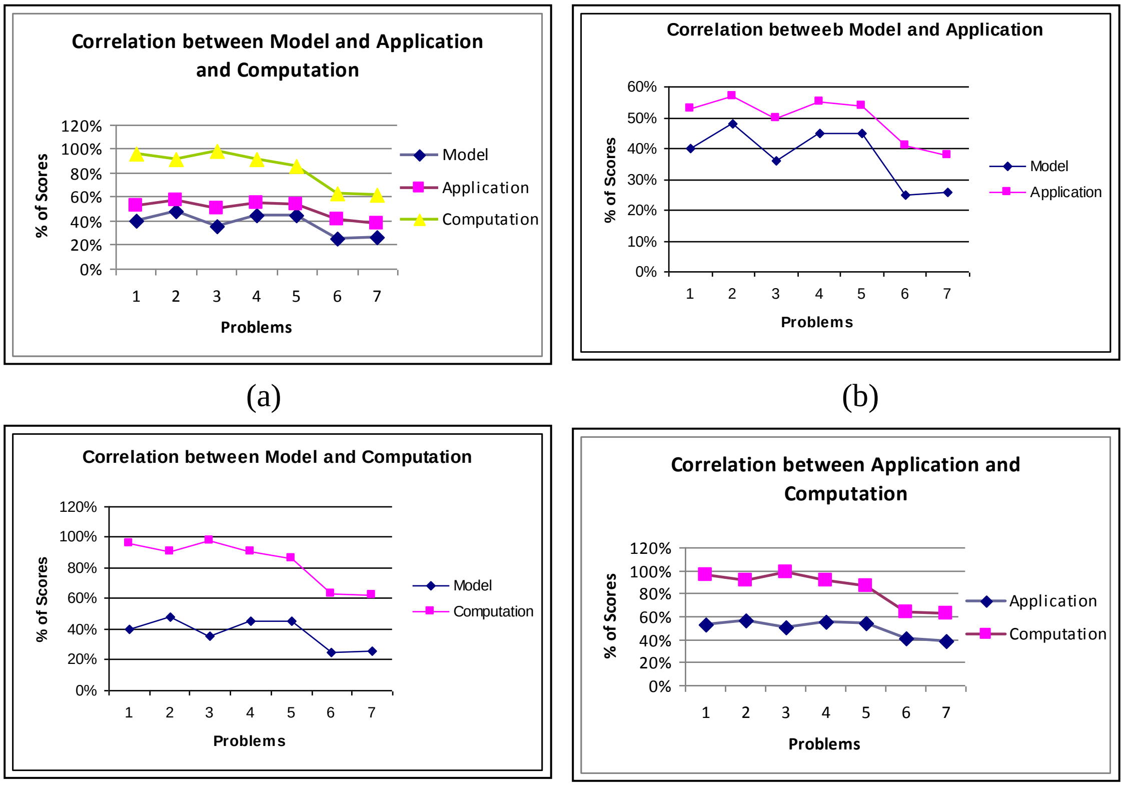


Figure 12. **Correlation between model, application, and computation.**

Figure 12 (a) shows overall graphic patterns in correlations between model, application, and computation. Figures 12 (b) – (d) exhibit the correlations between the three components separately: Figure (b) illustrates that the model line (up) is always positively associated with the application line (low), while the computation line (up) does not follow the changes of the model and application lines (low) in Figures 12(c) and (d).

The correlations between the three components of the MSA areas are also reflected in Chinese students' work. Figure 13 shows an example, demonstrating that a deep understanding of fraction concepts in multiplication led to a meaningful application.

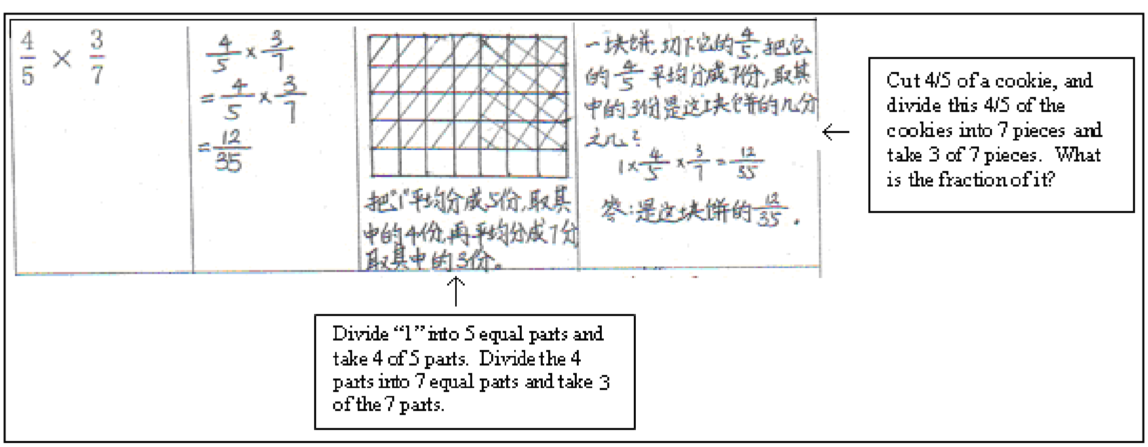


Figure 13. **Positive correlation between model and application.**

Figure 14 shows an example, illustrating that a good computation does not lead to a correct understanding and an appropriate word problem in an application.

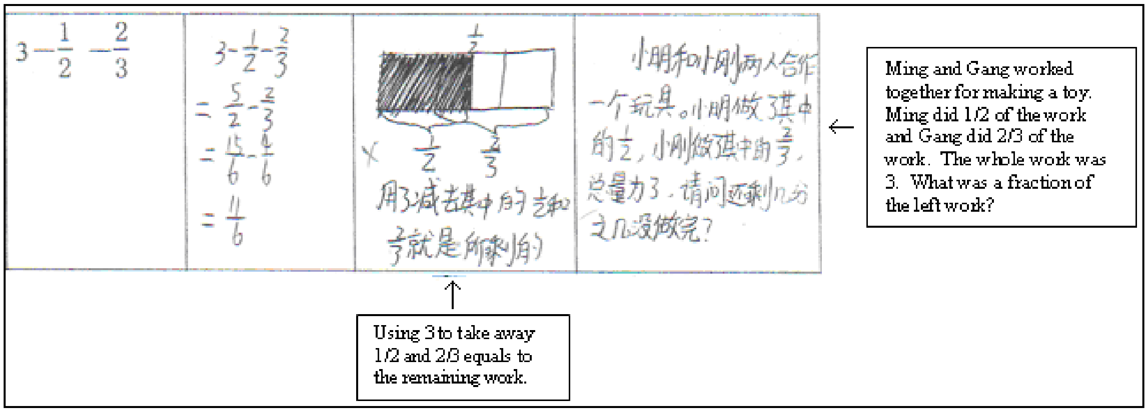


Figure 14. **Disconnection between computation, model, and application.**

Discussion

The Differences in Mathematics Proficiency in the Three Areas of the MSA Model

The results of this study show that Chinese students had a higher level of computations in fraction and decimal problems. The following Bar graphs in Figure 15 summarize the differences in model, application, and computation from the data in this study.

The visual bars in Figure 15 indicate that the computation level of the Chinese sixth graders is strongest among the three areas, whereas modeling is the weakest area, meaning some Chinese students did not have a deep understanding of concepts even though they had their highest levels in procedural fluency. In addition, although they did better in real world problem applications compared to the model area, they were still weak at developing word problems that make sense to them in fraction multiplication and decimal multiplication and division in this study. The strong ability in computation from Chinese students was well documented in various studies (Brenner, Herman, Ho, & Zimmer, 1999; Geary, Bow-Thomas, Fan, & Siegler, 1993; Gu, 1997). The influence of shaping Chinese students' strong ability in computation is indicated in historic beliefs in Chinese mathematics teaching and learning: Practice makes perfect (Li, 2006). Drawing on the observation data from Chinese mathematics classrooms, An (2004) found that Chinese teachers provided different levels of problems to build fluency in procedural development.

However, a variety of studies also documented that although Chinese students demonstrated a high level in computation and abstract thinking, they showed no superiority to U.S. students in visual representations, understanding tables, and open process problem solving (Brenner et al., 1999; Cai, 2000; Miura, Okamoto, Kim, Chang, Steere, & Fayol, 1994; Stevenson et al., 1990; Wang & Lin, 2005). These findings are consistent with the results in this study that Chinese students showed no advantage in developing visual modeling for demonstrating their deep conceptual understanding in learning mathematics.

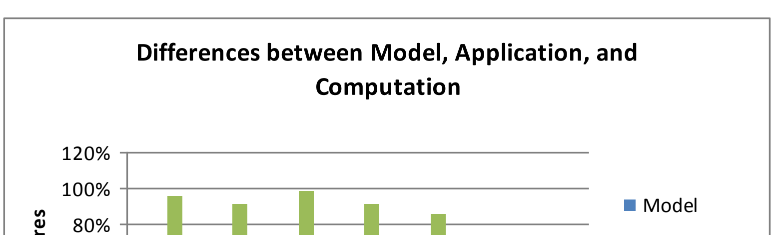


Figure 15. **Differences between model, application, and computation.**

Relationships between the Three Components of the MSA

This study found two aspects in the relationships between the three components of the MSA. First, procedural fluency without conceptual understanding yields non-meaningful and unconnected word problems in real-world applications. The examples of word problems from some Chinese students in this study showed their disconnection in mathematics learning to real-world applications in fraction addition. The example in Figure 14 also exhibits a fact that good computation does not lead to a correct understanding and an appropriate word problem in an application. The Chinese student showed his/her learning without understanding in the example in Figure 14. Although the student computed $3-\frac{1}{2}-\frac{2}{3}$ correctly, he/she mistakenly took away $\frac{1}{2}$ of 3 from the model instead of subtracting $\frac{1}{2}$ from 1. With the leftover part, the student was not aware that there was not enough for $\frac{1}{2}$ of 3 taking away $\frac{2}{3}$ of 3. The misconception in modeling led to an inappropriate word problem in an application exhibited in Figure 14.

The second relationship of the MSA drawn from the data in this study is that a better understanding is strongly associated with an appropriate and meaningful word problem application. The example from a Chinese student in Figure 13 presents this relationship. The Chinese student not only drew a correct area model for fraction multiplication, but also explained the process of modeling: Divide “1” into 5 equal parts, and take 4 out of 5 parts; divided the 4 parts into 7 equal parts and take 3 of the 7 parts. This good understanding resulted in an appropriate word problem in real-world applications in this study. Xu (2004) also observed the effects of using different representations in developing Chinese fourth graders’ understanding and problem solving.

The evidence of the above two relationships of the MSA approach was well supported by Wu and An’s studies (2006, 2007, 2008) for pre-service and classroom teachers. According to Wu and An (2007), the three components of the MSA model build upon each other; thus, ignoring one or another will result in ineffective teaching and learning mathematics.

Implications

This study compared the differences in Chinese sixth grade students’ mathematics proficiency by measuring their knowledge based on the MSA model of conceptual understanding, procedural fluency, and competence in word problem applications. In addition, it also examined the correlations between each component of the MSA.

The results of this study reflect a pattern of Chinese student learning using traditional textbooks: Learning mathematics focused on procedural development and following the belief, "Practice makes perfect" as addressed in Li's study (2006). As a result, Chinese students' high fluency in procedures did not automatically produce a deep conceptual understanding, thereby yielding their lacking in a connection between mathematics and real-world applications, and weakening their mathematics proficiency in all five areas as defined by NRC (2001) and RAND (2003). Although spending substantially more time on practicing computation problems (An, 2004; Stigler & Perry; 1988) was helpful for preparing the Chinese students for the National Exam, the cost was too high as evident by results in this study, that more than 50% of Chinese students still lacked a deep conceptual understanding of how to draw correct visual models. Numerous studies indicate the benefits of deepening conceptual understanding, as it assists students to make connections and make sense of mathematics as well as supporting students' mathematics proficiency (Carpenter & Lehrer, 1999; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier & Human, 1997; NRC, 2001; RAND, 2003; Shafer & Romberg, 1999).

The results of this study also implicate a pattern of Chinese teaching using traditional textbooks: focusing more on procedural development, resulting in an unbalance in teaching mathematics. The findings from Wu and An's studies (2006) indicate that the three components of the MSA model are all interrelated and equally important; learning mathematics while focusing only on procedures without acquiring conceptual understanding will result in students' inability in applying knowledge to create and solve word problems in applications.

The MSA approach from this study provides mathematics educators with a new means of assessment that is an effective and quantified assessment tool for measuring student mathematics proficiency and exploring their cognitive learning progress.

In conclusion, to maximize students' capacity for learning mathematics, a mathematics teacher must learn to balance the three components of the MSA approach. Focusing on one and ignoring another will result in a lower proficiency in mathematics in general. Further study is needed to investigate the difference between using traditional textbooks and using new textbooks. Further study is also needed to investigate the differences between using the MSA approach among students with diverse backgrounds.

References

An, S. (2004). *The middle path in math instruction: Solutions for improving math education*. Lanham, MD: Scarecrow Education.

Brenner, M. E., Herman, S., Ho, H. Z., & Zimmer, J. M. (1999). Cross-national comparison of representational competence. *Journal for Research in Mathematics Education, 30*, 541-557.

California State Dept. of Education. (2006). *Mathematics framework for California public schools, Kindergarten through grade twelve (2006)*. Sacramento, CA: Author.

Cai, J. (2000). Mathematical thinking involved in U.S. and Chinese students' solving of process-constrained and process-open problems. *Mathematical Thinking & Learning, 2*, 309-40.

Cai, J. (2001). Improving mathematics learning: Lessons from cross-national studies of Chinese and U.S. students. *Phi Delta Kappan*, *82*, 400-04.

Carpenter, T., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics classroom that promote understanding* (pp. 19-32). Mahwah, NJ: Erlbaum.

Carpenter, T., Fennema, E., Fuson, K., Hiebert, J., Human, P. & Wearne, D. (1999). Learning basic number concepts and skills as problem solving. In E. Fennema & T. Romberg (Eds.), *Mathematics classroom that promote understanding* (pp. 45-61). Mahwah, NJ: Erlbaum.

Carpenter, T., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97,* 3-20.

Geary, D. C., Bow-Thomas, C. C., Fan, L., & Siegler, R. S. (1993). Even before formal instruction, Chinese children outperform American children in mental addition. *Cognitive Development, 8*, 517-529.

Gu, W. (1997). *The differences of mathematics achievement between American children and Chinese children.* Unpublished master's thesis, Winona State University, Winona, Minnesota.

Fennema, E., Sowder, J., & Carpenter, T. (1999). Creating classrooms that promote understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics classroom that promote understanding* (pp. 185-199). Mahwah, NJ: Erlbaum.

Fuson, K. C., & Smith, S. T. (1995). Complexities in learning two-digit subtraction: A study of tutored learning. *Mathematics Cognition, 1*, 165-213.

Glaser, R., & Silver, E. A. (1994). Assessment, testing, and instruction: Retrospect and prospect. In L. Darling-Hammond (Ed.), *Review of Research in Education.* Vol. 20, (pp.393-419). Washington, DC: American Educational Research Association.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and student learning in second grade. *American Educational Research Journal, 30*, 393-425.

Hiebert, J., & Wearne, D. (1996). Instruction, understanding, and skill in multidigit addition and subtraction. *Cognition and Instruction, 14,* 251-283.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding,* Portsmouth, NH: Heinemann.

Lapointe, A. E., Mead, N. A., & Askew, J. M. (1992). *Learning mathematics.* Princeton, NJ: Educational Testing Service.

Li, S. (2006). Practice makes perfect: A key belief in China. In F. K.S. Leung, K. D. Graf, & F. J. Lopez-Real (Eds.), *Mathematics education in different cultural traditions: A comparative study of East Asia and the West, The 13th ICMI Study.* New York: Springer.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.

Miura, I. T., Okamoto, Y., Kim, C. C., Chang C. M., Steere, M., & Fayol, M. (1994). Comparisons of children's cognitive representation of number: China, France, Japan, Korea, Sweden, and the United States. *International Journal of Behavioral Development, 17,* 401-411.

Murray, H., Olivier, A., & Human, P. (1992). The development of young students' division strategies. In W. Geeslin & K. Graham (Eds.), *Proceedings of the sixteenth international conference for the Psychology of Mathematics Education: Vol. 2* (pp. 152-159). (ERIC Document Reproduction Service No. SE 055 811)

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* In J. Kilpatrick, J. Swafford, and B. Findell (Eds.), *Mathematics learning study committee, Center for education, Division of Behavioral and Social Sciences and Education.* Washington, DC: National Academy Press.

Shafer, M., & Romberg, T. (1999). Assessment in classroom that promote understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics classroom that promote understanding* (pp. 159-184). Mahwah, NJ: Erlbaum.

Stein, M., Smith, M., Henningsen, M., & Silver, E. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development.* New York: Teacher College Press.

Stevenson, H. W., Lee, S. Y., Chen, C., Lummis, M., Stigler, J. W., Fan, L., & Ge, F. (1990). Mathematics classrooms in Japan, Taiwan, and the United States. *Children Development, 58,* 1272-1285.

Stigler, J. W., & Perry, M. (1988). Mathematics learning in Japanese, Chinese, and American classrooms. *New Directions for Child Development,41,* 27-54.

Swoder, J., Philipp, R., Armstrong, B., & Schappelle, B. (1998). *Middle-grade teachers' mathematical knowledge and its relationship to instruction.* New York: State University of New York.

RAND Mathematics Study Panel. (2003). *Mathematics proficiency for all students: Toward a strategic research and development program in mathematics* education. Santa Monica, CA: RAND Education.

Wang, J., & Lin, E. (2005). Comparative studies on U.S. and Chinese Mathematics learning and the implications for standards-based mathematics teaching reform. *Educational Researcher*, *34*(5), 3-13

Wu, Z., & An. S. (2006, April). *Using the model-strategy-application approach to developing pre-service teachers' pedagogical content knowledge.* Paper presented at annual meeting of AERA. San Francisco, CA.

Wu, Z., & An. S. (2007, January). *Sharpening teaching ability in mathematics classrooms.* Paper presented at Association of Mathematics Teacher Educators (AMTE) 11th Annual Conference. Irvine, CA.

Wu, Z., & An, S. (2008, July). *Using the model-strategy-application approach to developing pre-service teachers' knowledge and assessing their progress in math methods courses.* Paper presented at 11th International Congress on Mathematics Education. *Monterrey, Mexico*.

Xu, B. (2004). The effects of different representations on mathematics learning of Chinese children. In L. Fan, N. Wang, J. Cai, & S. Li. (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 483-500). Singapore: World Scientific Publisher.

Zhou, Z., & Peverly, S. T. (2004). Cross- and within-cultural variations in children's understanding of distance, time, and speed interrelationships: A follow-up study. *The Journal of Genetic Psychology, 165*, 5–27.

Zhou, Z., Peverly, S. T., Boehm, A. E., & Lin, C. (2000). American and Chinese children's understanding of distance, time, and speed interrelations. *Cognitive Development, 15*, 215–240.

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