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Argumentation and Proving in Multicultural Classes: A Didactical Experience with Chinese and Italian Students

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The paper explores, through different theoretical/experimental investigations, a particular complex field of study such as the analysis of the teaching/learning phases in a multicultural setting in class. Through a linguistic-cultural approach it is a further close examination on the didactic thematic related to a possible comparison between East and West and particularly between Chinese mathematical thought and Italian thought, in some particular aspects related to the phase of argumentation and proving. To carry out the research, we chose to investigate, in a qualitative and quantitative way, the behavior of Chinese and Italian students of different ages (13-15 years old) attending the Secondary School of Palermo, on a number theory problem as pre-algebraic problem ("the game of prime numbers").

Key words: argumentation and proving, Jiuzhang Suanshu, algebraic thought, Chinese written language, Fermat problem.

Introduction and Theoretical Framework

The research works published in recent years on argumentation and conjecturation are now manifold. Some of this research essentially contemplates a historical- epistemological analysis of demonstration and therefore it describes its formative value for students. Others, instead, treat the problematic through a critical discussion of the learning environments of the demonstration phase; they are therefore particularly focused on the delicate passage from the phase of reasoning to that of demonstration. Our study is centered in this field of research as a further reflection on the treated problematic, also taking into account a possible didactic multicultural milieu: A real context in the today's school situation. In the scholastic year 2007-08 the non-Italian citizen pupils within the national scholastic system corresponded to 574.133 units. The phenomenon of immigration, increasing in our country, is continually reflected in the Italian school. The impact is so big that the affiliate students of foreign origin have increased to over 500,000 units in the last ten years. One of the most interesting didactic problems that today is defined in our classes is then the possibility to compare ourselves with the "diversability" in situations of multiculturalism, a reality in our society, in continuous socio-cultural change, and therefore a central point for the research in didactics. To analyze cognitive styles in different cultures, underlining those that can be the schemes of reasoning, the behaviors, the beliefs and the conceptions in respect to the acquisition of a particular concept, it is certainly a complex operation, but it can be the key to more careful didactics to the different abilities and therefore to the respect of others. In recent years, the research didactics of mathematics appeared very sensitive to the treated problematic. Different research studies allowed the highlighting of the role of history of mathematics as a tool of observation and analysis of teaching/learning situations in a multicultural setting; the role of the language and logic in the development of discipline and autonomous mathematical thought; the role of the socio-cultural context of the milieu in which the situation of multicultural teaching/learning situation is inserted and analyzed. In this sense, what we analyze when we discuss the binomials mathematics-culture, fundamental relation for this work and for a large tradition of research in the didactics of mathematics, is not only the presentation of specific techniques through which a certain group of people treat mathematical knowledge, but a critical discussion of possible correlations between the epistemological context of these people and the mathematical concepts treated which are elaborated and obtained in classes. This is the approach that we are following, in our work, even if in a first approximation.

Our research is therefore aimed at exploring a particularly complex field of study in which it is possible to find numerous "variables" in play, interconnected among themselves as for instance the knowledge, the socio-cultural context in which the knowledge is inserted and on which the mediation happens, the epistemology of the considered cultures, the schemes of reasoning typical of the origin culture of the students and so forth. In the specific case of the didactic problematic considered, we would like the paper specifically to be a further close examination of the didactic thematic related to a possible comparison between Chinese mathematical thought and Italian thought on the phases of argumentations and proving in a pre-algebraic mathematical context. In our opinion, the "variables" previously discussed, analyzed in two different cultures such as the Chinese and the Italian, could indeed turn out to be, in fact, interesting in many significant aspects (logical-argumentative, algorithmic, linguistic, etc.). The aim of this article is so to discover, in an experimental way, if some of the cultural differences verified in the Italian and Chinese cultures (we refer to the historical-epistemological aspect of the two cultures) could tacitly influence the didactical sphere of the discipline, the teaching/learning phases, highlighting analogies and differences on the basic nature of the cognitive process utilized by the culturally different students, placed in the same classes, on mathematical (pre-algebraic) problem solving. The first research questions from which we started for the experimental work were as follows: Is it possible to find, in a experimental way, an equivalence between the analogies and differences verifiable in the Chinese and Italian historical cultural tradition and the patterns of reasoning found today in multicultural situations of teaching/learning of mathematics? Can the definition of these differences referred to the phases of argumentation and proving, be significant for the mediation of particular mathematical knowledge in all classes between Chinese and Italian students? Also, as regards the choice of the problematic situation, is it possible to give a cultural "definition" of argumentation and proving? Can a particular mathematical context such as Number Theory, as a pre-algebraic mathematical context, help us, in this sense, to answer this question? These questions are not simple and they would deserve greater close examinations of a historical-epistemological nature. In this context we will treat the problematic only in a first approximation, underlying, through the experimental phase, how what constitutes the way to proving varies from culture to culture, as well as from age to age (to prove to convince, to prove to verify, to prove to generalize, etc.). Which can be the dynamics of reasoning highlighted by the Chinese and Italian students in the phase of reasoning and conjecturing in non-standard pre-algebraic problem solving? Aware of the impossibility to treat, in an exhaustive way, the framework of the variables in play, particularly “complicated” for the Chinese student placed in our Italian classes, we think, as a function of the experimental data discussed below, that it could be useful to present briefly, as an instrument of observation and analysis of multicultural learning/teaching situations with foreign Chinese students, the book of the Jiuzhang Suanshu as canon for the construction of mathematics (1st Cent. B.C.-1st Cent. A.D.).

After that in the following paragraph, we discuss, even if in a first approximation, the role of written natural language in the possible development of mathematical thought for the Chinese students. The main references in this sense are the works of Chemla (2003, 2007), Chemela and Guo, 2004, and Needham (1981) as regard the particular typical reasoning scheme for the Chinese Mathematical students. Parallel consideration has to be taken into account as regards the term Socrate/Platone/Aristotele for occidental civilization and Confucius/Tao/Buddha for the oriental one. Terms that, according to us, could influence the Italian and Chinese mathematical thought.

The Nine Chapters on Mathematical Art (J iuzhang Suanshu)

The Jiuzhang Suanshu or Nine Chapters on the Mathematical Art is a practical handbook of mathematics consisting of 246 everyday problems of engineering, surveying, trade, and taxation. It played a fundamental role in the development of mathematics in China, not dissimilar to the role of Euclid's Elements in western mathematics. It is thus a principal reference for mathematics in Chinese education, a canon both for the construction of mathematics (1st Cent. B.C.-1st Cent. A.D.) and for the teaching/learning of the same in the various historic periods. Among the most notable is the commentary of Liu Hui (263 A.D.) presented in the collection of the Mathematical Canons of the Tang Dynasty (618-907 A.D.). The key concept that organizes the description of the Jiuzhang Suanshu is the concept of “class” or “category” (lei) that plays a fundamental role in the commentaries. The elements that we find relevant for understanding the specificity of the book and so the related Chinese culture in reference to the Italian one (as in the Aristotelian/Euclidean tradition) are these: the typology of the problematic situation covered in the book, the modus operandi (shu) described, the evident algorithmic approach to mathematical problem solving and contextually the generalization of the different problems opportunely categorized and classified, the calculus instruments, the way to demonstrate the correctness of the problems (in the Chinese sense of term) and its epistemological values. The structure of the book is gradually articulated from the simple given of a problem (wen) related to a particular category, to its solution, “generalized” step by step through analogical reasoning, through a variable mutation strictly connected to the proposed context in a holistic vision (Nisbett, Peng, Choi, & Norenzayan, 2001). It is through a work on the procedure, on the resolute algorithmic that is possible to define the situation classes. The solution process is almost an adductive process where deduction and induction are together in a unique reasoning scheme. Perfection is defined in terms of simplicity and generality through a global vision of the problematic. So, what is the meaning of argumentation and proving, therefore, on the Jiuzhang Suanshu? According to the Chinese thought of the Jiuzhang Suanshu, to prove the validity of a mathematical reasoning, means to "demonstrate" the correctness of a procedure (using for example simple properties of arithmetic operations) with an algorithm approach in a possible combination of an iteration and of chosen "conditionals" and assignment of variable.

As Nisbett et al. (2001) declares

> The social worlds of East and West today reflect to a substantial degree their origin in Chinese and Greek culture, respectively…the social differences influence cognitive processes…we might expect to find cognitive differences among contemporary peoples that parallel those found in ancient times. (Nisbett et al., 2001, p.37)

Some of the differences that Nisbett pointed out and that we found, even if in a first approximation, in our Secondary school classes in which Chinese and Italian pupils study together, are specifically referred to: the algorithmic approach used by the Chinese students to argue and prove mathematical problems, a different verbalization from the Italian students in terms of generalization and relationship between the mathematical field and the object in the phase of argumentation; the organization into categories and the covering rules, instead of the organizing in terms of similarities and relationships (typical for the Chinese culture); the evidence that the "Western students" emphasize the non-contradiction, whereas Easterners emphasize on the "Middle Way" value of choice.

Observations on the Chinese Written Language

The organization of the Chinese written languages is complex: the different written characters are defined and classified in various categories or "meta-rules" of composition that are interesting from a mathematical point of view because, in our opinion, the activities of reading and writing this type of characters communicate a holistic mathematical thought, strongly algorithmic and relational-functional (en example is the contemporary relation "part-whole" on a written sign) that could influence the way of learning mathematics in many aspects; in particular in Algebra and transversally on the phase of augmenting and proving. Needham (1981) reports the classification in six classes and he discusses them in this way: a) Xiàngxíng (象形) "Forms of images, pictograms:" tree 木; sun 日; moon 月; mountain 山; horse 马; bird 鸟; b) Zhǐshì (指事), "Indicators of situation, indirect symbols"; c) Huìyì (会意), "Union of ideas, composition by association or logic composition". 80% of the ideograms are of the associative kind (Needham). They represent a sort of mental equations (Needham, 1981) as semantic combinations of two or more characters that are composed by association. We could find different examples for this: 男 [nán] man= 田 [tián] "field" + 力 [lì] "strength; 林 [lín] (森林 sēnlín) forest = tree 木 + tree 木 (plus 木). Two 木 [mù] trees side by side; 休 [xiū] stop, rest = 亻 (人 rén) + 木 [mù] tree. A person stopping to rest under a tree. d) Zhuǎnzhù (转注), "Transferable sense, symbols that it is possible to interpret reciprocally". e) Jiǎjiè (假借), "Language or sound". These characters are defined in a determinate general manner: the radical is associated to a phonetic sign to indicate the category in which we have to find the meaning of the word. So a lot of words with the same sound are written without confusion. (Needham, 1981). Example: 园 [yuán] garden = 囗 (wéi) "surround", suggesting a garden fence, and (full form) 袁 yuán phonetic or (simple form) 元 yuán phonetic; 袁 [yuán] or (simple form:) 元 [yuán] phonetic, and 辶(辵 chuò) "go" (to go far) = "far" f) Xíngshēng (形声), "Loan, rented phonetics character". The formation is very similar to the preceding case, but the way to construct the character is different.

Two other transversal interesting observations are referred to: (1) the use of contradiction in a "contemporary of the opposites" (From the point of view of Fuzzy Logic (Kosko, 1995) within the language: 杯 [bēi] cup: from 木 [mù] "wood" and 不 [bù] "not" phonetic. From the association of these two characters and so from the idea of "opposition/contrast" is born the concept of cup. Everybody knows that cups are 不 (not) made of 木(wood). (2) II) the idea of a variable as a thing that is varying and a parametric system within the composition of many characters. Some simple examples could be referred to the use of the radicals [kǒu] (口) or [tián] (田).

These characteristics of the written language seem to highlight, from a mathematical point of view, an internal search for a use of a common strategy to define "different characters" in which the radicals part assume a role of a parameter (in the mathematical sense of the terms) and communicate the meaning or the sound of the Character. Algebra or pre-Algebra, therefore, is a possible carrying element for the construction of the characters of the ideogram type for association. It seems to us a sort of search for a possible fundamental algorithmic to construct different “words” in a sort of generalization of the procedure with the aim of thinking more and more ample and sophisticated “classes of signs” proceeding by "analogy" and working towards concepts of sound in a possible construction of algorithmic maps. As also much neuroscience research confirms, the Chinese student learns to read and to write these signs, in a continuous parallelism, in a continuous relationship, between “serial thought” and “global thought” about each single sign, therefore, in a relational-functional aspect of the concept of variability (Radford, 2004), from the first years of their life.

Methodology

Taking into account the analyses reported before, referred to some key epistemological elements that, according to us, are really interesting for the study of the phase of argumentation and proving of the Chinese and Italian students of different ages on the resolution of particular mathematical pre-algebraic problems, we propose, in this paper, the description of the experimentation that we conducted with some Chinese and Italian students of the Lower and Upper Secondary school classes in Palermo. The descriptions of the results highlighted some features that allow us to recognize some key aspects of argumentation conjecturation and proving typical of the different cultures of the students involved, bringing back to some epistemological value of the own culture (philosophy, language, logical argumentative schemes). In the article, the attention particularly on the Chinese student and the analogies and differences that we found in their reasoning schemes in relation to the Italian ones will be stressed.

To carry out this, research we chose to investigate a pre-algebraic problem formalized as a Number Theory problem called “the game of prime numbers.” The choice was made for two reasons: firstly, because, according to us, these kinds of problems could represent nodal points from which the students derive the concepts of the letters as unknowns or “things which are varying” (Malisani & Spagnolo, 2005; Radford 2004). This is an interesting aspect, as we described before, not only from the mathematical point of view (idea of generalities and veracity of an assertion) but also strictly linked to the written Chinese language and so to its internal structure and secondly, these kinds of problems, as a-typical mathematical problems, could give us the possibility to analyze the free phase of argumentation, conjecuration and phase of proving from different viewpoints: reasoning schemes, strategy of solution and possible generalization, justification of the strategies, etc. These are the behaviors variable that we analyzed in a qualitative and quantitative way.

The given mathematical problem has a historical background, namely the determination of all the prime numbers which are sums of two squares. The first formulation of the theorem was expressed by Pierre de Fermat (1601-1665) in a letter to Merin Mersenne (1588-1648). The problematic situation presented in class arose in an informal manner during a lesson about the Pythagorean theorem and its converse. The simple observation of some examples of triples as $5^2 = 3^2 + 4^2$ or $13^2 = 5^2 + 12^2$ got the teacher to ask if a same relation can hold for prime numbers also, namely, when can a prime number be written as a sum of two squares? For example, 5 and 13 are sums of two squares, since $5 = 1^2 + 2^2$, $13 = 2^2 + 3^2$, while 3 and 19 cannot be written as a sum of two squares, since, for example, one can check, for 19, that none of the differences $19 - 1^2 = 18$, $19 - 2^2 = 15$, $19 - 3^2 = 10$, or $19 - 4^2 = 3$ is a square. This question, apparently simple, greatly interested the pupils. Therefore, the teacher posed a general question to find the generic form of all the prime numbers which can be written as a sum of two squares. The proposed situation problem has, according to us, three interesting characteristics: it uses the concrete experience field of arithmetic, semantic and syntactic register that is well known by all secondary school students and so significant in the solution and justification procedure; it is a non-standard open problem, and finally, it has, in its linguistic formulation, the possibility "to encourage" in the students the production of argumentation and conjecturation in natural language or in another different semiotic register chosen in a free way by the students with the unique aim to find a "general rule". The strategies adopted by the students to solve this problem could be different.

The experimentation was carried out at three different public Upper Secondary Schools and in one Lower secondary school (student's age13-15) in Palermo. The classes involved were the third classes of the Lower Secondary school and the first year of the Upper Secondary schools. Everyone was able to understand the language by which the text of the problem was expressed[3]. The experimentation was conducted in groups (Italian and Chinese) after a first phase of autonomous work of conjecturation. So, the students, after a brief moment of autonomous conjecturation and discussion in small groups, shared their approached strategies to the problems, discussing with all the student of the class. The procedure was chosen to encourage, at final step of the experimentation, the mediation of knowledge and the validation of the solution procedure of the mathematical problem/game. The role of the teacher was as a mediator and coordinator. As regards the methodology of investigation, the questionnaire which presented the text of the problem/game was distributed at the same time to all the classes with a table of the first 500 prime numbers and a table of the first 500 squares.

The students were allowed 120 minutes to work. The students were free to use all instruments that they needed (paper and pencil, calculators, computers, etc.). The methodological framework is relative to the Theory of the Didactic Situations of G. Brousseau (1998). As regards the adopted statistic methods, we make reference to the works of Spagnolo, Gras, Suzuki, & Guillet (2008) on implicative and factorial analysis. In order to quantify the student's answers to the question, the research was carried out by using a-priori analysis of the pupils' possible behaviors as we report below in the table:

Table 1

A-priori Analysis of the Pupils' Possible Behaviors

S0	The student doesn't understand the text of the problem
S1	he student proceeds for attempts and numerical errors
S2	The student calculates only 10 select numerical cases chosen by random and doesn't justify the procedure
S3	The student proceeds in a methodic way (according to an analogical recursive thought) in the choice of the numerical cases
S4	The student proceeds not with justified arithmetic calculations
S5	The student proceeds in a arithmetic way considering composed numbers and prime numbers introduced in the charts
S6	The student opportunely proceeds in a arithmetic way considering prime numbers and square numbers introduced in the charts
S7	The student calculates the sum among square numbers in a random way and doesn't deduce the solution
S8	The student proceeds calculating the difference with following square numbers and tries a generalization in natural language
S9	The student tries a generalization of the decisive procedure used

	without an explicit use of the formalized algebraic language (he/she only characterizes some numerical relationships)
S10	The student tries a generalization of the used decisive procedure and considers in a random way some numbers, selected opportunely among the greatest
S11	The student tries a generalization of the used decisive procedure and considers in a random way some numbers, selected among the first numbers
S12	The student tries a generalization of the used decisive procedure using an opportune algebraic formalism
S13	The student tries a generalization of the used decisive procedure and puts in evidence a recognition of a unique structure for some numbers
S14	The student underlines a possible formalized parametric writing
S15	The student changes decisive procedure, abandoning that pseudo-algebraic one and he returns to the natural language e/o to the arithmetic calculation
S16	The student deduces the procedure of resolution followed through possible reasoning, congetturazione and theoretical demonstrations
S17	The student deduces the procedure of resolution followed through specific numerical examples

Results and the First Conclusion

Through the use of the software CHIC (Classification Hiérarchique Implicative et Cohésive), the quantitative analysis of the collected data of the proposed mathematical situation, was read not only in relationship to the relevant historical epistemological aspect underlined previously in the paper, but also, in a broader way, connecting it to other relevant works realized by the GRIM of Palermo in other different mathematical contexts and conducted in a multicultural milieu with Chinese and Italian students of different ages present in our classes in Palermo.

The statistical indices used in this sense were R. Gras' index of implication and Lermann's index of similarity. Below we report the obtained implication graph constructed by the preparation of an Excel table of the behaviors (as we report below) and the transposition of it for the Lermann *similarity* graph:

Table 2

Excel Table of the Pupils' Possible Behaviors

	Strategy S1	Strategy S2	Strategy S3	…
Student 1	0	1	1	…
Student 2	1	0	0	…
Student 3	1	1	1	…
Student 4	0	0	0	…
…	…	…	…	…

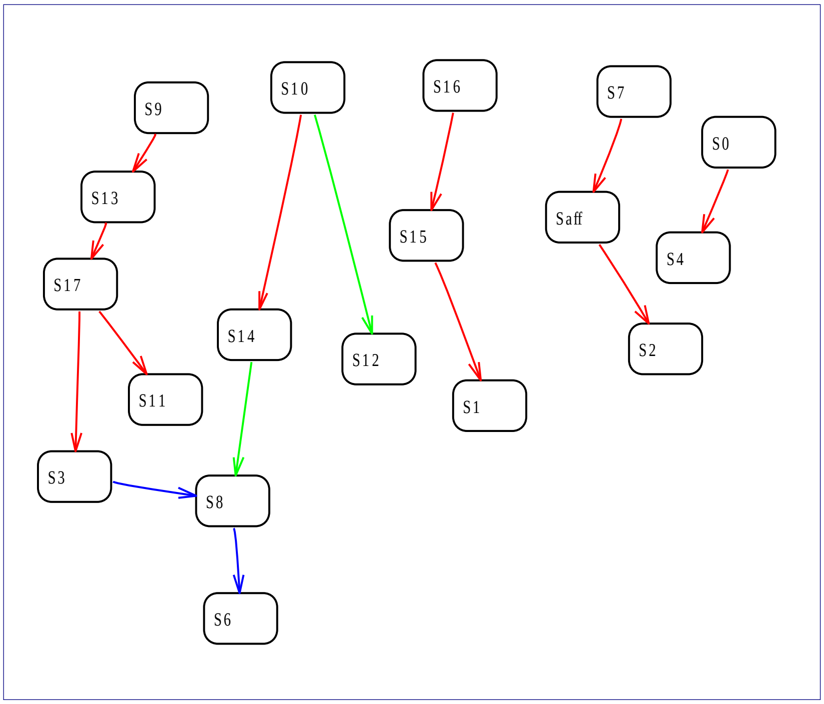

Figure 1. **Implicative Graph: 99, 95, 91, 85.**

The Italian students after a few random attempts and numerical errors, instead, tried to arrive directly at a possible formalization of the treated problematic, but they pointed out some difficulties with working in a formalized setting and so they always re-considered with a lot of attempts, as a help to their reasoning, the arithmetic strategies. The way to argue and prove some initial conjectuation on the problem/game is thus a continuing balance between arithmetic and algebraic thought. In this sense, particularly, in the protocols they show a continuous re-consideration of some sequence of large prime numbers as single cases of their reasoning. Formalizing equals, for them, considering that a property has to be true when they consider very large natural numbers. As regards this behavior, we could recognize two different interpretations. First, in order to prove the truth of the assertion concerning the posed problem, a student utilizes an "induction process" sui generis: i.e. he considers as initial quantities the examples presented in the text of the problem and he thinks that the decomposition of a prime number into the difference between two squares could be true for all the prime numbers if he is able to verify it for some large numbers.

Second, a student goes for abduction (according to Peirce) because he believes that verifying the truth of the assertion for a little set of large prime numbers the truth follows for the whole set of prime numbers. Regarding this aspect, the Chinese students highlighted, as first step, a holistic way to read and therefore interpret the table of prime numbers given to them. They sketched on it same particular single case and they tried to formalize a general form of prime numbers which could be written as the sum of two squares. General form was never expressed in terms of formalized algebraic language (which is instead possible to see in the Italian protocols). They initially paid attention to the particularity of each single case as a variable case in relation to the other possible cases, reading the table in a unitary vision. We didn't find the same procedure with the Italian students who tried to formalize the general case of this type of number directly. The structure of the procedure adopted seemed to highlight how gradually, from a first simple case given in the text of the problem, the Chinese students tried to solve the question "generalizing" step by step, through analogical reasoning, through a *variable mutation*, the proposed case and hence defining a "general" solution strictly connected to the proposed context of the problem, in a holistic vision (Nisbett et al., 2001). The solution process is almost an additive process. According to us, the reasoning adopted by the Chinese students in the search for the common construction of these prime numbers that could be written as a sum of two squares could be an interesting example of the meta-rule "*Making homogeneous and making equal*" (Chemla, 2003) which is a fundamental rule in the Chinese Mathematic and Philosophy. The adopted algorithm, once again, is an operation which "makes equal" and "makes homogeneous" (in the reduction to unity). This fundamental algorithm is, during the process of argumentation, combined itself several times always arriving at a sure argument. As Liu Hui observes, in this game of relationships between "serial thought" and "global thought" in reading and understanding a problem, particular attention must be given to the examination of the algorithm on the classes of possible homogeneous cases to be able to highlight its correctness. The mediation of the different reasoning schemes adopted by the Chinese and Italian students were discussed with the all classes in terms of possible approach to the problem. This final step of our experimental work was able to integrate, in classes, the pragmatic behavior of the Chinese students with the deductive logic and the search for generalization utilized by almost all the Italian students. The students, in an autonomous view, observe critically the passage between the possibility to prove for convincing, to prove to verify (typical Chinese strategy), to prove to find the generalization (typical Italian strategy) and so forth. In our opinion, the results achieved could be very interesting as regards the didactic implications for the multicultural classes in which foreign students of Chinese origin are present.

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