Phases of a Ten-Year Old Student’s Solution Process of An Insight Problem as Revealed By Eye-Tracking Methodology

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The study focuses on a 10 year old student's problem solving process on a mathematical problem. We hypothesized that phases of the solution process as revealed by eye-tracking methodology can be clearly identified, and the emergence and suppression of the student’s a-ha experience can be detected. In this research, one minute long observation of the solution process of a ten-year-old student were documented and analyzed. The results suggest that different phases of the solution process can be distinguished and described. Among the educational consequences of the results are the potential of merging objective eye-tracking data with qualitative narratives and identifying the suppression of “a-ha” experiences.

Keywords: Eye-tracking, a-ha experience, word problem.

Mathematical word problems are often considered as the archetype tools of measuring how students are able to apply their mathematical knowledge. Word problems have a long historical and educational route to become the unavoidable means of testing students’ mathematical knowledge. A huge body of the literature focused on the types of word problems and on the solution process. Insight problems where the so-called a-ha experience can be observed are in the focus of several investigations. The current research aims to provide evidence on the potential eye-tracking methodology may provide when revealing different phases of insight word problem solution process.

The Process of Mathematical Word Problem Solving

Mathematical word problems are verbal descriptions of mathematical problem situations (Verschaffel, Greer & De Corte, 2000). Previous research provided several classifications of simple arithmetic word problems (see e.g., Morales, Shute & Pellegrino, 1985; Riley and Greeno, 1998), and there can even developmental milestones be identified in the efforts to describe the related mental processes. Research on the mental processes that underlie
arithmetic word problem solving can be interlinked by three milestones in the field. First, Kintsch and Greeno (1985) proposed a model that describes the solution process as a set of sequential, well-defined steps and working memory limitations play a decisive role in their model. Second, Hegarty, Mayer and Monk (1995) suggested a model that permits ramifications and loops in the mental processes. There is one crucial step in the process: whether the problem solver succeeded in building an appropriate problem representation or not. The third line of research, hallmarked by the Leuven research team lead by De Corte and Verschaffel (1981), highlighted the metacognitive components of word problem solving. For instance, the need for verification of the outcome of an arithmetic operation may require metacognitive considerations (see also Pólya, 1945).

**Insight Problems and A-Ha Experience**

Within the various types of word problem, non-routine and puzzle-like word problems are often labeled as insight problems as opposed to move problems. Still little is known about the cognitive processes involved in solving insight problems (Batchelder & Alexander, 2012). The way how move problems are solved is labeled as ‘mental set’ (Öllinger, Jones, Knoblich, 2008), referring to routine procedures during the solution process, whereas insight problems require unconventional methods of solution. Insight problems give the opportunity of sudden cognitive transactions (Spivey & Dale, 2006) by means of changing the initial problem representation (Knoblich, Ohlsson, & Raney, 2001). Puzzle-like word problems offer ground for evoking a-ha experience which means that having induced an initial frustration caused by the lack of immediate solution, sudden changes in brain-correlated mind states can be detected (Spivey & Dale, 2006).

Some tasks are more likely to provoke sudden ideas than others (these tasks may be labelled “insight problems”), therefore when investigating the insight phenomenon it is the tasks characteristics that provide a first variable in research design. However, for those problem solvers who have already encountered with many insight problems, such problems may seem to be somewhat easier and even of routine-type. This principle relates to individual differences, and might be referred as the process of dimension of insight. The third dimension of the insight phenomenon as a psychological process is whether the problem solver experience an involuntary process called the a-ha experience (Öllinger & Knoblich, 2009).

The detection of an involuntary psychological process requires an objective method of observation. Eye-tracking may serve as one of the possible observation methods. For example, Johnston-Ellis (2012) used anagram problems, Knoblich, Ohlsson and Raney (2001) chose to apply matchstick arithmetic problems, Jones (2003) tested participants by a computerized version of a game about maneuvering a taxi in a car park. All these previous studies
have proven that eye-tracking has the unique sensitivity to inspect the insight phenomenon (Knoblich, Öllinger, & Spivey, 2005). Previous research (Jacob & Hochstein, 2009) proposed eye-tracking-data-based quantitative description of the possible transition between psychological processes. In their research, while searching for identical cards on the screen, and matching them by clicking, the number of fixations tended to be changed before the conscious recognition of the pairs.

The aim of the current research is to observe and analyze the solution process of an insight problem. In line with the three dimensions of the insight phenomenon, a task that may provoke a-ha experience being solved by a student for whom the task is certainly a task of routine-type was used. The main research question is whether it is possible to identify different phases of the solution process of such a problem. It is hypothesized that eye-tracking data will clearly show different phases of the solution process, and it was also assumed that the possible emergence of the a-ha experience can be detected.

Our original intention was to find an objective method to describe the solution process of an insight problem. In previous publications we focused on the potentially reportable strategic processes of reading and mathematical abilities. It was experienced several times that students are unable to appropriately and accurately report on their own thinking processes. This might in part due to their poor vocabulary, but there are cases where even adults might suffer from finding a fluent description of their own thinking. As part of a quantitative educational research project on the role of number stimulus modality in mathematical problem solving, an insight problem was administered to students with the aim of revealing their possible a-ha experiences.

It was hypothesized that the emergence of the a-ha experience can be observed by means of eye-tracking methodology as opposed to students’ self-reports when post-hoc narratives may either embellish their real thinking process or students may be unable to verbally report on their thinking. Actually we hypothesized that the emergence of the a-ha experience would be observed by means of sudden changes in the eye-movement pattern.

**Methods**

**Rationale for Using Mixed Methods Approach**

Among methods for detecting the emergence of a-ha experience, collecting quantitative data on the thinking process seemed to be inevitable. In this mixed methods study (Johnson & Onwuegbumie, 2004) the numerical variables from an eye-tracking session were combined with qualitative interpretive analysis. According to Johnson and Onwuegbumie (2004), eight possible types of mono- and mixed methods can be distinguished. Our intention to objectively detect the emergence of the a-ha experience is clearly a research objective of quantitative nature. Also the data set was quantitative with time measures. However, we performed qualitative data analysis, since there was no
previous quantifiable criterion when and how the emergence of an a-ha experience can be revealed. The qualitative data analysis was based on a series of consultations with pre-service and in-service mathematics and reading teachers. In regular university courses, teachers watched the video footage analyzed in this study in silence without any previous commentary on the emergence of an a-ha experience. At the phases shown in Figures 7 to 9 in the Results section, the audience burbled, and during the discussion of the footage, the colleagues agreed upon the emergence and the temporal suppression of the student’s a-ha experience.

**The Task**
The task was presented on the screen of the eye-tracking machine. The distance from the screen was 70 cm, and the text seen in Figure 1 was displayed:

![Figure 1](image)

**Figure 1. Display of the puzzle-like task.**

Note. The translation of the text is as follows: “You pilot the airplane. In Berlin, 10 people board. In Budapest, 9 people get off, and 6 people board, and finally, in Vienna, 4 people get off, and 12 people board. How old is the pilot?” Following the ‘didactical contract’ (Brousseau, 1997), students are expected to find the task meaningful, and therefore realize that it is them who pilot the airplane therefore the answer should be their age. Furthermore, the task offers the compliance of the routine algorithm of collecting numbers and executing arithmetic operations. The procedure starts with inspecting the figures and the possible relations between them. Hence a possible (albeit meaningless) solution would be $10 - 9 + 6 - 4 + 12 = 15$. The question concerns the age of the captain, so instead of adding and subtracting the numbers of the passengers, students should find the semantic connection between the first and last sentences. Even in this latter case, the reality of the situation described in the word problem is seriously questioned not only because of the feasibility of such a trip, but mainly because of the impossibility of being a pilot at the age of 10 or 11. Consequently, there is no ‘correct’ answer for this task, but the semantic
connection between the first and last sentences makes it possible for the students to have an a-ha experience.

**Data Collection and Analysis**

Data collection took place in the school. Twenty-four students participated in the experiment, and they were tested in a quiet room. Having solved a warming-up task and four simple arithmetic tasks, they finally encountered the sixth shown in Figure 1. Eye movements were registered with a Tobii T120 eye tracker. The eye-tracking machine automatically record and store the data on students’ fixations on the screen. The number and length of fixations on the screen indicate conscious and non-conscious processes in thinking (see e.g., Rayner, 1983). Researchers usually define several areas of interests on the screen, and the number and length of fixations can be analyzed in view of these areas.

In usual (quantitative) data analysis procedures different fixation time measures are to be computed (see e.g., De Corte, Verschaffel, & Pauwels, 1990). These measures in our study have already been presented (Authors, 2013). Our aim was now to zoom in on one student’s problem solving process. Besides collecting those usual quantitative measures, we aimed for qualitative insights about the reasoning process.

**Participants**

One of the students, pseudo-named Oliver, proved to be very typical concerning the quantitative eye-tracking measures on the puzzle-like tasks, but on a previous arithmetic skill test, he achieved the second highest score in the group. Good arithmetic skills can compensate for even possibly low level of reading skills when solving arithmetic tasks (Nortvedt, 2011). We aim to analyze Oliver’s solution processes who has above the average counting skills, but has average reaction time measures on this puzzle word problem.

Oliver was 10 year and 9 months old at the time of the experiment. During this six task experiment, interestingly, he failed to solve two tasks where the number components of the word problems were written in number words and not in Arabic numerals. As for the puzzle-like task, he belonged to the small group of five who succeeded in a way that they could semantically connect the first and last sentences and claim that the pilot was as old as they were.

**Results and Discussion**

The results of the current study are organized according to the timeline of the video footage. As it was mentioned, besides having had a quantitative research objective and having collected quantitative data (frequency and length of fixations), the analysis was based on qualitative narratives agreed upon by pre-service and in-service mathematics and reading teachers. The number and length of fixations on different areas of interests will be presented by means of
several screenshots of the footage, and the qualitative analysis serves as the discussion of these results.

It took Oliver 1 minute and 5 seconds to answer the question: How old is the pilot? At the end of this interval, he answered: ‘10 years old, since it is me who the captain was’. The Results section contains a narrative analysis of this time interval.

**Phases of the Solution Process**

0:00 – 0:02 The first sentence was read during only 2 seconds. The two short words received only one fixation, and the two longer words required two fixations.

0:02 – 0:06 The second row required four seconds with seven fixations.

0:06 – 0:10 The third row consumed another four seconds with eight fixations.

0:10 – 0:15 Until this, the reading process was straightforward and sequential, without any rereading or without skipping any part of the text. From now on, Oliver started to jump in the text. The way of his thinking may be reflected by the way of the fixation visualization pattern. The first jump in the text is shown in Figure 2.

![Figure 2. Screenshot at 0:14.60.](image)

By that time, Oliver may have realized that his working memory was full of data: places and numbers were read, and before reading the last numeral, he decided to read the numeral of the second row again. The 15th and 16th seconds of the solution process provides even further evidence about using the ‘search for numerals’ strategy (Reusser & Stebler, 1997). Figure 3 shows how quickly the first three numerals were reread.
These three very quick fixations on the numerals clearly indicate a conscious strategy.

0:15 – 0:21 Oliver reads carefully the remaining numeral (note: the last figure has not been read previously because of the chosen regression seen in Figure 2. It lasted five seconds to read the last two numerals, having fixated not only them, but on the keywords telling “take-off” and “landing” passengers. Having read all numerals and the keywords carefully, Oliver finally chose to read the question. Until this point he seemed to be rather sure that the question will be meaningful, at least it concerns the number of passengers on board.

0:21 – 0:25 Oliver reads the question in the fifth row, exactly twice, fixating three times each, and a third rereading occurred on the words “the pilot”. Then he started to jump backwards on numerals as seen in Figure 4.
0:25 – 0:26 Having read the numerals 4 and 9 again (it was the third time for reading the 9 the 4), he decided to start reading the text from the very beginning again as seen in Figure 5.

Figure 5. Screenshot at 0:25.74.

0:26 – 0:29 There is no fixation point on the screen. Oliver looked above the screen for three seconds, and then restarted gazing upon the screen.

0:29 – 0:33 Oliver reads patiently (with the usual fixation frequency) the third row. But suddenly starts reading the first row again.

0:33 – 0:37 Reading continues in the second row at the usual pace. Reading seems to continue with the third row, but suddenly, as seen in Figure 6, Oliver jump back to the first word of the second row, and to the first word of the first row.

Figure 6. Screenshot at 0:36.87.

The sudden change in the pace of reading, and the regression to the beginning may indicate the recognition of the importance of the first sentence.

0:37 – 0:40 Reading the first sentence with the usual fixation points for the fourth time might even indicate the loss of interest or stepping down from the
solution process. But at the end of the first row, fixation on the word “vagy” [literally: you are] lasts much longer than before. The animated visualization shows that fixation on that word started at 0:38.33, and terminates at 0:40.07, with two long fixation periods on the same word. This long double fixation on the semantically most important key word could lead Oliver to the decision that instead of collecting numerical information, from now on he focused on the verbal components of the task.

**Emergence and Suppression of an A-Ha Experience**

The solution process seemed to be somewhat struggling for 40 seconds. The next seconds witness the emergence and suppression of an a-ha experience Oliver might undergo. The emergence phase has two parts. First, we see the insistence on fixating again and again on the key word in the first sentence. This insistence was already present at the double fixation interval, but it still continues. The second part is the fixation oscillation between the first and last rows. It can be clearly seen that Oliver possessed the solution; he realized that the semantic connection between the first and the last sentences was important. However, for some more seconds he suppressed that insight, and restarted the reading process.

Figure 7 illustrates the first phase of the emergence of a possible a-ha experience.

![Figure 7](https://example.com/figure7.png)

**Figure 7. Screenshots at 0:44.06 and 0:45.89.**

The second phase of the emergence of an a-ha experience is illustrated in Figure 8. Similarly to Figure 7, this twin figure shows two moments when it is clearly observable that fixations oscillate between the first and the last rows.

![Figure 8](https://example.com/figure8.png)

**Figure 8. Screenshots at 0:51.80 and 0:52.71.**

There was even a third quick transition in fixation between the last and first rows at 0:53.42, and there was a fourth quick transition at 0:56.21. During
these seconds only the first and last sentences received fixations, and none of the middle three rows.
Every time this footage was shown at university courses the audience burbled during these seconds. It was so obvious for everyone (colleagues, students with various background) that Oliver fully understand the task and he already knows the answer. Much to the audience’s surprise, the footage continued in an unexpected way. Oliver looked out of the screen to the right, and when coming back at 0:58.55 he started to read the middle passages that were thought (by the observers) to be recognized as irrelevant to the solution.
Figure 9 illustrates the disappointing moment when instead of finishing the task by providing an answer based on semantic connection of the first and last sentences, Oliver started to read numerals again.

![Figure 9. Screenshot at 0:59.41.](image)

Fortunately, and much to the observers’ relief, after some seconds, Oliver’s fixations disappeared from the middle rows. Figure 10 shows the final moment of the solution process. (When the researcher heard the expected solution, he terminated the probe and the video footage by hitting the space bar on the computer.) The last moment finds Oliver fixating on the key words of the first sentence.
Conclusion

In this case study we presented a narrative analysis of a 65 second long video footage (animated visualization) that showed a 10 year old boy’s eye fixations during a mathematical word problem solving session. It became clear from the eye-tracking data that the boy applied the well-known strategy of ‘search for numbers then connect them with mathematical operations’. It took 25 seconds (almost half of the complete solving process) to read the task. Some parts were read twice. The next 15 seconds witnessed a strategy terms, i.e. instead of jumping from numerals to numerals in the rereading phase; he decided to focus on semantic elements of the tasks. In the next 15 seconds we saw the emergence of an a-ha experience in two phases. First, it became obvious for the boy that the essential content element can be found in the first and the last sentence. Then oscillation in the eye-fixations between the first and the last sentence was observable. In the last ten minutes a seemingly disappointing rereading phase occurred (suppression of the a-ha experience), but finally the boy succeeded in finding a semantically appropriate (albeit from a mathematical point of view unrealistic) solution to the task.

The novelty of our research was providing a qualitative description of a student’s word problem solving procedure. It gave the opportunity to match theoretically established model phases and empirically supported data. Subsequently our aim was to enrich the case study methodology by means of amalgamating objective data and assigned narratives based on agreement among experts.

The generalizability of our results can be extended in two ways. Either the number of participants who supposedly have a-ha experience while solving a mathematical problem should be increased or the discussion of the current case study can be enriched by means of having quantitative data about the agreement among experts who watch the critical phases of the video footage. The question of generalizability is therefore obviously connected to the possible future extension of this research.

Methodological Considerations

In light of the 21st century shift in advocating scientifically-based research (the term has several facets, see Denzin, Lincoln, & Giardina, 2006) eye-tracking methodology may provide the means to meet the classical quantitative research criteria of objectivity, reliability and validity, and at the same time it provides enormous data sets that permit focusing on case studies and provide platforms for critical and interpretive narratives. In this research, eye-tracking was done by collecting objective and sensitive data about each hundredths of a second during a 65 second long interval. The animated
visualization was presented to various audiences, and independently of their status as pre- or in-service teachers, the same crucial moments were identified.

As for the three criteria of scientificity proposed by Hegedus (2010), we (1) attempted to attend to the empirical epistemology of inductive nature, (2) reflected on the novelty our claims proposed, and will soon reflect on the limitations of our ideas, and (3) in the next paragraph we try to identify the dynamic components of our investigation.

Our contribution to the “circular endeavor of scientific discovery” (Hegedus, 2010, p. 391) appears in an invitation to criticize possible holes in our argument in order to refine the methodology we used. Should we have examined more students’ solution processes without thinking of them as units in a sample? Since we had the opportunity to observe many people’s reactions while watching the 65 second video footage, can our approach be considered as a case of investigator triangulation?

The study certainly had some limitations. The students (including Oliver) had previously solved routine word problems before they were shown the puzzle-like task. Although, as Verschaffel, Greer, and De Corte (2000) summarized, contextual changes have only slight effects on the achievement of non-routine word problems, in this case the context of using an eye-tracking machine may have in itself caused changes in attitude and performance. A second methodological concern is of ethical nature. Since it is quite obvious that insight problems (by definition) induce frustration in longer or shorter periods of the problem solving process, some may argue that students should not be exposed to such puzzle-like tasks. While we agree in general that exposing students to frustrating experiences should be avoided, in the actual experiment students’ possible frustration experience was reduced by (1) employing well-trained educational researcher in the data collection process, and (2) reassuring them that their valuable contribution to an educational experience will not be connected to their current school performance and marks.

Educational Issues

The educational considerations are twofold. From a theoretical and methodological point of view, the evidence of the eye-tracking methodology yielded draws our attention to the potential what merging the qualitative narratives and the objective eye-tracking data may provide in describing mental processes. This methodological prospect has the potential to reveal other high-level processes and representational shifts that have practical relevance, and to which rich instructional methodologies and conceptual frames have already been developed, e.g. metacognitive strategies, aesthetic value forming.

The second aspect of the educational implications more closely relates to the current phenomenon we investigated. The emergence and temporary suppression of an “a-ha” experience while solving a word problem points to several didactical concerns. (1) The use of the routine algorithm of collecting figures in the text of the word problem without taking the semantic content into
account, (2) rereading some numerals (because of working memory constraints) even before reading the question of the task, (3) hesitating and struggling after the emergence of the a-ha experience are all consequences of the aims and the means why and how word problems are taught in the elementary schools. These three concerns are worth being embedded in teacher education and continuous professional development courses, since these ‘symptoms’ are results of teaching and learning processes that are not in accordance with declared curricular targets and aims!

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