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Evaluating the Effectiveness and Insights of Pre-Service Elementary Teachers' Abilities to Construct Word Problems for Fraction Multiplication

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This research investigated the ability of 127 U.S. pre-service elementary teachers to write word problems that represent symbolic expressions of fraction multiplication. Results indicated that a significant percentage of the pre-service teachers were unable to construct appropriate word problems for the given symbolic expressions of fraction multiplication. In terms of semantic structures, most of the pre-service teachers constructed word problems that were an extension of the basic "forming the n^{th} multiple of measures" structure. In terms of units of measure, the majority of pre-service teachers used food-related objects as their units of measure. Implications for teacher education and the need for more research were discussed.

***Key words*:** teacher knowledge, fraction, fraction multiplication, word problems.

Introduction

Recent literature in the area of mathematics education has raised attention on word-problem writing which can serve as an avenue for developing and assessing students' understanding of important mathematical concepts and for engaging students in meaningful mathematics (Barlow & Cates, 2007; Barlow & Drake, 2008; English, 1998; National Council of Teachers of Mathematics, 2000). However, limited attention has been paid to the discussion of teachers' abilities to write word problems.

This limited attention is highly unfortunate for at least two reasons. Firstly, teachers' mathematics content knowledge greatly impacts their students' mathematics achievement (Hill, Rowan, & Ball, 2005; Kulm, 2008). However, research has shown that U.S. teachers have a weak grasp of basic mathematics content knowledge as compared with their Chinese counterparts (Ma, 1999). More to the point, previous studies indicate that U.S. pre-service teachers possess limited mathematics content knowledge in concepts and operations of non-whole numbers (Azim, 1995; Ball, 1990a, 1990b; Graeber, Tirosh, & Glover, 1989; Tirosh, 2000; Simon, 1993). U.S. pre-service teachers often concentrated on remembering rules and mastering standard procedures, and lacked comprehensive understanding of mathematical ideas and procedures (Ball, 1990a).

In relation to word problems, Ball (1990a, 1990b) found that pre-service elementary teachers had significant difficulty in selecting an appropriate word problem for a symbolic expression. Simon (1993) also found that 70% of the pre-service elementary teachers in his study were not able to create an appropriate word problem for the fraction division of $\frac{3}{4}$ divided by $\frac{1}{4}$. It is interesting to know how deeply this problem writing difficulty is rooted. Will a similar pattern be identified from the pre-service elementary school teachers for the topic of fraction multiplication which is typically taught before the topic of fraction division?

Secondly, difficulty with fractions is a major stumbling block for further progression in mathematics as described by the National Mathematics Advisory Panel (2008). According to NCTM (2000), all children should understand fraction concepts in grades 3–5 and understand the meaning of fraction operations in grades 6–8. In some American states, upper elementary students are expected to be able to develop their understanding of the meaning of fraction operations.

Therefore, in order to support schoolchildren in their ability to interpret and to create word problems and to deal with the challenging transition from whole numbers to fractions and from fraction concepts to fraction operations, teachers themselves need to have a high level of ability to construct word problems and must possess a well-developed understanding of fraction concepts and operations. Because of this, learning what pre-service teachers know and how they interpret mathematics is critical. This information will guide teacher educators as they plan for the needs of their pre-service teachers. This information can be used to determine what teacher educators still need to know in order to effectively develop and provide better-quality teacher preparation and professional development to the pre-service teachers they serve.

Theoretical Perspectives

The theoretical framework for this research derives from the noteworthy discussion of subject-matter content knowledge by Shuman (1986, 1987). Mathematics content knowledge is rooted in knowledge and application of procedural facts, algorithms, and methods as well as the understanding of how these are interrelated (Kahan, Cooper, & Bethea, 2003). The task of writing word problems, which has been recommended as an avenue to assess the depth of students' mathematical understanding and application (Barlow & Drake, 2008), can serve as a tool for teacher educators to evaluate the mathematics content knowledge of their pre-service teachers. Teachers' abilities to write word problems reflect their mathematical literacy, which can be classified into a hierarchical system of performance levels. The highest performance level of mathematical literacy should include a contextual application and an understanding of mathematics (Kaiser & Willander, 2005; OECD, 2006).

Teachers' abilities to appropriately write word problems also reflects their mathematics content knowledge, which can be evaluated in terms of their paradigmatic and narrative knowledge as proposed by Bruner (1985, 1986) and referred to by Chapman (2006). Paradigmatic knowledge focuses on "mathematical models or mathematical structures that are universal and context-free" (Chapman, p. 216). In relation to word problems, individuals' paradigmatic knowledge can be assessed through analyzing the semantic structure of word problems, the mathematical models or structures that are evoked in a word problem. Narrative knowledge focuses on contexts such as characters, objects, situations, actions, and/or intentions (Chapman,). When contexts are considered, appropriate units of measure are the focus of assessment (Simon, 1993).

Purpose and Questions of Current Research

To recognize the quality and depth of pre-service elementary teachers' mathematics content knowledge, there is a need to explore the ability of pre-service teachers to translate fraction multiplication into words. To this end, the purpose of this research is to investigate the ability of pre-service teachers to represent the symbolic expressions of fraction multiplication in words.

More specifically, this research focuses on the following research questions: (1) What average score can pre-service elementary teachers make when asked to represent two symbolic problems of fraction multiplication as word problems? (2) What kind of word problems do pre-service elementary teachers construct to represent a symbolic problem of fraction multiplication?

Method

Participants

The participants were a group of 127 pre-service elementary teachers in the U.S. They were enrolled in an undergraduate early childhood and elementary education program that prepares pre-service teachers to teach children in pre-kindergarten through fifth grade. They were at the beginning of the mathematics methods course when participating in this research, but were already given classes in whole number operations and in fraction concepts.

Instrument

This research involved one instrument that includes two representative problems where the participants were asked to use words to interpret symbolic problems of fraction multiplication. The small number of problems was used to ensure that the participants were able to complete the instrument within the allotted time limit of the class.

The first problem in the instrument was developed to determine if participants were capable of using words to interpret the symbolic problem "$1\frac{2}{3}\times 4=?$." The second problem was developed to determine if participants were capable of using words to interpret the symbolic problem "$\frac{1}{2}\times\frac{1}{3}=?$." According to Izsak (2006), the first problem is a two-level problem of "parts of a whole," and the second problem is a more complicated three-level problem of "parts of parts of a whole."

Data Coding

The techniques of content analysis (Gall, Borg, & Gall, 1996) were utilized to develop coding categories for this research. First, to increase the reliability of the coding system, an explicit scoring rubric for evaluating the performance levels of collected word problems was developed (see Appendix). Each of the word problems was graded with one of five performance levels ranging from a failing level with a score "0" to a good level with a perfect score "1." The description for each performance level was developed based on the discussion of mathematical literacy by Kaiser and Willander (2005), as well as the nature of the given symbolic problems.

Second, the category of semantic structures was developed based on the discussion of number operations in the literature (Christou & Philippou, 1998; Izsak, 2006; Schmidt & Weiser, 1995; Simon, 1993; Van De Walle, 2007), as well as the nature of the given symbolic problems. For the first symbolic problem "$1\frac{2}{3}\times 4=?$," the semantic structures were categorized into "none" if no word problem was written by the pre-service teacher, "repeated addition," "multiplicative comparison," and "other structure." For the second symbolic problem "$\frac{1}{2}\times\frac{1}{3}=?$," the word problem was categorized into "none," "multiplicative comparison," "equal sharing of a fraction," "part of a fraction," and "other structure."

Third, the category of units of measure was developed based on the units of measure associated with the product. Since the category of units of measure is bonded to the contexts of collected word problems, it was not developed until the unit of measure was scanned. In this research, the units of measure were categorized into "none" if there was no word problem; "container," "pizza," "cake," "cookie," "other food," and "other unit."

Lastly, to ensure the reliability, the rubric and samples of coding data had been reviewed by an external evaluator who is an experienced researcher in the area of fractions. Based on the suggestions made by this evaluator, some adjustments for the rubric and data coding were made.

Data Analysis

The data analysis was completed in two phases. The first phase focused on analyzing the percentage of pre-service teachers who constructed appropriate word problems for the given symbolic expressions of fraction multiplication to answer the first research question. The overall descriptive statistics were analyzed. Frequency counts and percentages were ascertained for this research group of pre-service teachers, with respect to each performance level (i.e., failing, poor, weak, fair, and good).

The second phase focused on analyzing pre-service teachers' word problems in terms of their semantic structures and units of measure - to answer the second research question, "What kind of word problems do pre-service elementary teachers construct to represent a symbolic problem of fraction multiplication?" Then, frequency counts and percentages were ascertained for this research group of pre-service teachers, with respect to their word problems' semantic structures and units of measure.

Results

Overall Performance on Word-Problem Writing

The results confirm the earlier conjuncture that it is less difficult to construct a word problem for fraction multiplication with a whole number factor than a problem without a whole number factor. As shown in Table 1, when all of the scores of these pre-service teachers were averaged together, the pre-service teachers' overall mean score for correctly representing the first symbolic problem "$1\frac{2}{3} \times 4 = ?$" was .7539 and the second symbolic problem "$\frac{1}{2} \times \frac{1}{3} = ?$" was .3661. This means that a significant percentage of these pre-service teachers were unable to construct appropriate word problems for the given symbolic problems.

In terms of performance levels, Table 2 shows that only about half of the pre-service elementary teachers (58.3%) could correctly construct a "good" word problem to represent the first symbolic problem "$1\frac{2}{3} \times 4 = ?$" and about a quarter of them (27.6%) could correctly construct a "good" word problem to represent the second symbolic problem.

Table 1

Overall Performance on Word Problem Writing

Symbolic problem	*N*	Minimum	Maximum	Mean	*SD*
$1\frac{2}{3} \times 4 = ?$	127	.00	1.00	.7539	.33921
$\frac{1}{2} \times \frac{1}{3} = ?$	127	.00	1.00	.3661	.44964
Valid *N* (listwise)	127				

Table 2

Performance Levels on Word Problem Writing

Level	Frequency	Percent	Cumulative Percent
First symbolic problem "$1\frac{2}{3} \times 4 = ?$"			
Failing	13	10.2	10.2
Poor	4	3.1	13.4
Weak	25	19.7	33.1
Fair	11	8.7	41.7
Good	74	58.3	100.0
Total	127	100.0	
Second symbolic problem "$\frac{1}{2} \times \frac{1}{3} = ?$"			

Failing	72	56.7	56.7
Poor	5	3.9	60.6
Weak	4	3.1	63.8
Fair	11	8.7	72.4
Good	35	27.6	100.0
Total	127	100.0	

On the other hand, the remaining pre-service teachers constructed word problems at varying levels less than "good." Of greatest concern were the pre-service teachers who performed on a "failing" level because they could not write a solvable word problem using the given multiplication sentence (10.2% - first problem and 56.7% - second problem). For example, instead of writing a solvable word problem for multiplying the two unit fractions "$\frac{1}{2}$" and "$\frac{1}{3}$", one of the pre-service teachers wrote the addition problem, "Bobby has half of a sheet cake. Marla has a third of a sheet cake. How many do they have all together? "

Semantic Structures

To represent the first symbolic problem "$1\frac{2}{3} \times 4 = ?$," as shown in Table 3, 110 of 127 pre- service teachers (86.6%) constructed a "repeated addition" problem. Those "repeated-addition" word problems represent the most "basic intuitive meaning of multiplication" (Schmidt & Weiser, 1995, p. 66) to the pre-service teachers. Additionally, only five of 127 pre-service teachers (3.9%) constructed a "multiplicative comparison" problem. Regardless of whether the written problem was categorized as a "repeated addition" or a "multiplicative comparison" problem, the multiplicand was always the mixed number "$1\frac{2}{3}$," and the multiplier was always the whole number "4."

Table 3
Semantic Structures of Collected Word Problems

Semantic Structure	*N*	Percent	Mean	Std. Deviation
First symbolic problem "$1\frac{2}{3} \times 4 = ?$"				
None	2	1.6	.0000	.00000
Multiplicative Comparison	5	3.9	.9000	.22361
Repeated Addition	110	86.6	.8295	.24869
Other Structure	10	7.9	.0000	.00000
Total	127	100.0	.7539	.33921

Second symbolic problem "$\frac{1}{2}\times\frac{1}{3}=?$"				
None	35	27.6	.0000	.00000
Multiplicative Comparison	2	1.6	1.0000	.00000
Fair Sharing of a Fraction	19	15.0	.8684	.26833
Part of a Fraction	36	28.3	.7778	.30342
Other Structure	35	27.6	.0000	.00000
Total	127	100.0	.3661	.45074

To represent the second symbolic problem "$\frac{1}{2}\times\frac{1}{3}=?$," also shown in Table 3, it appears that the "fair sharing of a fraction" (15%) and "part of a fraction" (28.3%) structures are the most intuitive ones to this group of pre-service teachers. Similar to the word problems constructed for the first symbolic problem, very few pre-service teachers tried to use a "multiplicative comparison" (1.6%) structure to interpret the multiplication of a fraction by a fraction.

Furthermore, as shown in Table 3, all of the "other" structure problems are failing examples (as noted by mean scores of .0). None of the "other" structure problems is a "product-of-measures" (Van De Walle, 2007, p. 161) structure in which the multiplication product consisted of a two-dimensional unit, such as a two-lengths (length × width) unit for the product of an area. Also, none of the "other" structure problems applied probability concepts by counting how many events or what probability could be made with two number factors.

Units of Measure

This research found that most pre-service teachers used food as their unit of measure for multiplication products (68.5% - first problem and 60.6% - second problem). For the first symbolic problem, as shown in Table 4, 23 of the 127 pre-service teachers (18.1%) used "pizza" as their unit of measure, 19 (15.0 %) used "cake," 17 (3.4 %) used "cookie," and 28 (22.0%) used some "other food," such as pies or brownies as their unit of measure. Similarly, for the second symbolic problem, as shown in Table 4, among the units of measure involving food, the use of "pizza" is the most popular one.

Table 4
Units of Measure of Collected Word Problems

Unit of Measure	N	Percent	Mean	Std. Deviation
First symbolic problem "$1\frac{2}{3}\times 4=?$"				

None	2	1.6	.0000	.00000
Container	24	18.9	.9688	.11211
Pizza	23	18.1	.6630	.40317
Cake	19	15.0	.5658	.32105
Cookies	17	13.4	.7941	.36695
Other Food	28	22.0	.8036	.28347
Other Unit	14	11.0	.7500	.31009
Total	127	100.0	.7539	.33921
Second symbolic problem "$\frac{1}{2}\times\frac{1}{3}=?$"				
None	35	27.6	.0000	.00000
Container	6	4.7	.7917	.40052
Pizza	29	22.8	.4310	.45756
Cake	17	13.4	.6029	.46820
Cookies	9	7.1	.7778	.36324
Other Food	22	17.3	.3750	.43473
Other Unit	9	7.1	.4167	.50000
Total	127	100.0	.3661	.45074

Table 4 also shows that 24 of the pre-service teachers (18.9%) used "container" such as glasses, cups, or cans as a unit of measure in their word problems for the first symbolic problem, and 6 (4.7%) used it for the second symbolic problem. Still, most of the pre-service teachers who used "container" as a unit of measure also used food-related contexts in their word problems.

Moreover, although various "other" units of measure such as "land," "ribbon," and "reading time" were used, all of them were one-dimensional units of measure. None of the pre-service teachers created a word problem with a two-dimensional unit of measure, like an area problem, nor did they create a problem using probability. Different from the adoption of an everyday object as a unit of measure, one pre-service teacher adopted pattern blocks and used a hexagon shape to represent one whole by writing the word problem, "Catherine wants to play with pattern blocks; she wants to play with 1 hexagon and 2 parallelograms ($1\frac{2}{3}$). She also wants to play with 4 of her friends. She wants to give them the same amount she has. How many hexagons and parallelograms do all friends receive?" In this example, pattern blocks were adopted and a hexagon shape was used to represent one whole.

Discussion and Implications

This research highlights the need to promote the ability of pre-service elementary teachers to construct word problems as an avenue for developing and assessing students' understanding of critical mathematical concepts and for engaging students in meaningful, real-world mathematics. The results suggest that a significant percentage of the pre-service teachers in the study did not have the ability to construct a completely appropriate word problem and that, taken together, pre-service teachers' semantic structures and units of measure are limited.

Failure to Construct Appropriate Word Problems

There are at least two obvious reasons that any pre-service teacher may be unable to construct appropriate word problems for the current study. One reason may be that this pre-service teacher may not possess the mathematical content knowledge associated with multiplication of fractions. Another reason may be that this pre-service teacher may not possess the knowledge needed to construct word problems.

Regardless of the reason which caused their failure to construct appropriate word problems, this area of research needs more attention because teachers' knowledge greatly impacts students' achievement (Hill, Rowan, & Ball, 2005; Kulm, 2008). Teachers cannot give their students knowledge that they do not possess. To this end, teacher educators may need to re-arrange and re-structure the content and pedagogical courses for pre-service teachers in order to ensure that their limited knowledge of fraction multiplication and word problem construction are addressed. If teachers truly buy word problems as being an avenue for developing and assessing students' understandings and for engaging students meaningfully, then perhaps course rubrics and evaluations need to require word problem construction in addition to content and pedagogy. Teacher educators need to consider alternative approaches for improving the mathematics preparation of pre-service elementary teachers, and we must test the effectiveness of these approaches on our pre-service teachers. We must make time to push for the deep understandings that we wish our pre-service teachers to possess.

Limited Semantic Structures and Units of Measure

Also, this research shows that the diversity of pre-service teachers' interpretations of the symbolic expressions of fraction multiplication appeared limited. There was little variation in the structure and context of their problems. In terms of semantic structures, most of the collected word problems were an extension of the basic "forming the n^{th} multiple of measures" structure for a whole number operation. In terms of units of measure, the majority of pre-service teachers used a food related object as the unit of measure for their multiplication products. These findings raise the question: Can pre-service elementary teachers nurture children's construction and interpretation of fraction multiplication in a variety of ways or are they limited to the structure and unit of measure they used in this research?"

In order to be able to reach as many of their students as possible, it is essential for teachers to know how to provide alternative structures and contexts for a formal symbolic expression. It ensures that teachers have the breadth of knowledge and ability that they need in order to reach a variety of students. Further studies must be undertaken to recognize whether pre-service teachers have the ability to construct a broad range of word problems for fraction operations.

Future Research

In addition, some findings from this study could serve as starting points for future experimental studies. Answers to the following questions, for example, would be helpful in the development of efficient methods for instructing pre-service teachers. First, will increasing pre-service teachers' breadth of mathematics content knowledge motivate or enable them to construct a greater variety of word problems for fraction operations? This question was raised from the finding that all the appropriate word problems were limited to the basic intuitive semantic structures. It would be valuable to investigate whether pre-service teachers who have been given more training in bridging fractions with other topics such as probability and geometry, are more willing and capable of developing different meanings for those areas of fraction multiplication.

Second, will enhancing pre-service teachers' experiences with synthetic materials strengthen their abilities to construct word problems for fraction operations? This question is raised from the finding that one of the pre-service teachers adopted pattern blocks in her word problem as the unit of measure. The use of this kind of synthetic material, in comparison to the other everyday objects, suggests that the provision of experiences with a variety of materials may enable pre-service teachers to develop a greater variety of word problems, involving a greater variety of units of measure. Since it is often observed that mathematics cannot be modeled directly from our everyday environment (Skemp, 1987), teachers' abilities to use available synthetic materials, such as fraction circles, pattern blocks, and base-ten blocks, to construct the meaning of fraction multiplication become important. For further implications to teacher education and development, it is worthwhile for education researchers to explore which materials, as well as how effective a type of material, help pre-service teachers develop their abilities to construct word problems for fraction multiplication.

Third, will pre-service teachers perform better in writing word problems for fraction multiplication if they have more experiences in writing word problems? The pre-service teachers attending this research were at the beginning of their mathematics methods course. As previously stated, they may have lacked experiences in writing word problems. Thus, the results might more reflect their limitations in writing word problems than in understanding fraction multiplication. A follow-up experimental study would help to clarify whether the performance of word-problem writing is determined by experience in word-problem writing or by knowledge of fraction multiplication.

Fourth, will the order in which fraction multiplication problems are presented make pre-service teachers interpret that problem in a different way? It was found that all the pre-service teachers who wrote a word problem for the symbolic problem "$1\frac{2}{3}\times 4=?$" treated the mixed number "$1\frac{2}{3}$" as the multiplicand and the whole number "4" as the multiplier. It appears more intuitive for the pre-service teacher to deal with an integral repetition of a fraction than a non-integral repetition of a whole number. Does the order of presentation influence this interpretation? To draw conclusions, the order of presentation must be tested.

Lastly, will an emphasis on the transition from whole number multiplication to fraction operations enable pre-service teachers' construction of word problems for fraction operations? This question is raised from the finding that the pre-service teachers' performed much better for the first symbolic problem than for the second one. It means a significant proportion of the pre-service teachers cannot successfully transfer their knowledge of "fraction by whole number" multiplication to that of "fraction by fraction" multiplication. School mathematics curricula and teaching in the U.S. often fail to make effective connections across various rational number relations (Davis & Maher, 1993). It is worth knowing whether the ability of pre-service teachers to construct word problems for fraction multiplication can be increased if the connection between whole-number and non-whole number operations is emphasized.

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Appendix

Grading Rubric and Sample with Respect to Each Performance Level of Mathematics Literacy

Level	Description
Failing (Score=0)	(a)An individual does not write a word problem; or (b)An individual ignores basic mathematical concepts and methods; his or her word problem is not solvable by using the given multiplication sentence. *Sample #23-1:* Catherine $1\frac{2}{3}$ oranges. She wants to share

	them among her friends. How much oranges does each friend get? *Comment:* This problem is incomplete because it does not address the number of friends. It violates basic multiplication concepts and methods of the given sentence "$1\frac{2}{3}\times 4 = ?$".
Poor (Score=.25)	(a)An individual demonstrates a basic understanding of mathematical concepts and methods, but does not demonstrate the ability to unify and relate central mathematical concepts and methods. (b)The word problem is solvable by using a multiplication sentence, but does not match with the given numerical value. *Sample #118-2:* Jo made brownies and split the pan in $\frac{1}{2}$ for his sister. She ate $\frac{1}{3}$ of the $\frac{1}{2}$ of her part. If you took the total pan how much did she eat? *Comment:* This problem represents the sentence "$\frac{1}{3}\times\frac{1}{2}\times\frac{1}{2} = ?$" instead of the given Sentence "$\frac{1}{2}\times\frac{1}{3} = ?$".
Weak (Score=.50)	(a)An individual demonstrates an ability to unify and relate central mathematical concepts and methods, but does not demonstrate a contextual understanding and application of mathematical concepts and methods. (b)The word problem matches the given multiplication sentence, but includes a logical error or misleading context. *Sample #31.1:* Four children have each got $1\frac{2}{3}$ of a sheet cake. How many cakes do they have all together? *Comment:* There is a logical error for the description "$1\frac{2}{3}$ of a sheet cake" because it is not reasonable to have more than a whole out of a sheet cake.
Fair (Score=.75)	(1)An individual demonstrates a contextual understanding and application of mathematical concepts and methods, but does not demonstrate it in a clear or coherent manner sufficiently. (b)The word problem is logically and contextually correct, but does not show sufficient clarity or coherence. *Sample #97-2:* Mom made a cookie. Sally frosted $\frac{1}{3}$ of the cookie. On the frosted part, she put sprinkles on $\frac{1}{2}$. How

	much has sprinkles? *Comment:* The question, "How much has sprinkles?" is ambiguous. The question should have stated "how much of the cookie is covered with sprinkles?" or "what fraction of the cookie has sprinkles?"
Good (Score=1.00)	(a)An individual demonstrates a contextual understanding and application of mathematical concepts and methods in a clear and coherent manner. (b)The word problem is logically and contextually correct, and clearly and coherently described. *Sample #125-1:* Mary had $1\frac{2}{3}$ pieces of pie. Joe had 4 times as many pieces. How many pieces of pie does Joe have?

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