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Study on the Relationship between Individual's CPFS Structure and Problem Inquiry Ability

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The studies investigated CPFS (special cognitive structure in mathematics learning) and showed that: (1) There were significant correlations between the individual's CPFS Structure and problem inquiry ability. (2) There existed significant differences between the perfect CPFS Structure and the defective one in the performances of inquiry problems and proposing problems by intuition but no significant differences in proposing directional problems. (3) The question familiarity had the immediate influence on inquiry problems and no interaction with an individual's CPFS Structure. (4) With the external regulation, there existed significant differences between the perfect CPFS Structure and the defective one in the performances of moderate-level difficulty inquiry problems and low-level difficulty problems.

Key words: CPFS structure, problem inquiry, correlations, put forward problem.

Problem Posing

Many studies and achievements had been made on the relationship between cognitive structure and problem solving. Among these studies, an important field was to compare the ability of problem solving between experts and novices. Except for some other factors, a common result was that experts had better knowledge structure than novices. The idea of knowledge structuralized had become a standard concept of cognitive structure (Ericsson & Lehmann, 1996). Anderson (1995) described two characteristics of knowledge structure in mind, namely, activation and strength. The former referred to instantaneous availability of a knowledge element in memory structure, and the latter referred to its permanence during a relatively long period. In problem solving, knowledge elements of high activation and high strength were easily brought forward. Bielaczye, Pirolli and Brown (1995), Chi (1994) and Renkl (1997) studied the role of self-explanation in the process of problem solving. This self-explanation relied on the individual's cognitive structure. Those who had perfect cognitive structure could know their internalized knowledge structure clearly and explain their inner knowledge structure well. The study showed that the better individuals explained details of constructing knowledge and key structure, the better they extracted available information in solving problems.

Lawson and Chinnappan (2000) studied those participants whose ability was lower in solving problems. They found these participants couldn't extract useful information effectively in geometry problem solving. But if some suggestions were given, the situation would be changed and they could recall some relevant information. The researchers thought that knowledge organization was different for students having different levels of ability of solving problems. There existed a lot of studies about how individuals' knowledge organization affects their problem solving. Larken (1979) and Prawat (1989) found that there were close relationships between effectively solving problems and the quality of individuals' knowledge organization. Lawson and Chinnappan (1994) studied the role of knowledge connectedness in geometry problem solving. They examined the relationship between the quality of students' knowledge organization and the performance of their solving problems. Meanwhile, participants recruited from Grade Ten were classified into a high-ability group and a low-ability group to be administrated tests. The result showed that participants in the high-ability group could activate knowledge chains more automatically and extract useful information integrally and rapidly. Furthermore, suggestions on connectedness could better help students to solve problems than that knowledge.

Throughout these studies, we thought there existed two problems. One was that the definition of knowledge structure was relatively vague. More accurate description of this concept should be put forward according to the characteristic of special discipline. The other was that these studies were limited to considering the relationship between knowledge structure and problem solving while studies on the relationship between knowledge structure and probing problems were deficient.

According to mathematics characteristics, we put forward CPFS Structure theory that described mathematics knowledge representation (Yu, 2003a). CPFS Structure referred to a schema involving a concept field, concept system, and proposition field and proposition system constructed in mind by learners on mathematics learning. Concept Field was defined as the schema of all equivalent definitions of a mathematics concept, reflecting describing one concept from different angles, and revealing equivalently abstract relationships between concepts. Concept Field described the storing pattern of the knowledge network consisting of mathematical abstract relations between concepts. Likewise, Proposition Field was defined as the schema of a set of equivalent propositions, and Proposition System was defined as the schema of the network consisting of semi-equivalent propositions. Both of them accurately described the organization form in mind of mathematics propositions and their relationship. Therefore, the meaning of CPFS Structure was the following: (1) A mathematics knowledge network internalized in an individual's mind consisting of mathematics knowledge elements (concepts and propositions) having certain locations. There were some specific mathematics relationships (strong abstraction, weak abstraction or generalized abstraction relationships) between these knowledge elements. (2) Just because there were certain abstraction relationships among mathematical knowledge elements, these abstraction relationships themselves implied some thinking methods. The linking between mathematical knowledge elements containing mathematical methods, that is to say, “the linking set” was a method system. (3) It was a specific cognitive structure of mathematics and a knowledge structure internalized by the individual in accordance with mathematics logical characteristics.

CPFS Structure had a relatively delicate description of mathematical cognitive structure. Based on this, how individual’s CPFS Structure affected problem representation, how self-controlling and CPFS Structure affected mathematics achievement, and the relationship between CPFS Structure and transfer on mathematical problem solving had been studied (Yu, 2003b, 2004a, 2004b). In this paper, we would study the relationship between the individual’s CPFS Structure and ability of probing into problems, and what we are concerned were: (1) Whether the individual’s CPFS Structure affected problem posing. (2) Whether the individual’s CPFS Structure affected probing into problem properties. (3) Whether the individual’s CPFS Structure played a different role in solving problems of different difficulty or different familiarity for participants.

First, we defined what the ability of probing into problems was.

We thought probing into problems in mathematics learning includes: (a) Problem posing, namely, posing problems according to certain information or clues. Problem posing could be classified into two kinds. One was directional problem posing. That was to say, learners were directed to pose relevant problems with a specific objective and definite direction given. The other was non-directional problem posing. That is to say, with no specific objective and no definite direction given, learners were directed to pose problems in a kind of free atmosphere. Obviously, problem posing was classified into two levels, and the level of non-directional problem posing was higher than that of directional problem posing. As to mathematics learning in middle school, directional problem posing should be the main kind of probing into problems. So, “problem posing” in this paper only means “directional problem posing”. (b) Probing into problem properties according to the problems posed by oneself or others.

Based on the above, the individual’s ability of probing into problems included the ability of problem posing and the ability of probing into problem properties. The ability of problem posing referred to one in which students posed some meaningful mathematics problems utilizing mathematical methods with problem consciousness by themselves. Specifically, students had formed problem consciousness through knowledge accumulating and they could raise problems by generalization and induction, or analogy, or intuitive reasoning, or problem solving itself. The ability of probing into problem properties referred to the ability that students solve problems posed by themselves, or probing into problems unfamiliar and posed by others (such as probing into the background, the property and the domain). Obviously, probing into problems more relied on the strength of logical thinking.

Method for Study 1

Participants

The participants were 95 students recruited from Grade Ten in a high school in Changzhou, Jiangsu province. 45 of them were female and the others were male. Their performance was at an average level of all the students in Grade Ten.

Materials

The materials consisted of two tests based on "function". One was designed for assessing the participants' CPFS Structure (called Test I), and the other was for measuring their abilities of probing into problems (named Test II).

Test I included 12 sets of problems representing the different CPFS Structures of different "Function" topics, such as "*the domain*", "*the range*", "*the relationship between a function and its graph*", "*the odevity*", "*the monotonicity*", "*the inverse function*", "*the periodicity*", "*the graph conversion*" and so on.

For example, in order to assess the participants' concept field and concept system on *the domain of function*, the 1st set was designed to consist of 7 problems about the domain of special functions, including radical function， fractional function, logarithmic function, and the inverse function, as well as one after function operation, all that were essentially used to find out whether the participants had gotten a general idea about function domain. The 3rd set contained questions as follows:

1) *Which knowledge has to do with the properties of a quadratic function* $y=ax^2+bx+c$*? Please write them out.*
2) *What conclusions (including equal relations and inequality relations) about coefficient a,b,c could you get if the graph of* $y=ax^2+bx+c$ *is given, and* $x=\frac{1}{3}$ *is its symmetric axis? Please write them out as much as possible.*

Both were to assess if the participants had developed the CPFS Structure of quadratic equations, quadratic inequality and quadratic function. Since each question had not only one answer, those who got more answers would be recognized as the ones who had the more perfect CPFS Structure. The 7th set was designed to test the concept system on *the odevity of functions*. Each respondent was asked to judge that the result of two odd functions or two even functions was odd or even by four fundamental operations of mathematics.

For each set of problems in test I, there were two kinds of problems. One was gap filling. A score of 1 was given for a correct answer and 0 for false. The other kind were open-ended questions. A score of 1 was given for each right response. The more answers, the higher score total.

Test II contained 8 items (23 problems in total). The participants were asked to raise questions by generalization and induction, or analogy, or intuitive reasoning, or logic reasoning at first and then to probe into the problems. Thus it was composed of 4 sub-tests. Familiar and unfamiliar functions alternated within these sub-tests. Take the 1st item for example, which was to assess the ability of posting problems by generalization.

Item 1: *Observe the following function expressions. Each expression can be divided into the sum of the function P(x) and the function Q(x).*

1) $f(x)=x^3=\frac{1}{2}[x^3+(-x^3)]+\frac{1}{2}[x^3-(-x^3)]=P(x)+Q(x)$;

2) $f(x)=x^2+x=\frac{1}{2}[(x^2+x)+[(-x)^2-x]]+\frac{1}{2}[(x^2+x)-[(-x)^2-x]]=P(x)+Q(x)$;

3) $f(x)=a^x=(a^x+a^{-x})+\frac{1}{2}(a^x-a^{-x})=P(x)+Q(x)$;

4) $f(x)=\sin x=\frac{1}{2}[\sin x+\sin(-x)]+\frac{1}{2}[\sin x-\sin(-x)]=P(x)+Q(x)$;

Please answer the following questions:

1) *Is P(x)/Q(x) respectively odd or even?*

2) *Could you find any rules from the above expressions? Can you prove your conclusion?*

For another example, item 3 was to assess the ability of proposing problems by analogy.

Item 3: *We have known that the sum of two even numbers is even, the sum of two odd numbers is even, and the sum of an odd number and an even number is odd. If we change the rules with "function" instead of "numbers", what conclusions could you get? Are they all correct? Please prove them.*

As to properties probing items, there were two types, one by intuition developed through the process of solving problems such as item 4, and one by analyzing the problems composed of unfamiliar concepts such as item 8.

Item 4: 1) *Prove the inverse function of* $f(x)=\frac{1-x}{1+x}$ *is itself.*

2) *Could you find more functions that are similar to such a function?*

Item8: 1) *Given the function* $y=f(x)$, (x) *means the distance from x to the integer that is nearest to x. Please search into the nature of this function, such as symmetry, monotonicity and odevity, and then draw its graph.*

The function had not yet appeared on the middle school textbook, thus it's unfamiliar to participants.

A score of 2 was given for a correct response in the condition of closed questions, and a score of 1 in the condition of open questions.

A pilot study had been made with 95 students in Grade Ten in a high school in Nanjing. As we could see from Table 1, it indicated its good structural validity. Correlation coefficients between sub-tests and test II were significantly higher than correlation coefficients between sub-tests. In other words, the four sub-tests contribute to test II, and they were independent from each other.

Table 1

Correlation Coefficients Between Sub-tests and Test II

	Y	Y_1	Y_2	Y_3	Y_4
Y	1	0.594**	0.368**	0.728**	0.665**
Y_1	0.594**	1	-0.010	0.307**	0.183
Y_2	0.368**	-0.010	1	0.110	-0.028
Y_3	0.728**	0.307**	0.110	1	0.251*
Y_4	0.665**	0.183	-0.028	0.251*	1

* Y represented test II, Y_1 posing problems by intuitive reasoning, Y_2 posing problems by generalization and induction, Y_3 posing problems by analogy, Y_4 inquiring properties by logic reasoning.

Procedure

The participants were administrated Test I in 50 min. on the first day, and Test II in 90 min. on the next day.

The study was a 2×2 within-subject design where factor A stood for CPFS Structure and factor B for the level of what extent participants were familiar with the materials. In detail, A was divided into two levels: high CPFS Structure (HC) and low CPFS Structure (LC); and B was also divided into two levels: familiar questions and unfamiliar questions.

According to their average performance (53.6) on Test I, the participants were classified into two groups. Those whose score was higher than 58 (=54+4) (including 58) belonged to the HC group, and those whose score was lower than 50 (=54-4) (including 50) belonged to the LC groups. Considering the results of test II, 4 groups were formed: HC with unfamiliar questions, HC with familiar questions, LC with unfamiliar questions, and LC with familiar questions.

Results for Study 1

Relationship between CPFS Structure and Creative Thinking

The correlation analysis between test I and test II showed that individuals' CPFS Structure correlated highly with problem inquiry ($r = 0.371, p = 0.008 < 0.01$) ability. Furthermore, the correlation coefficients between CPFS Structure and proposing questions by intuitive reasoning, by generalization and induction, by analogy, and inquiring properties by logic reasoning was respectively 0.247*, 0.210, 0,175, 0.334*. It indicated that the relationship between CPFS Structure and posing questions by intuitive reasoning, and inquiry properties by logic reasoning were closer than that between CPFS Structure and proposing questions by generalization and induction, or by analogy.

Difference in Problem Inquiry Ability between HC and LC

As could be seen from Table 2 (a result of a T-test between HC and LC about test II), the variance was equal, and there existed significant differences between the HC group with average score 26.65 and the LC group with average score 20.18 on test II. With regard to the ability of probing into problems, the former was obviously higher than the latter.

Table 2

Comparisons Between HC and LC About Test II

	F	Sig.	*t*	*Df*
equal variance assumed	1.994	0.168	3.177	65
unequal variance assumed			3.202	64.593
	Sig.	Mean Square	Std. Error	
equal variance assumed	0.002	6.46964	2.03614	
unequal variance assumed	0.002	6.46964	2.02053	

We further examined the differences between HC and LC from the four aspects measured by the sub-tests. As shown in Table 3, it was the ability of proposing problems by intuitive reasoning (marginal significance) and probing properties by logic reasoning (remarkable significance) that made the significant difference. On the other two aspects, the difference was not so remarkable. Then, the way an individual's CPFS Structure influenced the ability of probing into problems was mainly through intuitive reasoning and logic reasoning.

Table 3

Comparisons Between HC and LC About Sub-tests in Test II

	proposing problems by intuition	proposing problems by generalization	proposing problems by analogy	probing properties by logic reasoning
HC	6.686	4.771	6.686	8.143
LC	5.125	3.938	5.094	5.813
P	0.058	0.264	0.094	0.019

Result about analysis of variance

The main effect of the experiment was remarkable (see Table 4). First, the main effect of Factor A was remarkable. That was to say, an individual's CPFS Structure directly affected problems' inquiry ability. Second, the main effect of Factor B was remarkable, too. That is to say, the level how students were familiar with the questions directly affected problems' inquiry ability. Third, the cross action between A and B was not significant.

Table 4

Analysis of Variance

Source	Sum of Squares	*df*	Mean Square	*F*	Sig.
Main Effect	3997.563(a)	3	1332.521	47.706	0.000
A	349.844	1	349.844	12.525	0.001
B	3562.279	1	3562.279	127.534	0.000
A×B	43.055	1	43.055	1.541	0.217

Method for Study 2

Purpose

The purpose of Study 2 was to further examine the relationship between the individual CPFS Structure and abilities of probing into problems. For the participants of the LC group, Test II was set under a certain condition of external regulation and control to explore whether their performance could reach the same level as that of the HC group, and to reveal the effect of individual CPFS Structure on the development of creative thinking ability from a more deep review.

Participants

The participants were 116 students recruited from Grade Ten in a high school (57 from one class, and 59 from another) in Nanjing, Jiangsu province. 60 of them were male and the others were female. Their performance was at an average level of all the students in Grade Ten.

Materials

The materials consisted of three tests. Designed respectively for assessing the participants' CPFS Structure and for measuring their abilities of probing into problems, Test I and Test II were the same as those of Study 1. The items of Test III were the same as that of Test II, but in Test III, with regard to each item, some corresponding suggestions were given on the right. All the suggestions were regarded as external factors of regulation and control for the participants to pose problems and solve problems.

Procedure

First, all the participants were administrated Test I in 50 min. According to their average performance (50.23) of Test I, they were classified into three groups. Those whose score was higher than 54 (=50+4) (including 54) belonged to the HC group (40 participants); those whose score was lower than 46 (=50-4) (including 46) belonged to the LC group (34 participants), and the others belonged to the middle CPFS Structure (MC) Group (42 participants).

Two days later, the HC group was administrated Test II and the LC group was administrated Test III. After that, certain participants were interviewed.

The course of Study 2 was as in Figure 1.

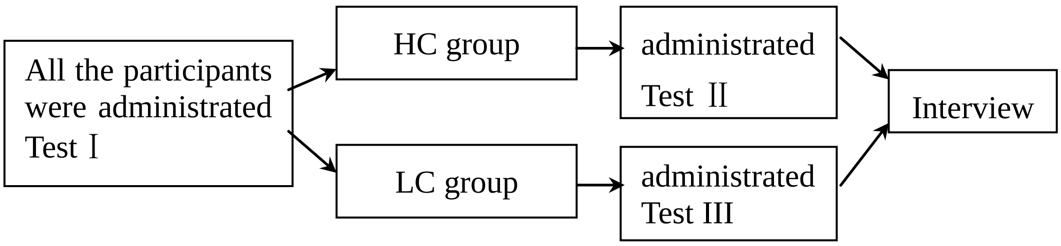

Figure 1. **Course of study 2.**

Results for Study 2

Relationship between CPFS Structure and abilities of probing into problems

As the participants were different from those in Study 1 and the LC group was given an external adjustment, we further verified the correlation between the individual CPFS Structure and abilities of probing into problems. The result showed that there existed a remarkably significant correlation between them (r = 0.449, p =0.000). This further verified the results of Study 1.

Difference of the performance between HC and LC group on Test II and Test III

See Table 5 (a result of a *T*-test between HC on Test II and LC on Test III). Despite the LC group being given external suggestions on Test II, the test score was still significantly lower than that of the HC group. And it further illustrated that the influence of individual CPFS Structures on abilities of probing into problems was remarkably significant.

Table 5

Comparisons Between HC on Test II and LC on Test III

	F	Sig.	*t*	*df*
Equal variance assumed	2.624	0.108	3.605	109
unequal variance assumed			3.677	109.000
	Sig.	Mean Square	Std. Error	
Equal variance assumed	0.000	5.702	1.582	
unequal variance assumed	0.000	5.702	1.551	

To more meticulously examine the relationship between individual CPFS Structure and the difficulty level of a problem, we divided the problems in Test II and Test III into three types according to their difficulty level, namely, high-level difficulty problem, medium-level difficulty problem and low-level difficulty problem. And then the average scores of the HC group and those of the LC group in these three types were compared. They are shown in Table 6.

Table 6

Comparisons Between HC and LC on Problems with Different Level of Difficulty

	low difficulty problem	medium difficulty problem	highly difficulty problem
HC group	7.330	9.425	2.675
LC group	4.735	5.677	1.735
P	0.002	0.001	0.110

Table 6 indicated that there existed significant differences between the perfect CPFS Structure and the defective one in the performances of inquiry in moderate-level difficulty problems and low-level difficulty problems. No significant differences existed in inquiry in high-level difficulty problems.

Discussion

Relationship between individuals' CPFS Structure and the ability of inquiry problems

The above study indicated that there was significant correlation between individuals' CPFS Structure and the ability to inquire in problems. This showed that a close relationship existed between them. The learners could not inquire in mathematical problems without knowledge and experience. As a mathematical problem, it was bound to have a logical relationship with other problems. The clearer the logical relations were in mind, the more easily would the relevant information be extracted. The lines between the knowledge-points revealing their relationships, and individual's CPFS Structure which contains information of mathematics methods would help learners to inquire in problems.

Therefore, with a perfect CPFS Structure, the participant's ability of inquiry problems was higher than that with a defective CPFS Structure. This had been illustrated by Table 2. On the other hand, because the participants in the study were required to raise the questions under certain goal guidelines, namely, the test was a kind of directional questioning, the participants did not need to have more information of knowledge, and mainly relied on their own abilities of summarizing and analogical reasoning.

So, the relationship between an individual's CPFS Structure and the ability of summarizing and analogical reasoning was not very close. However, raising questions intuitively and probing the nature logically necessarily involved a high degree of knowledge information and methods of problem inquiry. Only when participants activated related knowledge-points in their minds, could they quickly extract information. The CPFS Structure was an ordered structure formed in the individual's minds. Based on potential mathematical logic the learners could easily and quickly active knowledge-nodes so as to extract useful information to deal with current problems. So, the relationship between CPFS Structure and raising questions intuitively and the probing nature logically was becoming closer, which resulted in the abilities of participants with perfect CPFS Structure being higher than those with a defective one. Table 2 showed the conclusion.

Due to defective CPFS Structure, the participants' knowledge system stored in the minds was incomplete and their internalized knowledge could not be integrated effectively. They could not know concept and understanding proposition multi-angles and multi-levels. Thus, even with appropriate suggestions offered to them during the process of probing questions, it was difficult to activate their available resources. External suggestions could only make the participants do limited reasoning and hardly

have a continuous reasoning process .We had analyzed the test papers of the low

CPFS Structure group in Research 2 and found out the above condition.

For example, Question 2:

(1) *Please show examples of f(x) which satisfy the condition:*
$f(x+y)= f(x)+f(y)$.

(2) *Please show examples of f(x) which satisfy the condition:*
$f(x+y)= f(x) \cdot f(y)$.

(3) *Please show examples of f(x) which satisfy the conditions:*
①$x>0$，②$f(xy)=f(x)+f(y)$.

(4) *Please show examples of f(x) which satisfy the conditions:*
①$x>0$，②$f(xy)=f(x)f(y)$.

(*Suggestion: if f(x)=x,, which condition can be satisfied ?*).

Many of the participants only could show *f(x),* in condition (1) and did not know how to deal with the following.

Another example was the fourth question:

(1) *Prove that the inverse function of* $f(x)=\frac{1-x}{1+x}$ *is itself. Could you find more functions that are similar to such function? Namely its inverse function is itself. (Prompt: Consider similar expressions.)*

Because of the effect of the suggestions, most participants got the expressions such as $f(x)=\frac{2-x}{1+x}$ and $f(x)=\frac{3-x}{1+x}$ and they could not get the variant of the expressions. Thus, even with appropriate external monitoring offered to those with defective CPFS Structure, it was difficult to make them reach the same level in exploring questions as those with the perfect Structure.

Relationship between individuals' CPFS Structure and different kinds of inquiry problems

Table 4 showed that question familiarity had a direct influence on the problem inquiring. But whether the participants were familiar with the problems or not, there existed significant differences in the performances of inquiry problems between the high CPFS group and the low one. We believed that an individual's CPFS Structure had the growing instinct and the assimilation and conformance of its intrinsic mechanism would increase gradually with the development of CPFS Structure. With perfect CPFS Structure, when facing new problems, learners could activate the corresponding CPFS Structure following the information index. So, on the one hand, they strengthened the connection between the external information and internal representation and made judgments consciously to mathematics abstract relations between them. On the other hand, learners might be enlightened by the "method system "of CPFS Structure and transfer it to solve new problems. In Study 2, we had interviewed some of the high CPFS Structure group. In Question 8, though the interviewees were not familiar with the given function $y=(x)$, they nearly replied that they had thought of function y=[x] and therefore made a judgment that *y*=(*x*)was a piecewise function. And they used methods of discussing *y*=[*x*] to study *y*=(*x*). Yet for the low CPFS Structure group, even though suggestions were offered in the test, most of the participants still could not set up the connection between *y*=(*x*) with *y*=[*x*]. During the interview, many participants said they did not know the nature of the function *y*=[*x*] and its corresponding research methods. This showed that the CPFS Structure of function *y*=[*x*] had not been formed in their long-term memory.

There was a difference between the high CPFS group and the low CPFS group in inquiry problems of different-level difficulty. And the difference appeared mainly in moderate-level or low-level difficulty question probing and no difference in a high-level difficulty one (see Table 6). The study indicated that there were many factors resulting in the difference of problem inquiry ability which involve the learner's own ability in mathematics, meta-cognitive ability and non-cognitive factors. There was no remarkable difference between the performance of high CPFS and low CPFS groups when probing into the high-level difficulty problems. The reasons might be as follows:

Firstly, for inquiring into the high-level difficulty questions, not only would an individual's CPFS Structure affect it, but the capability in mathematics and self-control would affect it more. It involved complex and advanced mathematics thinking.

Secondly, because the low CPFS group accepted the external suggestions which invisibly reduced the level of difficulty of the problem, their performance would be enhanced. Further study would be needed for evidence of such a problem.

Conclusion

（1）There existed significant correlation between individuals' CPFS Structure and the ability of inquiry problems;

（2）There existed significant differences between the perfect CPFS Structure and the defective one in the performances of inquiry problems and proposing problems by intuition and no significant differences in proposing directional problems.

（3）The question familiarity had an immediate influence on inquiry problems and no interaction with individuals' CPFS Structure;

（4）With an external regulation, there existed significant differences between the perfect CPFS Structure and the defective one in the performances of moderate-level difficulty problems and low-level difficulty inquiry problems. No significant differences existed in high-level difficulty inquiry problems.

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