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A Study on the Development of Students' Epistemological Beliefs about Mathematics

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This study aims to explore the development of students' epistemological beliefs about mathematics and the differences of mathematical beliefs between students at different performance levels. Seventy-six primary school students, 62 junior high school students, 116 senior high school students and 96 college students were selected to complete a test with 12 hypothetical-situation questions and a questionnaire on mathematical beliefs. The results show that the student's attitude toward mathematics is closely related to the course content of mathematics as well as the implicit beliefs of mathematics that the student has acquired in the long-term learning process. The development of students' mathematical beliefs partially follows a V-shaped pattern. Remarkable differences are found among students at different performance levels.

Key words: epistemological beliefs about math, development, math learning, performance level.

Epistemological beliefs about mathematics are people's understanding and viewpoints of mathematics. Students' mathematical beliefs are considered an important component that affects mathematics education (Xu, 2006). Based on literature review, Liu and Chen (2004) posit that student's beliefs of mathematics consist of three parts: the view of the nature of mathematics knowledge, the view of mathematics learning process and the academic self-concept in the field of mathematics. Other scholars share this idea and believe that students' epistemological beliefs about mathematics involve three aspects: beliefs about mathematics, beliefs about mathematics learning and beliefs about themselves (Zhang, Li & Li, 2003).

Studies have shown that, as metacognitive knowledge，the students' beliefs of mathematics have great impact on their mathematical learning process, emotional experience, and learning outcomes (Muis, 2004; Schommer, Duell, & Hutter, 2005; Zhou & Wang, 2007). Meanwhile, student beliefs about mathematics are partly resulted from their experiences of mathematical learning at school. They are influenced by the curriculum, the classroom teaching and the school environment. A research group of the Chinese University of Hong Kong conducted pioneering research on the mathematical beliefs of students in Hong Kong and mainland China (Huang, 2002). After that, a number of studies have been devoted to revealing the mathematical beliefs of students at different stages (Liu & Chen, 2002; Liu & Niu, 2004; Yang & Li, 2005; Li, 2006). The study conducted by Liu and Chen shows: The primary school pupils are generally interested and confident in learning mathematics. However, as they move to higher grades, the positive attitudes towards mathematics begin to wane, and this change in attitude becomes evident between the 4^{th} grade and the 6^{th} grade (Liu & Chen, 2002). In another study, Li (2006) finds that four factors, namely, the interest of mathematics learning, the involvement of classroom learning, the views about mathematics, and the quality of classroom discourse, can explain the variance of primary school students' views of mathematics leaning. The former two factors decrease with the increasing grade of primary school students, while the latter two factors increase as the grade moves up.

A study conducted by Yang and Li (2005) shows that most junior high school students like mathematics and have high self-confidence in mathematics learning, but this positive attitude declines as the grade rises. Their beliefs about mathematics learning processes are basically constructive and exploratory, yet not very definite. Moreover, more students tend to learn mathematics as a fact as they move towards higher grades. In a study of college students, it is found that some students from teacher education programs do not have a comprehensive understanding about 'what is math' and their views of mathematic research are largely superficial. Many believe that mathematics is just as valuable as a tool and a way to train one's thinking ability (Liu & Niu, 2004). An international study also finds that the pupil's positive attitude to mathematics goes down gradually as they move to higher grades (Eccles, Wigfield, Harold, & Blumenfeld, 1993).

Above all, the studies mentioned above mainly focus on students at a specific stage, while the trajectory of the development of students' mathematical beliefs from primary school to college remains unclear. How do students' beliefs about mathematics change over the years? How do they differ between students at different levels of academic performance? Few studies have been done to answer those questions.

This study aims to explore the development of the students' beliefs of mathematics and the differences of mathematical beliefs between students at different academic levels.

Methods

Participants

The participants include 76 fifth-grade students of Liaocheng Democratic Primary School, 62 second-year junior students and 116 second-year senior students from Liaocheng No.2 Middle School, 96 sophomores from School of Mathematics Science of Liaocheng University, among which, 162 are boys and 188 are girls. Based on the records of students' mathematical learning performance provided by their mathematics teachers, the top 27% students are identified as excellent students, the bottom 27% students as under-achievers, and the 46% students in between as average students. As a result, three levels of mathematic learning performance —excellent, average and poor, are identified, with 95, 160 and 95 students in each bracket respectively.

Test Instruments

Two instruments were used to investigate the students' beliefs about mathematics. One is the research materials developed by Huang (2002), which includes 12 problems in hypothetical-situations. The 12 questions concerns estimation of word numbers, time recording, observations in life, fraction comparison, length measurement, calculating with a calculator, weight estimation, jigsaw in life, observation, selection, inference decision, and mathematical graphics. The students are asked to determine whether the solution to each problem depends on mathematics. If the answer is "yes", 1 point is recorded, if "No", 0. The other instrument is a questionnaire which was developed based on Rude Liu's framework of mathematical beliefs. The questionnaire consists of three dimensions. The first dimension concerns the students' beliefs about the nature of mathematical knowledge, including 7 items (Cronbach's $\alpha = 0.731$) which aimed to examine the students' view of the certainty, simplicity, and sociality and the value of mathematical knowledge. For instance, the students are asked to decide whether the mathematical knowledge is evolving or fixed and determined, whether it is isolated or closely linked with each other and with reality.

The second dimension deals with the beliefs of mathematical learning process.

The 11 items (Cronbach's α = 0.720) aim to examine how students understand mathematics learning or the problem solving process, its strategy and influencing factors (such as ability and effort). For example, the questions include whether the progress of mathematics learning is quick or slow, and whether mathematical ability is innate or learned. The third dimension examines the beliefs of the academic experience in mathematics. The eight items (Cronbach's α = 0.904) aim to examine the students' mathematical self-concept, that is, how the students perceive their abilities to learn mathematics and solve problems, whether they are aware of the relationship between mathematics learning and its applications and what is their emotional experience in mathematics learning. The Cronbach's α of the whole questionnaire was 0.864. The five-level Likert scale is adopted in the questionnaire: Not at All True, Slightly True, Partly True, Mostly True, and True. 1, 2, 3, 4, 5 point(s) are recorded respectively.

The students' answers to problems in hypothetical-situations in the first instrument can reflect their naive, intuitive, implicit view of mathematics, while the questionnaire on mathematical beliefs is intended to discover students' systematic view about mathematics adopted and formed in the long term mathematics learning process and through the interaction with the teachers. By combining the two complementary instruments, the test is expected to reveal a more comprehensive picture about how students' mathematical beliefs differ and develop.

Procedure and Data Analyses

The students completed the questionnaires in class under the supervision of the researcher. The whole process took 20-30 minutes. SPSS16.0 was used to process the data. The mean value, percent value, analysis of variance, and LSD post hoc comparison are examined.

Results

The Participants' Answers to the Questions in Hypothetical-situations

Table 1

The Percentage of the Participants' Agreeing on the Involvement of Mathematics in Hypothetical Situations (%)

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
Primary School	94.7	73.4	1.3	75	98.7	100	68.4	7.9	0	94.7	97.4	98.7
Junior High	80.6	67.7	4.8	98.4	96.8	93.5	69.4	17.7	4.8	32.3	29	85.5
Senior High	86.2	69.8	11.2	83.6	92.2	88.8	76.7	30.2	9.5	45.7	47.4	79.3
College	79.2	60.4	16.7	93.8	91.7	72.9	81.2	47.9	12.5	56.2	47.9	93.8
Total	85.1	72	9.4	87.1	94.3	87.7	74.9	28	7.4	56.9	55.1	88.6

Table 1 shows the percentage of the participants who agree that the hypothetical situation involves mathematics (%). In terms of the perceived relatedness with mathematics, the 12 questions range from the most related to least related as Q5> Q12> Q6> Q4> Q1> Q7> Q2> Q10> Q11> Q8> Q3> Q9. Among them, Q5, Q12, Q6, Q4, Q1 are perceived to be mathematics related by more than 80% students, Q7, Q2, Q10, Q11, 50% to 80%, while Q8, Q3, Q9, less than 30%.

Comparison between the Answers of the Participants of Different Grades

As Table 2 shows, the lower the grades, the following questions are more likely to be accepted as mathematics related: Q1, Q2, Q6, Q10, Q11, Q12. Generally, the acceptance level of primary school students is the highest. On the other hand, the higher the grades, the following questions are more likely to be accepted as mathematics related: Q8, Q9, Q3, Q4. In general, the lowest level of acceptance is from primary school students.

Table 2

Comparison of the Perception of Relatedness with Mathematics Between Students of Different Grades

Questions	*F*	LSD Post Hoc Comparison
Q1	3.161^{*}	a>b, a>d
Q2	8.687^{***}	a>b, a>c, a>d
Q3	4.701^{**}	c>a, d>a, d>b
Q4	7.744^{***}	b>a, d>a, b>c
Q5	1.863	
Q6	11.70^{***}	a>c>d, b>d
Q7	1.661	
Q8	13.90^{***}	d>a, d>b, d>c, c>a
Q9	3.744^{**}	d>a, c>a
Q10	26.63^{***}	a>d>b, a>c
Q11	32.37^{***}	a>b, a>c, a>d
Q12	7.225^{***}	a>b, a>c, d>c

Note: a represents primary school students, b represents junior high students, c represents senior high students, d represents college students. * P <0.05; ** P <0.01; *** P <0.001, the same below.

Comparison of the Answers between Students at Different Performance Level

Table 3

Comparison of the Perception of Relatedness with Mathematics between Students at Different Performance Level

Questions	*F*	LSD Post Hoc Comparison
Q1	5.319^{**}	Excellent>Average>Poor
Q2	1.728	
Q3	0.330	
Q4	1.174	
Q5	5.140^{**}	Excellent>Average>Poor
Q6	1.711	
Q7	2.249	
Q8	3.766^{*}	Excellent>Average>Poor
Q9	3.263^{*}	Excellent>Poor
Q10	0.487	
Q11	3.002^{*}	Excellent>Average>Poor
Q12	6.011^{**}	Excellent>Average， Excellent>Poor

Table 3 shows that there are greater differences in students' responses to Q1, Q5, Q8, Q9, Q11, Q12. The acceptance level of students with good academic performance is significantly higher than that of students with average and poor performance.

Questionnaire Scores of Students in Different Grades

Table 4

Questionnaire Scores of Students in Different Grades [*M* (*SD*)]

Beliefs	Primary	Junior	Senior	College	Total
Mathematical Knowledge	4.346 (0.448)	4.228 (0.522)	3.937 (0.540)	3.964 (0.517)	4.085 (0.538)
Mathematical Learning Process	3.988 (0.475)	3.872 (0.486)	3.713 (0.507)	3.953 (0.446)	3.867 (0.492)
Mathematical Academic Experience	4.607 (0.595)	4.129 (0.679)	3.266 (0.960)	3.602 (0.726)	3.802 (0.933)
Means	4.275 (0.425)	4.047 (0.462)	3.636 (0.524)	3.848 (0.439)	3.906 (0.527)

Table 4 shows that all the scores of students in different grades as well as the means are above the critical value (3 points) in every dimension.

Comparison of the Scores between Students of Different Grades

Table 5

Comparison of the Scores in Each Dimension of the Questionnaire between Students of Different Grades

Beliefs	*F*	LSD Post Hoc Comparison
Mathematical Knowledge	13.21^{***}	a>c, a>d, b>d, b>c
Mathematical Learning Process	6.595^{***}	a>c, b>c, d>c
Mathematical Academic Experience	50.92^{***}	a>b>d>c
Means	30.69^{***}	a>b>d>c

Table 4 and Table 5 show that the change of students' mathematical beliefs manifests a partial V- shaped trajectory. That is, primary school students' scores are the highest, and the scores decrease in junior high years, reach the lowest point in senior high years, then increase again in college. Significant differences between grades are found in all three dimensions of the questionnaire.

Questionnaire Scores of Students at Different Performance Levels

Table 6

Questionnaire Scores of Students at Different Performance Levels [*M* (*SD*)]

Beliefs	Excellent	Average	Poor	Total
Mathematical Knowledge	4.262	4.094	3.826	4.085
	(0.395)	(0.558)	(0.554)	(0.538)
Mathematical Learning Process	4.041	3.879	3.605	3.867
	(0.401)	(0.459)	(0.576)	(0.492)
Mathematical Academic Experience	4.089	3.856	3.275	3.802
	(0.876)	(0.853)	(1.015)	(0.933)
Means	4.115	3.930	3.563	3.906
	(0.430)	(0.500)	(0.553)	(0.527)

Table 6 shows that all the scores of students at different levels of academic performance as well as the means are above the critical value (3 points) in every dimension.

Comparison of the Scores between Students at Different Performance Levels

Table 7

Comparison of the Scores in Each Dimension of the Questionnaire between Students at Different Performance Levels

Beliefs	*F*	LSD Post Hoc Comparison
Mathematical Knowledge	13.92^{***}	Excellent>Average>Poor
Mathematical Learning Process	16.98^{***}	Excellent>Average>Poor
Mathematical Academic Experience	17.06^{***}	Excellent>Average>Poor
Means	25.04^{***}	Excellent>Average>Poor

Table 6 and Table 7 show that there is remarkable difference in mathematical beliefs between students with different academic performance. That is, the scores of students with good academic performance are significantly higher than that of students with average and poor performance.

Discussion

The Overview of the Students' Epistemological Beliefs about Mathematics

In the hypothetical-situation questionnaire, the questions involving length measurement, mathematical graphics, calculations with a calculator, fraction comparison, and estimation of word numbers are perceived by over 80% students as mathematics related. Those questions are covered in the elementary math textbooks. The questions concerning jigsaw in life and simple observation are least accepted as relevant to mathematics. Those questions are less likely to appear in math textbooks. The questions with the medium level of acceptance are related to estimation in life, the record and expression of time, choices in life, etc.. These questions are included in the primary school textbooks after the adoption of the New Curriculum Reform, but never before.

The results of the questionnaire on mathematical beliefs show that students' scores in all dimensions are above 3 points. The students' understandings about the nature of mathematical knowledge, the learning process, and academic experience appear to be considerably mature. But their responses to hypothetical-situation questions reveal that they are unable to identify the relationship between mathematics with many problems in their daily life, such as observation, recording, reasoning, and decision-making. One possible explanation for this inconsistency that the questionnaire reflects the students' theoretical mathematical beliefs received from their teachers at school; while the situational questions reflect the students' intuitive, implicit, naive understanding about mathematics. These two understandings on different levels lead to two different types of answers. Xu, Liu and Chen had a similar analysis in their own studies (Liu & Chen, 2002; Xu, 2006).

The Mathematical Beliefs of Students of Different Grades

There are significant differences in the answers to the hypothetical-situation questions between students of different grades. The primary school students are more likely to identify questions concerning estimation of word numbers, the record and expression of the time, calculations with a calculator, choices in life, reasoning, decision-making and mathematical graphics as mathematics related. Such knowledge is included in primary school mathematics textbooks after the adoption of the New Curriculum Reform. The current primary school mathematics textbooks contain four major areas: number and algebra, space and graphics, statistics and probability, practice and integrated application (Ministry of Education of the People's Republic of

China, 2001). In particular, the newly added contents such as statistics and probability, number and algebra, emphasize the ability to make estimations. Therefore, it is not surprising that more primary school students believe the questions involving choose and decide, reasoning, number estimation are mathematical than do students of other grades. In the interview with the students, many primary school students mentioned that "there are similar topics in our textbooks and exercise books", while high school students and college students held different opinions. "They are just common sense, too simple to need mathematical solutions, so they cannot be regarded as mathematics." It shows that the students' attitudes toward mathematics are closely related to the course contents of mathematics and their implicit view formed in the long-term learning process.

The questionnaire results suggest that the development of the students' mathematical beliefs tend to follow a V-shaped course. The change of course content and evaluation of mathematics learning from primary school to college may account for this tendency. In terms of curriculum, the primary school mathematics textbooks pay more attention to relating mathematical knowledge to real life situations with which the students are familiar. The contents are introduced through the description of real life situations and a number of open-ended questions (the question with incomplete conditions or with more than one answer, in contrast with the traditional close-ending questions) are included into the textbooks. This enables the primary school students to understand more clearly that the mathematical knowledge is dynamic and closely related to real life. Therefore, they got higher scores in the dimension of the beliefs of mathematical knowledge. In terms of evaluation of mathematics learning, the primary school students are under much less academic pressure than are the students at higher stages.

Basically, the assessment of primary school students' mathematics learning is not solely results-oriented; instead, it is focused on various aspects, including knowledge and skills, processes and methods, attitudes, emotions and values. Furthermore, mathematics at the elementary level is easy to learn, and the students' confidence and interest in mathematics learning remain at a high level. Therefore the primary school students score the highest in the dimensions of beliefs about mathematical learning process and academic experience. At the senior high school age, the mathematics course content becomes much more abstract and logical than that of primary school and junior high school. The gradually deepened learning contents, coupled with the enormous pressure posed by the college entrance examination, intimidate students who are not very confident of their mathematical ability.

Many students study mathematics only for the purpose of getting a good score on college entrance examinations, and they often find mathematics learning meaningless and tedious. At this stage, mathematics learning begins to take on a utilitarian color. Test scores are the only measurement to evaluate the students' learning and the performance ranking prevails within class, grade and even among schools. Under their constant strain, senior high school students tend to lose confidence and interest in the mathematics learning. Their inadequate understanding of the nature of mathematical knowledge, lack of initiative in mathematics learning process, as well as an unfavorable academic experience result in the lowest scores in every dimension of the questionnaire. At the college level, a variety of mathematics courses are offered, which reflect the common link between mathematical knowledge. The curriculum becomes more flexible for there are both required courses and optional courses. In terms of evaluation, college teachers seldom compare students' mathematics scores, and the frequent class ranking is gone. Students are allowed to construct their own mathematical beliefs in a relatively relaxed environment through new experiences of the nature of mathematical knowledge and mathematical learning process. There is a significant improvement in the college students' scores in each dimension of questionnaire, compared with those of the senior high school students.

It is worth mentioning that, although the questionnaire scores of college students and primary school students are both at a high level, their cognitive difference is obvious. It is easy to find by interview that the understanding of college students is more mature and rational than that of the primary school students. For example, the college students are more aware of the applicability of mathematics and "the essential role that mathematics plays in economic development and technological progress". The primary school students' understanding of the usefulness of mathematics is limited to daily activities like "shopping or bookkeeping". With regards to the academic experience in mathematics, the college students talk more about "the ecstasy after the problem is solved with painstaking effort". The primary school students tend to make remarks like: "I thought out a good method to solve the problem and was praised by the teacher, I felt very happy." On the learning process of mathematics, the college students believe that "thinking hard by yourself to find a solution to the problem is more helpful than listening to the teacher's explanation". The primary school students talk more about: "when facing a problem, the teacher often asks us to think first, and then discuss it together."

The Mathematical Beliefs of Students at Different Performance Levels

The result of hypothetical-situation tests show that there are more remarkable differences between students at different performance levels in the answers to questions involving estimation of word numbers, length measurement, graphics in life, observations in life, reasoning, decision making, abstract mathematical graphics. This content, compared with the more common mathematical operation, equation calculations, geometric constructions, geometric reasoning, share a small portion in mathematics textbooks. Excellent students tend to have wider range of knowledge and study more extensively, so they are more likely to perceive these questions as mathematics related than other students.

Questionnaire results show that excellent students score significantly higher than poor students and average students in all dimensions of mathematical beliefs. The reason might be that the excellent students are more interested and motivated to learn mathematics, and usually put greater effort into it. In the process of intensive and extensive mathematics learning, they gradually become able to appreciate that the mathematical knowledge is inter-related, actively constructed, relative, changing and developing, and highly valuable in real life practice. The success in mathematics learning further confirms their beliefs that the mathematical learning is achieved through self-regulated active construction through various strategies, and reinforces their positive academic self-concept. As the students find it exciting and meaningful to explore mathematics problems, they are more inclined to believe that the study of mathematics is interesting and valuable. In short, the students who have better achievement in mathematics learning tend to have more mature and positive views about mathematics than less successful students.

In addition, although this study reveals the trajectory of the development of students' mathematical beliefs and the differences of mathematical beliefs between students at different academic levels, but the deep mechanism of mathematical beliefs remained unclear. The use of regression analysis or structure equation modeling would be helpful to explore how mathematical beliefs affect the learning process and, thus, the performance.

Conclusions

Conclusions are as follows: (1) The student's attitude toward mathematics is closely related to the course content of mathematics as well as the implicit beliefs of mathematics that the student has acquired in the long-term learning process. (2) The development of students' mathematical beliefs from primary school to college tends to follow a partial V-shaped pattern. That is, primary school students' scores in the mathematical beliefs questionnaire are the highest, and the scores decrease in junior high school and reach the lowest level in senior high school. Then, the scores increase again in college. (3) Remarkable differences are found in every dimension of mathematical beliefs among students at different levels of academic performance. The scores of excellent students are significantly higher than that of the students with average and poor academic performance.

Above all, students' beliefs about mathematics develop gradually since the day when they begin to learn mathematics, and they are strongly related to the students' learning outcomes. A student's belief about mathematics is an important component of the student's learning experience since it affects the student's involvement in the mathematics learning activities as well as his/her mathematics achievement. Being aware of how the students view mathematics, the teachers can provide more appropriate and effective instruction for the students in mathematic learning. Teachers may be able to help the students with poor performance become more interested in mathematics by cultivating their mathematical beliefs. How to change students' beliefs about mathematics is an important topic of future research.

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