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# An Exploratory Study of Primary Two Pupils' Approach to Solve Word Problems

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*This paper describes the systematic approach used by a sample of 30 Primary 2 pupils from one primary school to solve two non-routine problems. The study focused on (a) the relationship between pupils' academic mathematics achievement and problem solving abilities, (b) the differences in problem solving abilities among the pupils for each problem item, and (c) the different heuristics they used to solve these two non-routine problems. This study found a very low correlation between achievement in academic mathematics and success in solving problems. From this we can deduce that even if a pupil performs well in academic mathematics, the pupil may not be proficient at solving non-routine problems.*

***Key words:*** mathematical non-routine problem solving, systematic approach, heuristics.

In the past two decades, a greater emphasis has been placed on pupils' mathematical problem solving as reflected in standards for school mathematics (e.g., National Council of Teachers of Mathematics, 1989, 2000). Despite numerous studies and books written about problem solving, the mathematics education research community seems to show little understanding of problem-solving experiences. In fact, Lester (1994) lamented that "problem solving has been the most written about, but possibly the least understood topic in the mathematics curriculum" (p. 661). McGinn and Boote (2003) reasoned this lack of understanding could be due to research on problem solving generally and mathematical problem solving in particular; both had focused on rather narrow theoretical perspectives. Moving in the same direction, Singapore's Ministry of Education revised the syllabus for the mathematics curriculum in schools with a primary aim to enable students to develop their abilities in mathematical problem solving (Ministry of Education, 2007).

## Background Issues

### Problem-Solving Process

Polya (1957) summarized four executive processes used by mathematicians when solving problems: (1) understand the problem, (2) devise a plan, (3) carry out the plan, and (4) look back at the problem. This 4-step model has impacted enormously on the teaching of problem solving in schools over the past half century. Other researchers have similar descriptions of cognitive activities used in the process of solving problems. For instance, Burton (1984) identified four phases in the problem-solving process: entry, attack, review, and extension. Similarly, Suydam (1980) identified these steps: understand the problem, plan how to solve it, solve it, and finally review the adequacy of the solution as carried out by effective problem solvers in solving problems. There are others who have identified five or more phases. Newman (1983) developed a five level hierarchy to summarize responses to verbal arithmetic problems that are presented in written form. According to Newman, all respondents to such questions, in their search to provide satisfactory solutions, would: decode the problems, comprehend them, transform them from the written, verbal form to an appropriate mathematical form, apply the necessary mathematical process skills, and encode an answer in a way that satisfies the original questions. Though there are slight differences in the approaches, most problem-solving models recommend that problem solvers: clarify for themselves what is problematic about a particular problem, draw up a plan of attack, carry out the plan and revise if it is necessary, and (d) review the processes employed and check the solutions obtained (Clements & Ellerton, 1991). If mathematics teachers are to involve students in the process of doing mathematics, they need to customize their teaching so that carefully-selected problems have suitable content, context, level of difficulty and interest for students in order to provide for optimum effectiveness (Lester, 1980). It seems that three factors: (1) mathematical problem-solving tasks, (2) difficulty level of required heuristics, and (3) the difficulties with mathematical problem solving should dominate the design of problem-solving curricula at the school level.

In problem solving, the process of arriving at the answer is as important as the final answer itself. Polya's (1957) model of problem solving, which involves four phases, is a broad framework encompassing different cognitive processes required for successful problem solving. Lester (1980) commented that Polya's model was rather a proposal for teaching pupils how to solve problems than a description of how successful problem solvers think. In other words, Polya's model provides a guide in organizing instruction of problem solving, but not a guide in identifying problem solvers' difficulties in problem solving or their mental processes involved in successful problem solving. However, many pupils at all school levels have difficulty solving unfamiliar or so-called "non-routine" problems. In a study based on their first-person perspective on problem solving and personal experience with the problems, McGinn and Boote (2003) identified four primary factors that affected perceptions of problem difficulty:

- categorization – ability to recognize that a problem fits into an identifiable category of problems which runs in a continuum from easily categorizable to un-categorizable,
- goal interpretation – figuring out how a solution would appear which runs in a continuum from well defined to undefined,
- resource relevance – referring to how readily resources were recognized as relevant, from highly relevant to peripherally relevant, and
- complexity – performing a number of operations in a solution.

McGinn and Boote (2003) suggested that the level of difficulty of the problem depended on problem solvers' perceptions of whether they had suitably categorized the situation, interpreted the intended goal, identified the relevant resources and executed adequate operations to lead toward a solution. This was also evident in Singapore's study conducted by Kaur (1995) and Yeo (2009). Kaur indicated that Singapore's students experienced problem-solving difficulties such as: (1) lack of comprehension of the problem posed, (2) lack of strategy knowledge, and (3) inability to translate the problem into a mathematical form. Yeo (2009) explored difficulties faced by 56 Secondary 2 (13- to 14-years old) students when solving problems. Information obtained from interviews indicated that the difficulties experienced by Secondary 2 students who were prevented from obtaining a correct solution were: (1) lack of comprehension of the problem posed, (2) lack of strategy knowledge, (3) inability to translate the problem into mathematical form, and (4) inability to use the correct mathematics.

The Present Study

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In light of Polya's (1957) model of problem solving discussed above, and mindful of the experience of lower primary school pupils in solving word problems, it seems timely to propose an 8 step problem-solving framework to guide the 30 primary 2 (8 years old) pupils in solving word problems. Therefore, the research questions to be answered from the analysis of the results in this study are:

- What is the relationship between the Primary 2 pupils' academic mathematics achievement and their problem-solving performance?
- What are the differences in problem-solving abilities among the Primary 2 pupils for each of the non-routine problems?
- Which of the mathematical problem-solving heuristics were used by Primary 2 pupils to solve problems?

In this study, heuristics referred to specific tactics, such as "number manipulation" or "draw diagrams", which were used by Primary 2 pupils to solve non-routine problems.

## Research Methodology

### Participants

The participants in this study were 30 Primary 2 (8 years old) pupils from an intact class in a typical government-aided, coeducational primary school in Singapore. Out of the 30 pupils, there were 12 girls and 18 boys. These Primary 2 pupils had not been explicitly exposed to Polya's (1957) model of problem solving or any specific problem-solving heuristics in their curriculum at the lower primary level. This was confirmed by their mathematics teacher.

### Instruments

An important criterion in choosing suitable mathematical tasks for this study was that they had to be "problems" for a majority of Primary 2 pupils. By the definition of a "problem" it had to be reasonably complex but approachable and requiring no specific high level mathematics. They are usually referred to as "non-routine or non-standard" process problems in the mathematics classroom context. It should also require the problem solver to use heuristic strategies to approach the problem, to understand it and to proceed to a solution. The two non-routine mathematical problems in the Problems Test (see Appendix A) were selected, adapted and modified from the school textbook. The language of the tasks was appropriate for Primary 2 pupils. In order to give credence to the Problems Test instrument, validation was done by the researcher and the mathematics teacher who taught the Primary 2 class. The teacher was explicitly told to look into the content and face validities of the instruments. Furthermore, the researcher and the mathematics teacher also examined the appropriateness of the non-routine problems and whether or not they tested problem-solving ability as defined in this study. In addition, Table 1 shows the 8-step problem-solving framework adapted from Polya's (1957) model that was used to guide the pupils in their thinking and problem solving process (see also Appendix A).

*Table 1*
**8-Steps Problem-Solving Framework**

| Steps | Types of Actions | Description of Actions |
|---|---|---|
| Step 1 | Read the question | Pupils read the word problem at least once to identify the main points. |
| Step 2 | Highlight the information | Pupils were then required to highlight or underline the important information. |
| Step 3 | What is the goal? | Pupils must then look for the relationship between the givens and the goals. |
| Step 4 | What information is given? | Pupils break the word problem into smaller parts and note the implicit information. |
| Step 5 | Find the link and the relationship. | Pupils look for ways to reduce the problem into smaller and simpler parts so that it is easier to solve. |
| Step 6 | Plan | Pupils try for ways to solve the word problem and plan out the steps. |
| Step 7 | Work it out | Pupils carry out the computations to solve the word problem. |
| Step 8 | Check if the answer makes sense | Pupils are required to verify the accuracy and reasonableness of the solution of the word problem. |

**Scoring**

The Problems Test has a maximum score of 12 points; that is, six points each for each item. They were scored by a modified version of the Analytic Scale for problem solving developed by Charles, Lester and O'Daffer (1987). Separate scores were assigned to each of the three stages in problem solving. The criterion for awarding points in each phase is shown in

*Table 2*

**Analytic Scoring Scale (Charles, Lester and O'Daffer, 1987)**

| Stages of Problem Solving | Score | Characteristics |
|---|---|---|
| Understanding the Problem | 0 | Complete misunderstanding of the problem. |
| | 1 | Part of the problem misunderstood or misinterpreted. |
| | 2 | Complete understanding of the problem. |
| Planning a Solution | 0 | No attempt or totally inappropriate plan. |
| | 1 | Partially correct plan based on part of the problem being interpreted correctly. |
| | 2 | Plan could have led to a correct solution if implemented properly. |
| Getting an Answer | 0 | No answer, or wrong answer based on an inappropriate plan. |
| | 1 | Copying error, computational error; partial answer for a problem with multiple answers. |
| | 2 | Correct answer and correct label for the answer. |

Journal of Mathematics Education
December 2011, Vol. 4, No. 1, pp.xxx-xx 

## Results

Out of the 30 pupils, there were 12 girls (40%) and 18 boys (60%). The mean academic achievement score, as measured through the Semestral Assessment (SA), was 85.08 points for the girls and 84.39 points for the boys and this difference was not statistically significant (t = 0.303, p= 0.764). The Problems Test score, as measured by the total mean score of all two problem items, was 7.83 points for the girls and 7.11 points for the boys. The maximum score for the total of the two problem items was 12 points. Similarly, for the mathematics academic achievement, there was no significant difference in the Problems Test score between the girls and the boys ($t$ = 1.061, p = 0.298). The pupils' mean score for the academic mathematics achievement as measured by the SA was 84.67 points (*SD* = 6.059), out of a possible score of 100 points and the mean score for the Problems Test as measured by the total score of all the two non-routine problem items was 7.40 points (*SD* = 1.831), out of a possible maximum score of 12 points.

*Table 3*
**Correlation of Pupils' Academic Mathematics Score and Problems Test Score**

| | | Problems Test ($n$ = 30) | Significance (2-tailed) |
|---|---|---|---|
| Academic Mathematics Score | Pearson | -0.062 | < 0.001 |
| | Spearman | -0.066 | < 0.001 |

The correlation between the pupils' academic mathematics scores and Problems Test scores were calculated at a 0.01 significance level. Table 3 shows the Pearson correlation coefficient and Spearman's rho correlation coefficient of the academic mathematics score and Problems Test score. The Problems Test score was not significantly correlated ($p < 0.01$) with the academic mathematics score; the correlation coefficient was near to zero. The range of scores for each item was 0 to 6. The mean score for the coin problem item was 4.57 points (*SD* = 1.040) and for the storybook problem item was 2.83 points (*SD* = 1.234). A score of 3 or less for a storybook problem indicated that the primary 2 pupils was unable to solve the non-routine problem correctly and had only partial understanding of the use of heuristics to solve the non-routine problem.

*Table 4*
**Mean Scores and Standard Deviations (SD) for Problems Test Items Scores**

| Non-Routine Problems | Mean | Standard Deviation |
|---|---|---|
| Problem 1: Coins Problem | 4.57 | 1.040 |
| Problem 2: Storybook Problem | 2.83 | 1.234 |

Table 5 shows the mean scores for the each stage of problem solving ranged from 0.27 to 1.97 points while the standard deviations ranged from 0.183 to 0.681. The mean scores decreased from "understanding the problem" to "getting an answer". Although the pupils were successful in understanding the two problems, a large of proportion of pupils was not able to get the correct answers. This is specially so for the storybook problem.

*Table 5*
**Mean Scores and Standard Deviations (SD) for Stages of Problem Solving**

| Stages of Problem Solving | Coins Problem | | Storybook Problem | |
|---|---|---|---|---|
| | Mean | Standard Deviation | Mean | Standard Deviation |
| Understanding the Problem | 1.97 | 0.183 | 1.77 | 0.504 |
| Planning a Solution | 1.47 | 0.681 | 0.80 | 0.664 |
| Getting an Answer | 1.10 | 0.548 | 0.27 | 0.521 |

Table 6 shows the correlations between the academic mathematics score and the problem-solving items score. Both Pearson's r and Spearman's $\rho$ were computed. The two problem items are not statistically significantly when correlated ($p < 0.01$) with pupils' academic mathematics achievement score as shown in Table 6. Their correlations were all weak.

*Table 6*

**Pearson's r Correlation and Spearman's rho Correlation coefficient of Pupils' Academic Mathematics Score and Problems Test Items Score**

| | *Pearson's r Correlation* | | *Spearman's rho Correlation* | |
|---|---|---|---|---|
| | Coins | Storybook | Coins | Storybook |
| Academic Mathematics Score | -0.122 | 0.011 | -0.149 | 0.044 |
| Significant (2-tailed) | 0.520 | 0.955 | 0.433 | 0.816 |

The problem-solution heuristics used by the 30 Primary 2 pupils to solve the two problems, coins and storybook, were analyzed. The pupils' written solutions were analyzed according to the types of heuristics used. The coding of the solutions was based on the written work only, as none of the pupils were interviewed. Four major types of heuristics were identified: guess and check, draw a diagram, number manipulation and tabulation.

1. **Guess and Check** – solution was obtained by making a guess and checking it.
2. **Draw a Diagram** – solution was obtained by using diagrams to model the process of solving the problem.
3. **Number Manipulation** – solution was obtained by simply manipulating the numerals in the problems using the four operations (+, -, /, x).
4. **Tabulation** – solution was obtained by organizing the data into a table.

The category "**No Response**" was used to classify solutions which were not attempted.

*Table 7*

**Heuristics Used by Primary 2 Pupils to Solve the Problems**

| Heuristics | Frequency (%) | |
|---|---|---|
| | Coins Problem | Storybook Problem |
| Draw a Diagram | 50 | 20 |
| Guess and Check | 3.3 | 0 |
| Number manipulation | 43.3 | 66.7 |
| Tabulation | 0 | 10 |

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Note. No Response: Coins = 3.3%, Storybook = 3.3%

Table 7 shows the percentage frequencies for the various heuristics used by 30 Primary 2 pupils to solve the two problems. The findings in Table 7 show that some heuristics such as "guess and check" and tabulation were problem specific while "draw a diagram" and "number manipulation" were common to the two problems. From Table 7, it seems that the main heuristic used by the Primary 2 pupils to solve the coins problem was the "draw a diagram" heuristic. A large proportion of those who used this heuristic were successful in obtaining the solution to the coins problem. Only one Primary 2 pupil attempted to solve the coins problem using the "guess and check" method. Approximately 40% of Primary 2 pupils made an effort to use "number manipulation" to solve the coins problem; they were relatively unsuccessful in obtaining the solution. Only one Primary 2 pupil solved the coins problem which was classified under "no response". Table 7 indicates that the heuristic used in high proportions of Primary 2 pupils to solve the storybook problem was "number manipulation". This is not surprising as all the Primary 2 pupils had been taught the four operations (+, -, x, ÷) five months prior to their participation in the study. This finding lends support to the observations by Burkhardt (1988), Bastow, Hughes, Kissane, and Mortlock (1984), and Resnick (1989) that most pupils are only comfortable solving problems which employ the mathematics they have learned some time ago. Most of the pupils who used "number manipulation" to solve the storybook problem appeared to be unsuccessful in obtaining the correct solution. The second most common heuristic used by the Primary 2 pupils to solve the storybook problem was "draw a diagram".

## Discussion and Conclusion

The findings in this study indicated that pupils' academic mathematics performance (semestral assessment) is not linked to success in problem solving in a clear-cut way. In particular, problem-solving ability will not simply follow from the development of general mathematical competence. The low correlation in the relationship between problem-solving abilities and academic mathematics achievement could involve many linked factors. One of the factors could be the ability to transfer knowledge and skills to non-routine problems. Another factor could be that the development of understanding and thinking of the pupils was still at an elementary stage and there is potential for these areas to be developed. Perhaps Primary 2 mathematics teachers could explore ways to incorporate non-routine problems into their mathematics curriculum so that the desired outcome of understanding and thinking could be observed. Partially correct solutions are particularly useful because the pupils can learn from the mistakes and find ways to improve on the solutions, hence deepening their knowledge of mathematics. The pupils' work can be collected and classified into meaningful codes as a joint effort between teachers and researchers, and this collection will be a useful resource for classroom teaching and teachers' professional development. The results have shown that the 8-step problem-solving framework is a systematic approach to help the pupils understand the problem. However, the pupils found that to proceed to solve the problem, they must be able to arrive at a network of knowledge and expertise that matches the present problem otherwise, they cannot propose effective problem-solving strategies or plans.

This paper has provided some evidence to show that Primary 2 pupils had different levels of success in using four of the common heuristics to solve mathematics problems. This difference may be explained by the interaction between the nature of the problems, the pupils' mastery of these heuristics, and the nature of the heuristics. In addition, the reliance on coping strategies such as "number manipulation" by Primary 2 pupils signals to the teacher that they are failing and need help.

In this study, pupils often use "number manipulation" and "draw a diagram" to solve problems. No trend were established as to when which were used. Further investigation is necessary in this area to help shed light on when pupils use "number manipulation" and "draw a diagram". Is the decision to use "number manipulation" and "draw a diagram" related to the type of problem being attempted? Future research should not rely only on the analysis of written work. The codes need to be validated through the use of complementary methods such as clinical interviews, error analysis, diagnostic tests, and observations and/or videotaping of pupils solving problems.

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Journal of Mathematics Education
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Appendix A
Problems Test

| | |
|---|---|
| 1. Peter has some \$2-notes and 50 cents-coins in his wallet. How many possible ways can he make up \$5?<br>Step 1: Read the question<br>Step 2: Highlight the information<br>Step 3: What is the goal?<br>Step 4: What information is given?<br>Step 5: Find the link and the relationship<br>Step 6: Plan<br>Step 7: Work it out<br>Step 8: Check if the answer makes sense.<br>Goal: ___________________________<br>Information:<br>Plan:<br>Work it out: | 2. Mary has a storybook. She starts reading from page 8. She reads the book for 5 days. Each day, she reads 4 pages of the book. At which page will Mary stop at on the $5^{th}$ day?<br>Step 1: Read the question<br>Step 2: Highlight the information<br>Step 3: What is the goal?<br>Step 4: What information is given?<br>Step 5: Find the link and the relationship<br>Step 6: Plan<br>Step 7: Work it out<br>Step 8: Check if the answer makes sense.<br>Goal: ________________________________<br>Information:<br>Plan:<br>Work it out: |

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