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Study on Effect of Mathematics Teachers' Pedagogical Content Knowledge on Mathematics Teaching

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Using Kawanaka and Stigler' s (1999)categories and definitions, we analyzed two mathematics lessons about one-variable quadratic inequality in this article. One was taught by pre-service high mathematics teacher Z, the other was taught by in-service high mathematics teacher Z1 who had 12 *years of teaching and was responsible for Grade Ten's mathematics teaching in a high school in Xiaogan. The results showed:* (1) *How a mathematics teacher' PCK effected his/her mathematics teaching could be seen not only from his/her teaching objects, teaching structure, notion explaining, but also from his/her education views, teaching emotion, teaching design, teaching language, students' mathematics thinking, students' learning attitude and so on.*(2) *A mathematics teacher' PCK was combined with his/her other knowledge, such as mathematics knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of students, and so on.*(3) *Just as "Knowledge is infinite,' teacher's teaching had his /her excellence and teacher's teaching should be improved in teaching practice.*

Key words: pedagogical content knowledge, mathematics teachers, mathematics teaching.

Introduction

Because the need for quality has replaced the need for quantity, teachers' professional development has become a hot topic in teacher education all around the world and one of its key contents is to study teachers' knowledge.

Teachers' knowledge is their internalized cognitive structure and many studies and achievements have been made about it.

Shulman (1987), Peterson (1992), Bromme (1994), Fennema and Franke (1992) studied how many elements there were in mathematics teachers' knowledge. They set forward different views. For example, Shulman proposed a framework for analyzing teachers' knowledge that distinguished different categories of knowledge: knowledge of content, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge (PCK), knowledge of students, knowledge of educational contexts and knowledge of educational ends, purposes and values. He emphasized PCK as a key aspect to address in the study of teaching.

In later studies, the notion of PCK was often combined with other theoretical ideas. For example, Klein and Tirosh (1997) evaluated pre-service and in-service elementary teachers' knowledge of common difficulties that children experience with division and multiplication word problems involving rational numbers and their possible sources. They summarized the findings saying that "most prospective teachers exhibited dull knowledge" of the difficulties that children's experience with word problems involving rational numbers and their possible sources, whereas "most in-service teachers were aware of students incorrect responses, but not of their possible sources."

Throughout these studies on the effect of mathematics teachers' PCK on mathematics teaching, we thought there were two problems. One was that teaching objects, teaching structure, and notion explaining were often noticed in these studies, whereas a teacher's education views, teaching emotion, teaching design, teaching language, students' mathematics thinking, students' learning attitude were not noticed fully. The other was that most teachers in these studies were primary mathematics teachers. In other words, how middle mathematics teachers' PCK or high mathematics teachers' PCK affected their teaching was not studied adequately.

Based on the above, we would analyze two mathematics lessons about one-variable quadratic inequality in this article. One was taught by pre-service high mathematics teacher Z, the other was taught by in-service high mathematics teacher Z1 who had 12 years of teaching and was responsible for Grade Ten's mathematics teaching in a high school in Xiaogan.

Method

Kawanaka and Stigler (1999) thought, in classrooms, speakers were mainly teachers or students. They asked questions, answered questions, provided feedback to answers, provided information, provided directions, and so forth. they developed coding categories to capture these behaviors:

soliciting, responding, reacting and utterances. The categories and definitions were listed as follows:

(1)Elicitation (E): A teacher utterance intended to elicit an immediate communicative response from the student(s), including both verbal and nonverbal responses.

(2)Information (I): A teacher utterance intended to provide information to the student(s) that did not require communicative or physical response from the student(s).

(3)Direction (D): A teacher utterance intended to cause students to perform immediately some physical or mental activity. When the utterance was intended for future activities, it was coded as information even if the linguistic form of the utterance was a directive.

(4)Uptake (U): A teacher utterance made in response to a student's verbal or physical responses. It was intended only for the respondent. When the utterance was clearly intended for the entire class, it was coded as information instead of uptake.

(5)Teacher Response (TR): A teacher utterance made in response to a student elicitation.

(6)Provide answer (PA): A teacher utterance intended to provide the answer to the teacher's own elicitation.

(7)Response (R): A student utterance made in response to teacher elicitation or direction.

(8)Student elicitation (SE): A student utterance intended to elicit an immediate communicative response from the teacher or from other students.

(9)Student information (SI): A student utterance intended by a student to provide information to the teacher or to other students that did not require an immediate response.

(10)Student direction (SD): A student utterance intended to cause the teacher or other students to perform immediately some physical or mental activity.

(11)Student uptake (SU): A student utterance intended to acknowledge or evaluate another student's response.

(12)Other (O): An utterance that did not fit into any of the previous categories or that was not intelligible.

Further, they classified elicitation requesting subject-related information into three categories to capture the cognitive demands of teachers' questions, and the definitions of the three categories were as follows:

(1)Yes/no (YN): Any content elicitation that requested a simple yes or no as a response.

(2)Name/state (NS): Any content elicitation that (a) requested a relatively short response, such as vocabulary, numbers, formulas, single rules, prescribed solution methods, or an answer for computation; (b) requested that a student read the response from a notebook or textbook; and (c) requested that a student chose among alternatives.

(3)Describe/explain (DE): Any content elicitation that requested a description or explanation of a mathematical object, nonprescribed solution methods, or a reason why something was true or not true.

Example 1. Yes/no

[E][YN]Teacher: Wait a second. Do you need this inside too?

[R]Student: This isn't needed.

Example 2. Name/state:

[E][NS] Teacher: How do you solve this?

[R]Student: You cross-multiply them.

Example 3. Describe/explain:

[E][DE] Teacher: So how do you think you should explain?

[SE]Student: Oh, my explanation?

[TR]Teacher: Yes. That's what we want to hear.

[R]Student: Um…one…one…un…so a figure in which this and this are connected …and…[explanation continues].

Next, we would use these categories to analyze how teacher Z's and teacher Z1's PCK affect her/his mathematics teaching.

Teaching Episode

Teaching Episode 1. Pre-service teacher Z

1[I] T (teacher Z): We will learn how to solve one-variable quadratic inequality in this lesson. (Teacher Z writes the topic on the blackboard.) First, let us to look at some questions:

a. Give the graph of $y=x^2-x-6$

b. Find the roots of $x^2-x-6=0$ through the graph of $y=x^2-x-6$.

c. Find the solution set of $y=x^2-x-6>0$ through the graph of $y=x^2-x-6$.

2[E][NS] T: The first question: give the graph of $y=x^2-x-6$. What is the graph of $y=x^2-x-6$?

3[R] S (Student): Parabola.

4[E][NS] T: Does the graph of $y=x^2-x-6$ intersect the x axis? If does, how many intersection points are there?

5[R] S: yes, there are two intersection points.

6[E][YN] T: Could you find the two intersection points?

7[R] S: yes.

8[E][NS] T: What are they?

9[R] S: −2, 3.

10[E][NS] T: (Teacher Z draws the coordinate axis, then draws (-2,0) and (3,0))What is the direction of $y=x^2-x-6$?

11[R] S: Up.

12[U]T:(Teacher Z draws the graph of $y=x^2-x-6$ approximately (Figure 1)) Good!

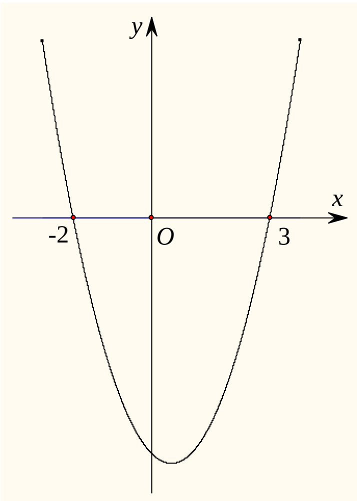

Figure 1. **The graph of $y=x^2-x-6$.**

13[E][NS] T: The second question: find the roots of $x^2-x-6=0$ through the graph of $y=x^2-x-6$. That is to say, in the expression of $y=x^2-x-6$, order $y=0$, then, ……

14[R] S: −2, 3.

15[E][YN] T: The third question: find the solution set of $y=x^2-x-6$ through the graph of $y>x^2-x-6$. That is to say, in the expression of $y=x^2-x-6$, order $y>0$, all right?

16[R] S: yes.

17[I] T: $y>0$ means the graph locates the upside of the x axis (Teacher Z marks the corresponding graph with a red chalk.) and the corresponding x is this section or that section (Teacher Z points to the graph of $y=x^2-x-6$) That is to say, $x<-2$ or $x>3$.

18[E][YN] T: Yes or no? Is the result true or not true?

19[R] S: No.

20[I] T: The result should be the solution set$\{x|x<-2$ or $x>3\}$. Let us summarize the findings: First, we draw the graph of $y=x^2-x-6$. Second, find the corresponding x. Third, find the solution set of $y=x^2-x-6$. That is to say, the solution set of $y>x^2-x-6$ is found through the graph of $y>x^2-x-6$. So, one-variable quadratic equality and one-variable quadratic inequality can be studied through corresponding graph, and it embodied the mathematics idea that a function is combined with its figure closely. This is a specific example. Next, we will study its general instance $ax^2+bx+c>0$ ($a>0$) or $ax^2+bx+c<0$ ($a>0$)……

(The all time is 6:38)

Teaching Episode 2. In-service teacher Z1

1[I] T (teacher Z1): We have learned the quadratic function and learned its graph, its properties and its solution. Next, let us to solve three questions:

a. Solve the equation of $x^2-x-6=0$.

b. Give the graph of $y=x^2-x-6$.

c. Find the solution set of $x^2-x-6>0$ through the graph of $y=x^2-x-6$.

2[E][NS] T: First, solve the equation of $x^2-x-6=0$. What about?

3[R] S: $x=3$ or $x=-2$.

4[U] T: $x=3$ or $x=-2$.

5[E][NS] T: Second, give the graph of $y=x^2-x-6$. How do we draw the graph?

6[R] S: list table, draw points, link points.

7[I] T: We omit the process of draw points and only draw some specific points.

8[E][NS] T: corresponding coordinate, this is …

9[R] S: $(-2,0)$.

10[E][NS] T: The coordinate of this point is…

11[R] S: $(3,0)$.

12[E][NS] T: The coordinate of this point is…

13[R] S: $(0,6)$.

14[E][NS] T: The coordinate of this point is…

15[R] S: $(\frac{1}{2}, -\frac{25}{4})$.

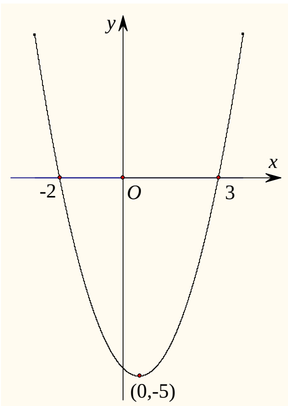

Figure 2. **The graph of $y=x^2-x-6$.**

16[E][NS] T: (Teacher Z1 draws the graph of $y=x^2-x-6$) Please observe this graph. The graph is given by listing table, drawing points and linking points. So, if there is a point and its horizontal coordinate is 4, what is its vertical coordinate?

17[R] S: 6.

18[U] T: 6.

19[E][NS] T: If a point of this graph is given and its vertical coordinate is found(Teacher Z1 points to the corresponding graph), we can see its vertical coordinate ……

20[R] S: Its vertical coordinate is bigger than 0.

21[U] T: Its vertical coordinate is bigger than 0, all right?

22[E][NS] T: If a point of this graph is given and its vertical coordinate is found (Teacher Z1 points to the corresponding graph), we can see its vertical coordinate ……

23[R] S: Its vertical coordinate is smaller than 0.

24[U] T: Its vertical coordinate is smaller than 0. Good!

25[E][NS] T: What about these two points? What are their vertical coordinate?

26[R] S: 0.

27[U] T: 0. Good!

28[I] T: The vertical coordinates of two points is 0 respectively. That is to say, (Teacher Z1 points to the corresponding graph and says) when $x=-2$ or 3, its corresponding y is 0. And if a point is given in this place, its corresponding y is bigger than 0, if a point is given in that place, its corresponding y is smaller than 0

29[E][NS] T: Can we find the solution set of $x^2-x-6>0$ through the graph of $y=x^2-x-6$?

30[R] S: Yes.

31[E][NS] T: $x^2-x-6>0$.…What is the relationship between it and $y=x^2-x-6$?

32[R] S: In $y=x^2-x-6$, $y>0$.

33[R] T: In $y=x^2-x-6$, $y>0$

34[E][NS] T: Through the graph, $y>0$ means the corresponding graph locates in…

35[R] S: Up.

36[I] T: Yes, $y>0$ means the graph locates the upside of the x axis.

37[E][NS] T: So, the value of y corresponding to this graph is…

38[R] S: $y>0$.

39[E][NS] T: This?(Teacher Z1 points to the corresponding graph)

40[R] S: $y>0$.

41[I] T: That is to say, $x^2-x-6>0$. If a point in this graph is selected, its vertical coordinate is bigger than 0, vice versa.

42[E][YN] T: Does this point have the property (Teacher Z1 points to the graph)

43[R] S: No.

44[E][YN] T: Does that point have the property?(Teacher Z1 points to the graph)

45[R] S: No.

46[E][YN] T: This point?

47[R] S: No.

48 [E][YN] T: These points? Are these points' vertical coordinate bigger than 0?

49[R] S: Yes.

50[E][NS] T: If these points are put in the x axis, what can we get?

51 [R] S: $x>3$ or $x<-2$.

52[I] T: We can describe $x>3$ or $x<-2$ in the x axis with red chalk.

53[E][NS] T: So, what is the solution set of $x^2-x-6>0$?

54[R] S: $\{ x|x>3 \text{ or } x<-2\}$.

55[E][NS] T: And, what is the solution set of $x^2-x-6>0$?

56[R] S:$\{x|-2<x<3\}$.

57[I] T: The solution sets of $x^2-x-6>0$ and $x^2-x-6<0$is found through the graph.

This is a specific example. Then, how about the general instance? This is what we will learn today, $ax^2+bx+c>0$ ($a>0$) or $ax^2+bx+c<0$ ($a>0$).

(Teacher Z1 writes the topic on the blackboard)

Analysis

Through previous teaching episode 1 and teaching episode 2, we could see pre-service teacher Z had finished her teaching from the special example to the general instance within six minutes. However, in-service teacher Z1 used ten minutes when he taught the same content. Based on Kawanaka and Stigler (1999) coding categories, the differences about the questions posed by teacher Z and teacher Z1 were as follows:

(1) The sequence for posing three questions was different, especially the former two questions.

First, teacher Z let students draw the graph of $y=x^2-x-6$; second, let students find the roots of $x^2-x-6=0$; third, he let students find the solution set of $x^2-x-6>0$. On the other hand, teacher Z1 first let students solve the equation of $x^2-x-6=0$; second, he let students draw the graph of $y=x^2-x-6$; third, he let students find the solution set of $x^2-x-6<0$ through the graph of $y=x^2-x-6$.

From the sequence for posing three questions by teacher Z and teacher Z1, we could see teacher Z thought the latter two questions were solved based on the first question. That was to say, the latter two questions could be solved through the graph of $y=x^2-x-6$. But, teacher Z1 thought the roots of $x^2-x-6=0$ should be found before the graph of $y=x^2-x-6$ was drawn, namely, the roots of $x^2-x-6=0$ were special points of the graph of $y=x^2-x-6$ (This view could also be seen from 7th row to 11th row in the teaching episode about teacher Z1). Thus, the different teaching design between teacher Z and teacher Z1 was reflected.

(2) As to the question "finding the roots of $x^2-x-6=0$" or "solving the equation of $x^2-x-6=0$," this question was easy for students in high school. So, two teachers let students answer this question directly. (This view could be seen from 13th row to 14th row in the teaching episode about teacher Z and from 2th row to 4th row in the teaching episode about teacher Z1). The only difference was that teacher Z1 often repeated students' answers.

(3) As to the question "drawing the graph of $y=x^2-x-6$," this question was also not difficult for students in high school.

Teacher Z asked 5 questions (Please saw 2th row to 12th row in the teaching episode about teacher Z): What was the graph of $y=x^2-x-6$? ——Did the graph of $y=x^2-x-6$ intersect the x axis? If it did, how many intersection points were there? ——Could you find the two intersection points? ——What were they? ——What was the direction of $y=x^2-x-6$?

Teacher Z1 also asked 5 questions (Please see 5th row to 15th row in the teaching episode about teacher Z1): How did we draw the graph? ——drew some specific points——The coordinate of this point was……——The coordinate of this point was……——The coordinate of this point was……——

From the first question, we could see teacher Z's and teacher Z1's different emphasis. Teacher Z particularly emphasized visual thinking and let students recall the form of the graph. On the other hand, teacher Z1 particularly emphasized logical thinking and let students recall the step about drawing the graph of function.

Next, teacher Z let students find the two intersection points between $y=x^2-x-6$ and the x axis and the direction of $y=x^2-x-6$. Then, teacher Z drew the graph $y=x^2-x-6$ approximately under right-angle coordinate system without unit length. On the other hand, teacher Z1 let students find some specific points, including two intersection points between $y=x^2-x-6$ and the x axis, one intersection point between $y=x^2-x-6$ and the y axis and the vertex of the parabola. These specific points could ascertain the approximate form of the graph. Then, teacher Z1 drew the graph $y=x^2-x-6$ under right-angle coordinate system with unit length.

It was a good question set forth by teacher Z when he asked students to recall the form of the graph. Actually, I once met such a thing in one mathematics teaching lesson: when I asked students to draw the graph of a function, they knew how to list table and how to draw points, but didn't know how to link points, namely, they didn't know these points were linked by linear form or by curvilinear form and didn't know these points were linked by continuous form or by discontinuous form. Thus, some mistakes were made when students were linking points. For example, some students didn't notice the function domain and the linking exceeded the function domain. So, if possible, mathematics teachers should let students ascertain the approximate form of the graph before they draw the graph since it is helpful for students learning the process of drawing the graph. Nevertheless, teacher Z didn't pay attention to cultivating students' precise mathematics thinking. For example, teacher Z didn't notice the importance of the vertex of the parabola in ascertaining the graph of $y=x^2-x-6$. Furthermore, teacher Z drew the graph of $y=x^2-x-6$ under right-angle coordinate system without unit length. All these teaching behaviors would lead students to form bad learning habits when they are drawing the graph without necessary precise demand. Just as one teacher said, "Although it is not crucial for students learning in this lesson, drawing the graph is an important mathematical skill. For example, drawing a graph accurately is helpful for solving questions in solid geometry. Furthermore, drawing graphs accurately also shows mathematics beauty. "

Teacher Z1 emphasized the step of drawing graphs from the beginning and paid attention to cultivating students' logical thinking. However, he didn't let students recall the form of graph. Through our talking with teacher Z1, we knew that teacher Z1 thought this question was easy for his students and needn't spend much time on it, but this question should be noticed according to students' learning content in other occasions. As to the latter two questions, teacher Z1 didn't list the table and draw points casually, whereas he let students find some key points that ascertain the form of the graph. From these teaching behaviors, teacher Z1's mathematics understanding about function graph could be seen. Furthermore, teacher Z1 noticed necessary criterion about drawing graphs, such as three elements of right-angle coordinate system, including origin, unit of length and positive direction. He was also aware of students' enjoying mathematics beauty from the graph.

(4) As to the question "finding the solution set of $x^2-x-6>0$", how to solve this question was crucial for students' mathematics learning in this lesson, and was helpful for students understanding the solution set of $ax^2+bx+c>0$ ($a>0$) or $ax^2+bx+c<0$ ($a>0$).

First, teacher Z used a "Y/N" question and gave the solution set by herself. Then, she used a "Y/N" question again and reminded students to notice how to write the solution set. After the two questions were answered, teacher Z summarized the findings and gave the general instance (This view could be seen from 15th row to 20th row in the teaching episode about teacher Z).

However, teacher Z1 spent much time on the process of finding the solution set of $x^2-x-6>0$ and posed many questions to accelerate students' thinking. These questions were as follows (This view could be seen from 16th row to 56th row in the teaching episode about teacher Z1): choosing a special point of the graph and letting students to think about its horizontal coordinate and vertical coordinate—— letting students to think about some points in some scope, including these points' vertical coordinate were bigger than 0, equal to 0 and smaller than 0—— summarizing the findings—— asking students to find the solution set of $x^2-x-6>0$—— leading students to analyze the relationship between $x^2-x-6>0$ and $y=x^2-x-6$—— leading students to observe the relationship between the value of x and the value of y —— giving three reverse examples and one positive example to strengthen the relationship —— leading students to get the relationship between the function value and the value of x —— getting the result. After all this learning, teacher Z1 didn't summarize these solving processes immediately but posed another question: "what is the solution set of $x^2-x-6<0$?" If students could get the solution set of $x^2-x-6>0$, they should get the solution set of $x^2-x-6>0$. That meant the solution set of $x^2-x-6>0$ could promote students to understand the solution set of $x^2-x-6>0$. Only when all these were answered, teacher Z1 summarized the findings and extended the findings to the general instance.

Based on the above, we thought teacher Z1 holds better PCK than teacher Z about this question.

First, transition. Teacher Z didn't give any transition from the former question to this question; namely, she posed this question directly after the former question. But teacher Z1 gave some transition from the former question to this question in order to lead students to discover the findings.

Second, relationship: Although teacher Z asked students to find the solution set through the graph, teacher Z didn't lead students to think and many students didn't know how to do this. Maybe some students who were good at mathematics would know how to do, but most of the students didn't know how to find the solution set through the graph. Specially, the process of finding the solution set through the graph was just the key for students' mathematics learning in this lesson. Obviously, teacher Z omitted this difficulty in student's mathematics learning. On the other hand, teacher Z1, with more than 10 years of teaching, especially paid attention to this difficulty in student's mathematics learning. He led students to discover the relationship among one-variable quadratic equality, one-variable quadratic inequality and quadratic function. Furthermore, he gave three reverse examples to explain this relationship. Based on these learning, students could understand the relationship and get the solution set of one-variable quadratic inequality through the graph of quadratic function.

Third, the manner of question: Two questions posed by teacher Z were "Y/N" question and only needed students to answer yes or no. Thus, the scope of students' thinking was limited invisibly. On the other hand, all questions posed by teacher Z1 were "N/S" question, which demanded students to search corresponding knowledge from their cognitive structure. So, the scope of students' thinking was widened greatly and the extensity and flexibility of students' thinking were developed too.

Fourth, uptake: There were more than 4 times uptakes in teacher Z1's teaching (This view could be seen from 18^{th} \\$21^{th}$\\$24^{th}$\\$27^{th}$ row in the teaching episode about teacher Z1). These "uptakes" were helpful for forming better classroom atmosphere where teacher Z1 and students had harmonious emotion communicative and students thought mathematics questions actively. But teacher Z gave more attention to her own instruction and students' answers seemed to respond to her instruction. Thus, the principal status of students in mathematics learning was weakened and the atmosphere in the classroom was inactive and tedious. So, teacher Z had to say: "Please follow me, otherwise, I will …" Based on the above, the two teachers' education views could be seen, namely, teacher Z1 was aware of the importance of students' initiative, but teacher Z wasn't.

Fifth, mathematics language: When teacher Z was narrating mathematics language, she didn't notice the correlative criterion about mathematics language. For example, from the 8th and 9th row in the teaching episode about teacher Z, we could see that when students said the intersection point was -2, 3, she didn't correct the mistakes. Obviously, she didn't notice that a point should have its horizontal coordinate and vertical coordinate. However, teacher Z1 noticed the correlative criterion about mathematics language and no mistakes appeared in his teaching. Thus, two teachers' different mathematics understandings were reflected.

Certainly, we could also see, any "D/E" question didn't appeared in the teaching of teacher Z and teacher Z1. There were many reasons for it. One reason dealt with the learning content. Because students had learned one-variable quadratic equality and quadratic function in middle school, the difficulty in this lesson was: (1) understanding the relationship among one-variable quadratic inequality, one-variable quadratic equality and quadratic function; (2) understanding the solution set; (3) permeating function thought. So, two teachers emphasized the relationship and the mathematics thought and didn't use "D/E" question. Furthermore, they didn't also use "SE" "SI" "SD" "SU" questions.

Conclusion

(1) How a mathematics teacher's PCK affected his/her mathematics teaching could be seen not only from his/her teaching objects, teaching structure, notion explaining but also from his/her education views, teaching emotion, teaching design, teaching language, students' mathematics thinking, students' learning attitude and so on.

(2) Mathematics teacher's PCK was combined with his/her other knowledge, such as mathematics knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of students and so on.

(3) Just as "Knowledge is infinite," a teacher's teaching had his /her excellence and a teacher's teaching should be improved in teaching practice.

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