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The Relationship between High-School Mathematics Teachers' Beliefs and their Practices in Regards to Intellectual Quality

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This study examines the relationship between mathematics teachers' beliefs and instructional practices related to learning, pedagogy, and mathematics in regards to components of intellectual quality for eight high-school mathematics teachers. Research has demonstrated that the higher the degree of intellectual quality for instruction is rated the higher student achievement is on standardized assessments. The findings in this study demonstrate a consistent pattern between teachers espoused beliefs and their instructional practices. Even though teachers' practices changed as they wrote curricular units to be more in line with intellectual quality characteristics, their beliefs stayed consistent over an 18 month period and were correlated to their instructional practices at the beginning and end of the project.

Key Words: Beliefs; instructional practices; mathematics, intellectual quality.

Over the past 25 years, research has prompted several calls for significant changes in students' mathematical understanding and achievement and classroom level instructional practices (CBMS, 2012; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000; NMAP, 2008). Recent policy initiatives in the U.S., such as the wide-spread adoption of the Common Core State Standards for Mathematics (CCSS-M) (NGA, 2010) and associated accountability measures, are built upon this research and have increased the pressure for teachers, schools and districts to meaningfully examine classroom instructional practices to determine whether students are learning the type and depth of mathematics called for by these new standards and assessments (Schmidt, 2012). Both the CCSS-M and assessment consortiums have made it clear that in order to be considered 'proficient' in mathematics, students should be able to reason and communicate with mathematics, make connections within and outside of mathematics, and become better problem solvers (Burkhardt, Schoenfeld, Abedi, Hess, & Thurlow, 2012; NGA, 2010).

While there has been extensive focus and efforts from the mathematics education community on shifting mathematics instructional practice over the last two decades, there is still little evidence of change. Hiebert and Stigler

(2000) examined 8th grade U.S. teachers mathematical practices and found (1) teachers typically state mathematical concepts without developing students' understanding, (2) little evidence of student reasoning and proof, and (3) 'work time' was primarily devoted to practicing procedures. Kane and Staiger (2012) in a more recent large-scale examination of fourth through eighth grade teachers' instructional practices, found the majority of teachers ranked low on mathematics observation constructs related to richness of the mathematics, student meaning making and reasoning, and connections to science or other topics. In light of results from mathematics education research and recent policy initiatives, there is a need to examine the factors that influence teachers' instructional practices in order to better understand the relationships.

Nearly twenty years ago, Newmann and Associates (1996) examined the factors influencing how teachers and schools fostered intellectual quality in mathematics, science, and social studies through instructional practices that lead to the type of increased student achievement called for in the CCSS-M. Newmann et al. (1995) defined intellectual quality as "the extent to which a lesson, assessment task, or sample of student performance represents construction of knowledge through the use of disciplined inquiry that has some value or meaning beyond success in school." They developed three criteria related to a social-constructivist learning theory by which to judge the quality of teachers' pedagogical practices and students' academic work: construction of knowledge, disciplined inquiry, and value beyond school. Then, they examined whether a relationship existed between the intellectual quality of teachers' pedagogical (a combination of task and instructional characteristics) practices and the intellectual quality of their students' work. The researchers found that the intellectual quality of students' performances and products was related to the degree of intellectual quality of teachers' pedagogical practices (Newmann & Associates, 1996). Their research provides a useful framework for examining the teacher factors that may influence the meaningful implementation of the CCSS-M.

In Newmann and Associates (1996) framework, mathematics teachers' instructional practices rated high in intellectual quality, for example, when their students engaged in mathematical analysis and reflective conversations that were focused on increasing students' understanding of the mathematical topic, and when students explored mathematical topics in sufficient detail that they could make connections from what they were studying to other mathematics and note the relevance of the task to something beyond getting a grade. Similarly, students' written work was found to include more intellectual quality when it included evidence of mathematical analysis, students' understanding of key mathematical concepts being assessed, and well-articulated explanations and arguments when needed (Newmann & Associates, 1996; Newmann et al., 1995).

More currently, educational research has examined why teachers are slow to change their instructional practices to be more in line with current learning theories and national recommendations (Kennedy, 2004). Desimone et al (2005) describes beliefs as one factor influencing teachers' practices. Beliefs become an important factor to understand if Pajares (1992) is correct in that beliefs control action more than knowledge. This gives rise to the importance of studying teachers' beliefs about learning, pedagogy, and mathematics in relation to notions of intellectual quality. In other words, if teaching with intellectual quality has positive effects on students' mathematical achievement and if beliefs leverage action, then it is important to examine this relationship between beliefs and instructional practices. What is still unknown is what beliefs affect practice and the degree to which they affect practice (Aguirre & Speer, 2000).

It may be possible that teachers' beliefs reflect components of intellectual quality, but their practices do not because teachers have stronger beliefs there is not enough time to teach towards intellectually quality, it takes too much energy, or there is too much uncertainty involved. By analyzing teachers' beliefs about learning, pedagogy, and mathematics in relation to the components of intellectual quality, reasons for finding or not finding these types of pedagogical practices will be exposed.

The purpose of this study, then, is to examine the relationship between high-school mathematics teachers' beliefs—about how students learn mathematics, teachers' mathematical practices, and mathematics itself as related to the components of intellectual quality—and the intellectual quality of their instructional practices. Hence, this study focuses on the following research question: What is the relationship between the intellectual quality of high-school mathematics teachers' beliefs and their instructional practices?

Method

Participants

This study focused on eight high-school mathematics teachers who were each part of a two to three person cross-departmental STEM team selected through a national search to develop a two- to four-week curricular unit that met the standards of intellectual quality. The teams were selected by their members' past experience in writing informal curricular units, their understanding of reform documents their schools' resources and administrative supports, and their demographic region. The seven teams chosen with the highest ratings were located in three urban, two suburban, and two rural sites across the U.S.

Curriculum Writing Project

The participants were part of an eighteen month curriculum writing project, which consisted of four major events. First, during the initial summer, participants were part of a week-long institute where they learned about the intellectual quality standards and wrote a draft unit. Second, each teacher taught the unit sometime during the academic year. Third the teams met again the following summer for a week to finalize their units based on assistance from mathematics educators, their reflections on teaching the draft unit, and readings related to standards. Fourth, the teams taught the unit again the following fall.

The eight study teachers' beliefs and practices were tracked over this year and a half period. Each teacher's instructional practices were observed two to four times during the first academic year and two to four times in the fall of the second year. Teacher interviews were conducted on the same day as each observation.

Data Collection and Analysis

In order to acquire sufficient information to address the research question, data were collected using observations and interviews. The first instrument was a teacher interview protocol used to investigate teachers' beliefs regarding intellectual quality. The second instrument was an instructional observation scale used to examine the intellectual quality of teachers' practices (adapted from Newmann et al's (1995) standards).

Measuring the intellectual quality of teachers' beliefs. Two semi-structured interviews were used to elicit and classify teachers' beliefs. Throughout the two interviews teachers were asked about their general and specific beliefs about learning, teaching, and mathematics in regards to the components of intellectual quality. All interviews were audio-taped and subsequently transcribed for data analysis.

Once the interview data set had been transcribed, it was cooked (Erickson, 1986). This process entailed reading and rereading the data corpus to code the data externally and internally. The nine constructs (based on the 3x3 matrix) were used to externally code the data. For example, a teacher's response was coded as CL when the teacher described his or her view about how a student *learns* in the context of *Construction of Knowledge*.

The data were then reread in a search for themes and relationships in teachers' beliefs within those nine constructs. Here, a second more discerning code was attached to each passage, creating a set of internal codes, which were formed from existential evidence (teachers' responses) and theoretical positions (based on the previous review of literature) and included two separate sets of internal codes—one pertaining to beliefs related to notions of intellectual quality and one to more traditional notions.

In order to characterize teachers' beliefs about each of the nine constructs, we used the procedure of creating descriptive levels of teachers' beliefs and instructional practices (Fennema et al., 1996). The procedure included: (a) coding the data; (b) aggregating the number of instances within each construct per teacher; (c) characterizing each teacher's set of statements for each construct as being intellectual, mainly intellectual, both, mainly traditional, or traditional; (d) confirming and disconfirming the classification by rereading the data and testing whether the characterization fit; and (e) classifying the set of statements into one of the five levels.

Samples of each teacher's interview data were given to two independent researchers to check the validity of the classifications. If differences arose, the two researchers discussed the differences until a consensus was reached. Each of the nine constructs was given a score from one to five matching the five levels of classification. Because teachers' instructional practices were given three scores from one to five for (a) Construction of Knowledge, (b) Depth of Knowledge, and (c) Value Beyond Instruction, belief scores were given scores for these same dimensions creating a combined score from three (traditional) to fifteen (intellectual).

Each set of beliefs was placed into a category by using the following reasoning. If all instances were coded as intellectual, the set of beliefs for that construct was categorized as being *intellectual*. At the other extreme, if all instances were coded as traditional, the set of beliefs was labeled *traditional*. If most of the instances fit one category—intellectual or traditional—but had a few deviant instances (for specific reasons), the set of beliefs was labeled as being *mainly intellectual* or *mainly traditional*. When the number of instances was fairly equal in both the intellectual and traditional categories and when the statements had equally strong reasons for each divergent case, the set of beliefs was categorized as being *both* intellectual and traditional.

Measuring the intellectual quality of teachers' instructional practices. The intellectual quality of a teacher's instruction was measured by observing and scoring classroom lessons using the Newmann et al. (1995) scales. Each teacher's instructional practices were observed over two time periods throughout the project and were from the same curricular unit. Before the second visit the unit had been revised to increase the unit's level of intellectual quality. During each site visit a teacher's classroom practices were observed between two and four times by two independent observers. The observers rated each lesson using the instructional practice scales. Once away from the site each rater aggregated the scores from each subscale for the visit. These scores were then discussed between the raters until agreement was reached. In other words, each teacher ended with one score per subscale per visit. There was little difference in ratings across the two time periods; therefore, the observation scores were aggregated.

The dimensions on which these teachers' classroom practices were observed and scored were: Construction of Knowledge, Disciplined Inquiry, and Value Beyond Instruction. *Construction of Knowledge* measured the extent to which the instruction involved students in higher order thinking or mathematical analysis. *Disciplined Inquiry* included Depth of Knowledge and Substantive Conversation. Depth of Knowledge measured the extent to which students were required to make connections and explore relationship with central ideas within mathematics. Substantive Conversation measured the extent to which students were engaged in extended conversational exchanges with the teacher and/or with their peers about mathematics in a way that built an improved and shared understanding of ideas or topics. *Value Beyond Instruction* measured the extent to which students were required to make connections between mathematics and either public problems or personal experiences.

Each category was scored on a five-point scale where the higher the score the more a teacher's practice was rated intellectual for that dimension. Disciplined Inquiry contained two subcategories. These two scores were averaged, obtaining one score from one to five. Each teacher, then, received a total rating of the intellectual quality of their classroom practices by aggregating the three dimensions listed above and receiving a score from three and fifteen.

Results

Our initial question asked, what is the relationship between the intellectual quality of high-school mathematics teachers' instructional practices and their beliefs? This section describes the degree of intellectual quality of teachers' beliefs and instructional practices and their relationship to each other. We first present a general picture of the degree of teachers' beliefs and instruction toward components of intellectual quality and then provide a more in-depth picture of the spectrum and consistency of these beliefs and instruction for two cases: one traditional and one intellectual.

A General Account of Teacher Beliefs

The results of the interview data demonstrate in great detail which types of beliefs were more prevalent while also showing consistent patterns of beliefs for the eight teachers and consistent patterns among the teachers. Tables 1 and 2 summarize the number of comments made within each construct and the ratio of intellectual to traditional comments by each teacher.

Table 2 demonstrates there were nearly an equal number of comments made among the eight teachers that were coded as either intellectual (392 comments) or traditional (374 comments). For both Construction of Knowledge and Depth of Knowledge the number of intellectual and traditional comments was fairly equal except for in the pedagogical domain where there was a slight increase in the number of traditional comments. This is accounted for by the large number of comments made by a few teachers who stated repeatedly that their instruction should engage students in memorization and use of algorithms without understanding. Also, the number of intellectual and traditional comments for Value Beyond Instruction weighs more heavily toward intellectual quality. This trend may be accounted for by the selection of mathematics teachers who volunteered to write curricular units that integrate mathematics with other disciplines. Hence, they might tend to believe that instructional tasks should have practical and personal value to the students.

Table 2 depicts the ratio of intellectual to traditional comments made by each of the eight study teachers for the three intellectual quality constructs. These data demonstrate the consistency of comments made by each of the teachers.

Teachers were consistent in the types of comments they made about Construction of Knowledge. Intellectual comments centered on finding patterns, exploring or making sense of mathematics, and mathematics as a way of reasoning or organizing information. These comments were made mostly by two of the nine teachers: Lenny and Boe. Traditional comments suggested that students follow directions or listen to the teacher and that mathematics is a set of tools. Four teachers—Brittany, Mae, Patsy, and Latisha—tended to make more of these comments. The other two teachers, Henry and Anne, made an equal number of comments regarding both positions.

Table 1
Number of Comments Made Per Construct*

	Belief Constructs									
	CL	CP	CM	DL	DP	DM	VL	VP	VM	Total
Authentic	57	68	15	55	70	11	35	65	26	392
Traditional	43	102	18	60	98	7	16	20	10	374

*The nine constructs are created by the 3x3 matrix shown in Table 1. The labels for each cell are created by the row (L — Learning; P — Pedagogy, and M — Mathematics) and column (C — Construction of knowledge; D — Depth of knowledge; V — Value beyond instruction) titles.

Table 2
Ratio of Belief Statements toward Authenticity to away From Authenticity per Teacher

	Authenticity Constructs		
Teacher*	Construction of Knowledge	Depth of Knowledge	Value Beyond Instruction
Lenny	13:1	8:1	24:1
Boe	3:2	5:2	4:1
Henry	1:1	1:1	3:1
Anne	1:1	2:3	3:1
Latisha	1:3	1:6	1:1
Patsy	1:7	1:3	7:1
Mae	1:11	1:13	4:1
Brittany	1:4	1:7	1:1

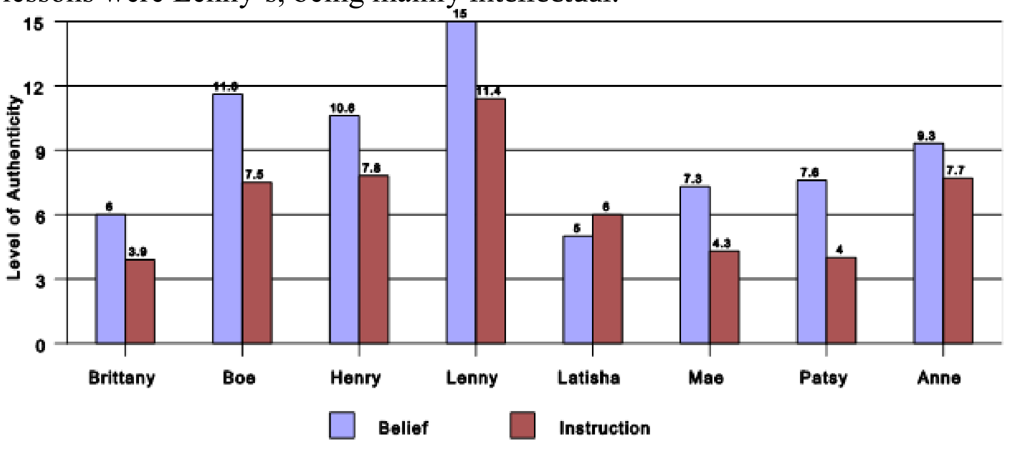
*Pseudonyms are used.

A similar pattern for Depth of Knowledge was found. Lenny and Boe continually made more intellectual statements, focusing their comments on making and discovering connections both within mathematics and among topics outside of mathematics and to some degree on encouraging substantive conversations. They commented that mathematics is a highly interconnected discipline. Traditional comments, focusing on memorizing and using procedures or algorithms in trivial ways and concentrating on teacher-centered communication, were made consistently by Mae, Brittany, Latisha, and Patsy. They tended to comment that mathematics was linear or sequential. Henry and Anne again made an equal number of comments.

The third component of intellectual quality, Value Beyond Instruction, tended to have more comments leaning toward an intellectual view. These statements focused on ideas of learning mathematics through contextual situations or situations that had value to the students. This might be explained by their position as STEM teachers. Mathematics was also overwhelmingly described as intricately connected with the real-world. Traditional comments focused on learning mathematics through non-contextual or isolated contexts and on a mathematics that can be described as separate from reality. Six of the teachers—Lenny, Patsy, Boe, Mae, Henry, and Anne—made more intellectual statements than traditional, while Brittany and Latisha made an equal number of intellectual and traditional comments.

A General Account of Teachers' Instructional Practices

Figure 1 shows the ratings for teachers' beliefs and instructional practices. For instruction the lowest set of scores was 3.9 (Brittany) and the highest was 11.4 (Lenny). A score of three represents a traditional lesson, six a mainly traditional lesson, nine a mixed lesson, twelve a mainly intellectual lesson, and a fifteen an intellectual lesson. In this manner, the lessons for teachers Brittany, Patsy, and Mae were rated traditional. Lathisha's lessons were mainly traditional, while teachers Anne, Boe, and Henry's lessons demonstrated both traditional and intellectual characteristics. The highest rated lessons were Lenny's, being mainly intellectual.



15 = Authentic, 12 = Mainly Authentic, 9 = Mixed, 6 = Mainly Traditional, 3 = Traditional

Figure 1. *Level of authenticity for teacher beliefs and classroom instruction.*

A Focused Examination of Teacher Beliefs and Their Instructional Practices

In order to understand in more detail the nature of teachers' beliefs and instruction regarding the components of intellectual quality, we focus on two extreme cases: Brittany (traditional) and Lenny (intellectual).

The traditional case – Brittany. Brittany taught in a large high school in an urban city along the west coast. The school contained a large high-risk and diverse population, which included mostly Latinos, African Americans, and Caucasians. She taught a range of courses including calculus and basic algebra.

Construction of Knowledge. Brittany's beliefs regarding Construction of Knowledge were traditional and consistent across notions of learning, pedagogy, and mathematics. She believed students learn by watching and listening to her and that instruction should focus on showing students algorithms. Here are two typical excerpts:

> All you have to do is show them what is going on.
>
> I have to show the students how to solve the problems. I will explain it to them until they get it.

In both cases she depicted students as passive learners, waiting for her to tell them what to do next or how to solve the problem. This traditional belief that students are passive learners affected her belief about instructional practices. For instance, when asked about the importance of using examples she exclaimed,

> I can't think of a time when it is not important to give clear examples. I think that when you are introducing a lesson, if you give them a clear example of where they are headed, they are able to tune into what is important. I think when you are developing the lesson and you start showing them some methodologies of attacking the problem, you need to have clear examples so that they know what they are supposed to be doing. When could you not use clear examples?

Here Brittany explains how important it is to focus on showing students what they need to know.

Her notions of mathematics were also consistent within Construction of Knowledge. She stated a number of times that mathematics is a tool: "You use algorithms to find answers . . . it helps to solve problems," and "There are certain basics in math as tools that students must know." Mathematics is believed to be passive, a set of tools, and is classified as being traditional.

Depth of Knowledge. Brittany's beliefs regarding Depth of Knowledge were characterized as traditional. Her notions of both learning and pedagogy focused on procedural knowledge. She believed, foremost, that students must know basic mathematical skills before they can take part in any mathematical analysis or solve complex problems:

> I guess the best way for a high-school student to become a good problem solver is . . . first of all, to be handed fairly easy routine problem situations, perhaps in a group of people where they can get a hold of it, understand it easily and get some decent success. Then, to move on to more difficult or obscure problems where there maybe isn't one particular right answer or they have to consider a number factors to take the problem apart and get it back together.

This passage typifies a core belief held by Brittany: students need to solve simple problems before they can work with complex ones.

Another belief for depth of understanding is communication. Brittany believed students should work in groups for the very reason that communication is important. However, she did not believe students should explain and defend their solution strategies:

> The students work together in class, so the only questions they have are the ones the bright kids can't answer. It makes no sense then to have students come up and explain a problem because they, of course, don't know how to do it.

This response describes a traditional belief: knowing how to do it, but not knowing how to explain the underlying structure of the mathematics.

When asked about whether she viewed mathematics as interconnected or as linear and hierarchical she stated,

> Well, I will have to say that I am leaning more toward hierarchical only because there are certain basics in math as tools that students must know before they can delve into other stuff. After that point, it is definitely interconnected. Once they've got those basics they can pull a strand from algebra and a strand from geometry, all related.

Her belief about mathematics was considered mainly traditional.

Value Beyond Instruction. Brittany believed contextual problems engaged students in low track classes. For students in the college-bound track, she felt problems isolated from contexts and their extraneous variables should be used. Brittany's beliefs within this domain vacillated between using contextual and non-contextual situations. She used contextual situations for one group of students—the non-college bound track. She declared that for these students, contexts were a way to motivate students and to afford them with tangible situations in order to think about abstract mathematical concepts. During an initial interview she stated, "These are kids who are not motivated. Anything I can do to motivate them is okay with me because they are going to get it a lot better if they are motivated."

The other, possibly more critical, aspect for intellectual quality is that contexts can deepen students understanding of important mathematical topics. Brittany stated:

> For people who are having difficulty in math or people who have a mental block against math, I always start out with concrete stuff. When I teach a calculus class, for example, kids who are familiar with math, they are able to deal with the abstract, all you have to do is show them what is going on.

Her belief about mathematics as being connected to the real-world was compromised:

> I see it both ways. I have enjoyed it immensely theoretically where it is just sheer math for math's sake. Personally, it is wonderful to get into it and do it. It never fails you. You always have the right answer. However, I think it is extremely naive that that is the way it should be taught to every student. But if you're talking about students' reality, sometimes it is theoretical. Does that make sense? It's like in a chemistry experiment: the experiment always fails, but the math doesn't.

She believed mathematics, at times, was abstract and isolated from realistic situations and, at other times, was directly related to the real-world. This notion relates to her earlier view that mathematics can be seen as a tool (pure and isolated from any confounding variables in reality), which is used to solve problems in reality; mathematics, for her, was easily separated from the messy confounds of real yet extraneous variables.

A typical lesson for Brittany. Brittany's instructional practices were observed during two different site visits and up to four times each visit. These lessons averaged a 3.9 rating on the instructional scales for intellectual quality. The following is a typical classroom episode.

The class began by the teacher correcting yesterday's homework, which consisted of checking to determine whether students had the correct solutions for twelve problems. She then spent twenty-five of the fifty-five minute class going over the homework before proceeding into the day's lesson, which was focused on slope.

Teacher: I'm checking homework. I'm really concerned about 6 and 8 because they missing from your papers or your graph. Across the board — one, you are not reading directions and two you don't understand graphing. You need to read the directions carefully and follow those directions because I'm checking. And you guys know that. So, when it says plot the points, graph the line, count it out, and you don't have that piece of information or the graph isn't the vertical line something is missed here. This we went through the last two days. What should be on your paper is the sequence of steps. Now does it matter where I put the points?

Student: Nooo!

Teacher: I'm going to start here. This is what I want to see on your paper. Okay? Because a lot of you are skipping this. Write this down. That was an issue that you were dropping signs when we were doing inequalities. We solved equations, when you added the opposite, you were dropping signs. So this is a theme that will appear throughout all of our discussion. Now, #8 said, a slope that is undefined, and we wrote those notes the last several days. Right? Horizontal we talked about remembering this and clues if H is across then m is 0. Vertical has no slope. When it asks you to draw this, what should you automatically know? What should the diagram look like if it is an undefined slope? [Calls on a student.]

Student: Straight down.

Teacher: Straight down. So I don't know where you are getting the other lines from. So there is some miscommunication what a vertical line is or how to draw this through a certain point. #8 says through (-2, 5) or (-2, -5) or whatever. It should be a straight line; up and down. What if I said a horizontal slope out with 0 through (0, 5). Where would I draw it? 1, 2, 3, 4, 5, and straight across that direction. Okay?

The teacher continues a similar line of explanations and questioning for the next three problems. Then, the day's lesson begins by passing out a worksheet for students to complete. The following excerpt begins a few minutes into to the discussion of how to solve the worksheet problems.

Teacher: …What could you describe this as? How many gallons of water our in this tank?

Student: 200,000 gallons.

Teacher: 200,000 gallons at what time frame? . . . [Student] put that away. #2, there are two things going on by the diagram you can see. What is happening simultaneously in this situation? I'm not asking for equations, I just want to you describe this situation. What is going on here at the same time? The whole development are draining water out of the tower. Simultaneously what is going on? Water is going into the tower. Hey, [student], do you get the picture here?

Student: Yeah.

Teacher: Look at the second diagram here. Can you see the K, K is our water tower. It holds 200,000 gallons. So to paraphrase this, at the same time the water is coming out to go to the housing developments, water is coming in [pause] from the inlets. Does that describe this pretty well? [Short pause, no student response.] Now, step #1 is what I really want to go through because a lot of you asked, 'Why did I get this marked wrong?' when it said define an inequality and write an expression and all that and there was nothing on your paper related to variables or inequalities. What does it mean, when it says define a variable? That is step one.

Student: That is like x =.

Teacher: Yes. And tell me what x is? Okay, [student]. Like what?

Student: Like X is the number of minutes that water has been flooded.

Teacher: Oh, that is a lot. Nice. Okay. Let x equal the number of minutes. Is that enough information to answer #1?

Student: Yeah.

Teacher: Yeah. Let's go to #2. 2 says write an expression representing the number of gallons that drawn from the first and second developments. Well, what is the relationship between the water and the gallons? [Student]?

Student: Gallons per minute.

Teacher: Is that what it says here for development 1? 1 gallon per minute.

Student: Yeah. 150 gallons.

Teacher: 150 gallons per minute. How would I write that relationship?

Student: 150 gallons.
Teacher: Okay. What is on this side?
Student: Total water.
Board: 1. Let x = # of minutes
y = # of gallons of water
2. Y = 150x
y = 250x
3. First inlet
Teacher: Okay. Total water or gallons. I'm gonna call that my 'y:' # of gallons of water. What about development #2? What comes out of the tank for development 2? [Student] can you find that on your paper?

The teacher continues working through the worksheet with the students in a similar fashion until the end of the class. She ends the day by telling students not to forget to do the homework.

Lesson rating. This lesson rated 3.5 on the scales for intellectual instruction. Under Construction of Knowledge, the lesson was rated a one. Most of the students, most of the time were engaged in lower ordered thinking. They were asked to recite information that they were to have memorized earlier. The teacher organized the lesson around her presentation of the material; students were to follow this explanation passively, only sharing pieces of knowledge that were elicited by the teacher.

For Depth of Understanding, the lesson received a rating of 1.5. First, the lesson was rated a two for deep knowledge. Although most of the mathematics students were asked to do was superficial they were discussing some important ideas related to slope, parallel lines, and equations. Second, the lesson rated one on substantive conversation. The conversations students had did not focus on any mathematical analysis, did not involve shared discussions with other students, and did not build on any student comments.

The lesson scored one point for Value Beyond Instruction. Although students encountered a context about two developments using water from a local water tower in the worksheet and filled out during the second half of the lesson, it was not used to connect to students' experiences or to a realistic problem they might face.

The lessons proceeding and following this lesson were nearly identical in how Brittany organized the sequence of events. Each day started with correcting the previous day's homework and followed by working through a worksheet and the assigning of more homework.

The intellectual case – Lenny. Lenny taught in a suburban high school near a large metropolitan area on the east coast. The school is located within a wealthy neighborhood and has little racial diversity.

Construction of Knowledge. Lenny believed strongly that students should learn by investigating and making sense of the mathematics and that his instructional methods should encourage students to learn by building on their own prior knowledge, and that mathematics is a way of understanding and making sense of the world:

> I think [students] learn best by discovery rather than just being told this is the algorithm and this is how you use it. If they understand the algorithms, then they will always know how to use it. If you just apply the algorithms, you'll get something done for the short term, but not knowing necessarily when you can ever use that algorithm again. He exclaimed that they do not learn by listening to the teacher but by discovering, practicing, playing, thinking, and reflecting.

His views about pedagogical practices matched his beliefs about learning.

> I believe that effective teaching isn't telling somebody what they should know, but rather I want to help them build their own model of understanding, rather than say, this is what you should think or know . . . Kids are terribly bright and that when we impose our thinking on them we rob them of their own innate way of looking at their work. . . . So, that is why I work that way. It is basically how I believe students learn. They learn by thinking and building their own understanding rather than being told this is what you should know.

Here Lenny tied his notion that students learn by being actively involved in mathematical analysis to his notion that instruction should foster this type of learning.

Lenny's belief that all students should learn through mathematical analysis was not compromised by time or other barriers. He retained the same belief for all students. It differed only in that he used more examples when students had difficulty analyzing the mathematics. However, he maintained his focus on arranging the activities so each student was able to make sense of the mathematics him or herself.

Similar to his notions of learning and pedagogy were his beliefs regarding mathematics. He stated that mathematics is "a way of problem solving and knowing math". Lenny viewed mathematics as an active discipline.

Depth of Knowledge. For this domain, Lenny's beliefs about learning, pedagogy and mathematics were characterized as intellectual. His main focus was for students to understand mathematics, to know the big ideas of mathematics, to understand how and why algorithms are used. For big ideas, he believed students need to understand the properties within mathematics and to be able to reason deductively and proportionally. Understanding algorithms was also important. The next two passages help to explain his position on algorithms:

> We always try to develop algorithms so that they know where they came from rather than ever be given them.

> I think that you need to think beyond [applying algorithms] or you need to think about the algorithm in order to get a better understanding of why you use it and why it works and how you can use it in different ways.

These comments demonstrate that algorithms are important, but only when students learn them conceptually. This notion of understanding how algorithms work relates to his position of knowing mathematical properties. By understanding mathematical properties, he believed, students could create, understand, and apply mathematical rules, operations, and algorithms.

These beliefs about understanding overlap with his beliefs about mathematical connections. He stated that during instruction he tried to reinforce the structure of a mathematical system. What you will want to do is have the students understand that math is about . . . there is some group of elements and how they are related and what are the properties that bind these elements together and then what are the operations that you can form on these elements in order to do something with them. . . . We see that this structure is fundamental to all math. You've got elements, you've got the things that make these elements a group and those are properties, and you've got operations that tell you how to deal with these elements and combine them to form some other purposeful construct.

It is particularly Lenny's belief that mathematics is much greater than its individual parts that enabled him to focus on multiple mathematical ideas.

He also had a belief that mathematics should be learned deeply and by making connections. He explained that

> I think we need more depth . . . so that students can think for a longer period of time about a certain group of concepts and so a long term understanding can be dealt with this . . . it is a very difficult challenge for a teacher to look at the textbook and take away from it what they feel is important, but not necessarily be a slave to the textbook. That is very time consuming and it is very easy to just go page by page rather than look at quality activities and the sequence of activities and the content of the activities.

His belief that mathematics should not just cover a vast amount of content in a linear fashion caused him to search elsewhere to find curricular materials concurring with his beliefs.

He also commented that students learn by thinking deeply about mathematical issues and that instruction should therefore engage students in reflective thinking. His belief about reflection is portrayed in this passage. He stated that students become good at mathematics

> by thinking about a problem and processing it several times rather than just trying to get an answer in a one shot deal. If you think about it and reflect on it and pose some solution and then analyze the solution and try to come up with a better solution, you'll be a better problem solver.

Lenny shared different ways in which he tried to help students reflect on the mathematics. One way would be by asking students to share their ideas. He stated students should "share insight into how they solve problems and maybe that helps other kids. So, I always ask them to justify or ask them to tell me how they know that". He also thought that writing encouraged mathematical reflection. He had students write, in their portfolios, every few days asking them to "reflect on the work, [to describe] what they understand, what gave them trouble, and also some calculation work where they have to show evidence of understanding on different levels". These beliefs about the importance of communicating and reflecting, of wanting students to understand big ideas in mathematics, and of focusing deeply on mathematical topics by making connections, classified his beliefs about learning and pedagogy for Depth of Knowledge as intellectual.

Not unlike his beliefs about learning and teaching, were Lenny's beliefs about mathematics. When asked to describe whether he would describe mathematics as being interconnected or linear, he referred to mathematics as a system, a structure where topics are interrelated. He also stated that mathematics is dynamic "developed and person-made" and "not static bodies of information". For Lenny, mathematics was alive and connected and students should share in its wonder, not being "forced into a rigid set of rules".

Value Beyond Instruction. Lenny believed that realistic contexts engage and motivate students, and allow them to understand mathematics more deeply. He believed that it was critical to begin each lesson with a real-world situation. When asked how, in general, he starts a lesson he stated,

> I start by motivating them in terms of a real-world situation and what occurs with examples. [I] ask them what they know about it, trying to bring out some places where this occurs in order to engage them in wanting to know what mathematics helps to describe it.

By providing students with realistic situations, Lenny wanted to encourage the students to become intrigued, to want to know how mathematics is used to explain phenomena.

To him, it made more sense to intertwine the mathematics with the context instead of teaching the mathematics first and the application second.

> If they understand more about physical phenomena that occurs, then they have a better notion of what is going on in the world about them and they can see interactions of how math helps them to understand that.

He believed contexts help students learn mathematics, enabling them to understand the relationship between mathematics and the real-world. Lenny described mathematics as being connected to the natural world in that mathematics is a way to think about and explain how things physically behave. Here was his depiction of the usefulness of mathematics:

> It should help people understand how their environment works, how the natural things that occur in their environment works, how they can control the influences in their environment by some mathematical understanding.

At another point in the interview, he stated this belief more bluntly, "The spirit of math is in making sense of the world". His goal was to create an instructional atmosphere that promoted this view.

A typical lesson for Lenny. Lenny's instructional practices were observed twice, once in February and again in November. During each site visit, his classes were observed four times. Over these eight observations, Lenny's instructional practices averaged 11.4 on the intellectual quality scales, which were characterized as being mainly intellectual. The classroom episode detailed below was typical of what the observers witnessed.

The students were in the middle of a unit on bridge building. Today's goal was for students to understand the concepts – tension, compression, and torque – and their mathematical properties in relation to beams. The class began with a demonstration of tension and torque and then moved into group work. The activity focused on setting up beams of different lengths and applying different forces to the beam or the span. Students worked in pairs on the computer, which has a program loaded, allowing them to test beams. The students were to mathematically evaluate how the beams react to the forces applied.

The students worked in pairs on the computer for about 30 minutes. Many of the students completed the experiment within 15 minutes and then spent the rest of the time manipulating variables to determine whether the change affected any outcomes. Once completed with the experiment students began to put their information in tabular form on flip chart paper. The next day's lesson began with student explanations of the mathematical relationships they found.

Group 1

Teacher	**Student**
What are you working on?	*We are testing the load on different spans.*
How do you use it?	*We select 70 cm for our length. We add two hinges to hold the length. Then we go to members and select Bass wood. Then we find the midpoint.*
Why find the midpoint?	*We use the midpoint to have a consistent breaking place.*
Ok, what did you do so far?	*We are setting up the load.*
What is happening in your picture?	*The structure is breaking in. It will give us span length and the measurements for the breaking load, structure rate, and the load to weight ratio.*
What part are you interested in?	*All of it. We have to add this to our data.*
How many span lengths are you testing?	*Five.*
What is the minimum?	*Four cm.*
. . . maximum?	*20 cm.*
How many do you have so far?	*We have 4 and 8 and this is 12.*
As the span length increases do you see any relationships?	*When the span length increases the breaking load decreases and the structure weight increases.*

Lesson rating. This particular lesson rated an 11.5 based on the instructional scales for intellectual quality. For Construction of Knowledge the lesson rated a four out of five points. There was one major activity (the group work) where students were engaged in higher order thinking. Students combined some initial ideas they had about tension, compression, and torque that they had gained from earlier lessons and experiments with spaghetti in order to make some working hypotheses, test them, and arrive at some initial conclusions. Lenny worked from group to group with a focused attention on determining whether students had general ideas about terminology and concepts and about the mathematical relationships.

Group 2

Teacher	**Student**
What are you doing?	*How the span length determines the breaking load.*
What are you using to test this?	*We are using six joists. We just discovered that the fixed joints can hold more weight and we are testing it at 16 cm right now to see how much weight they can hold. [The students had already completed the experiment with joists that were not fixed or nailed to a platform.]*
What is the piece that goes between the fixed joints called?	*Span.*
Now what are you doing?	*We are using Bass wood right now. Now I'm going to analyze the structure to see how much it can hold.*
What does analyze mean?	*It means putting the tension exactly in the middle of those joints to see how much it can hold.*
Does it bend?	*Yeah it shows how it bends and then breaks.*
Then it breaks?	*And it gives you the breaking load, the structure weight, and the load by weight ratio.*
So what information are you interested in?	*The breaking load and the structure weight to determine how much weight it can hold.*
Did you take any data yet?	*Yeah. We have 8 cm. And this one can hold less than 10 cm. So 16 cm can hold about half of what 8 can hold. It is breaking at about 4.6 lbs and the 8 cm is 9.5 lbs so it is about half.*
How does this compare to the spaghetti experiment that you did earlier?	*The spaghetti didn't hold as much as the bass?*
What about the relationship?	*The relationship is pretty much the same. Two fixed points to hold the spaghetti and applying force to the center to see how much weight it could hold. And the longer the span the easier or the less weight it takes to break it.*

The lesson averaged 3.5 points out of five for depth of knowledge. This construct consisted of two scales. First, the lesson scored a four on depth because it was structured and focused on the central idea of building understanding of the mathematical relationships of torque in relation to span length and other variables (in today's lesson) and in relation to tension and compression (in yesterday's lesson). Second, the lesson scored three points on substantive conversations, the second component of depth of knowledge. Although students discussed their ideas with each other in their groups, the conversation was mixed, in that, at times students focused the conversation on details of the computer program, and at other times, discussed their hypotheses and findings. To be rated a four or five this lesson would have to have focused more on building shared understanding. This, however, was not the focus of the day's lesson. In fact, the next day, where students reported on their findings and the teacher pressed students to explain and justify their claims against other students' claims, rated a five on this scale.

For Value Beyond Instruction, this lesson scored four out of five points. Students worked on a problem they saw as being a problem that structural engineers actually confront. The lesson would have rated a five if it had an additional feature where students were to present their findings to a group of engineers or parents or to use this knowledge in a way besides a grade.

There were similarities and differences for each of Lenny's lesson. He consistently asked students to take an active role in their learning by having them think about and discuss important mathematical ideas and how these ideas related. He did this, however, very differently from day to day. Depending on the topic and goals of the lesson, he would have them working individually or in groups or sharing their ideas with him, another student, or the whole class.

The Relationship between Beliefs and Instruction

To portray the relationship between beliefs and instruction, we used a Pearson correlation. The eight teachers' belief ratings for intellectual quality for both interviews were correlated with their average ratings for instruction for both site visits. The teachers' beliefs were significantly correlated with teachers' instruction ($r = .89$, $p < .01$, two tailed).

A consistent pattern emerges by comparing the relative degree of intellectual quality of teachers' beliefs to the degree of intellectual quality in their instruction. Using the interview data, Lenny's beliefs were found to be intellectually guided as was his instruction. Similarly, Brittany's beliefs rated the lowest on the scales of intellectual quality as was her instruction. A similar case could be made for Henry, whose beliefs and instructional practices contained both traditional and intellectual.

Beliefs Most Related to Instruction

Besides examining the aggregate of all teachers' beliefs to their practices, it is also important to examine how certain beliefs are related to the intellectual quality of instruction. Table 3

shows the correlations of teachers' beliefs for Construction of Knowledge, Depth of Knowledge, Value Beyond Instruction, learning, pedagogy, and mathematics, and the intellectual quality of their instructional practices for the two observation periods. Note that all eight study teachers' instructional practices were observed at the beginning of the study while six were observed at the end of the study due to financial constraints.

Table 3

Correlations of Instruction and the Major Belief Categories

	Correlations	
Beliefs	Instruction (Year 1)	Instruction (Year 2)
Construction of Knowledge	.87**	.97**
Depth of Knowledge	.75	.81
Value Beyond Instruction	.00	.65
Learning	.55	.91*
Pedagogy	.78*	.87*
Mathematics	.51	.78

*p<0.05, two-tailed; ** p < 0.01, two-tailed

Strikingly for this small number of teachers, there are some strong correlations and patterns. At the beginning of the study, two types of the beliefs were significantly related to instruction: Construction of Knowledge (r = .87), and Pedagogy (r = .78). Depth of Knowledge (r = .75), Learning (r = .55) and Mathematics (r = .51) held weaker, non-significant relationships and Value Beyond Instruction held no relationship at all (r = .00). A similar, but stronger, pattern was noted at the end of the study. Beliefs about Construction of Knowledge were significantly associated with instruction (r = .97), as were beliefs about learning (r = .91) and pedagogy (r = .87). Beliefs about Depth of Knowledge (r = .81), mathematics (r = .78) and Value Beyond Instruction (r = .65) were still highly related.

By examining these data, it is evident that a relationship exists between teachers' beliefs about learning, pedagogy, mathematics, Construction of Knowledge, Depth of Knowledge, and Value Beyond Instruction and their instructional practices. By examining the interview data, it appears that teachers' beliefs about Construction of Knowledge might be the best predictor of intellectual instruction. It is also evident that beliefs about learning and pedagogy are also highly related to intellectual levels of instruction.

Discussion

The results of this study reveal a consistent relationship between teachers' beliefs and their instructional practices regarding the components of intellectual quality (Newmann et al., 1995) even though the teachers under study had differing beliefs and practices among themselves. Using interview and observation data results, beliefs were found to be significantly related to instruction. The more a teacher's beliefs were rated intellectual, the more his instruction was rated intellectual.

The research question guiding this study focused on the relationship between teachers' beliefs and their instructional practices related to ideas of intellectual quality or intellectual quality. The findings demonstrated that the more intellectually-guided teachers' beliefs were, the greater the intellectual pedagogy found. Similar to Onosko's (1990) findings, teachers in this study whose beliefs were more elaborated about ideas of construction of knowledge or higher ordering thinking were found to support students' construction of knowledge through activities that promoted mathematical analysis. These findings are consistent with conclusions found in the research literature (Fennema et al., 1996; Onosko, 1990, 1991; Stipek, Givvin, Salmon, & MacGyvers, 2001; Turner, Warzon, & Christensen, 2011).

Teachers' beliefs and instructional practices were closely linked, each being within one classification of the other. For instance, Brittany's beliefs and instruction were rated traditional or mainly traditional on all measures at both periods in time. Similarly, Lenny's beliefs and instruction consistently leaned toward intellectual quality. There was one divergent case—Boe. His beliefs were classified as mainly intellectual while his instructional practices rated mainly traditional. This apparent difference might be explained by his openness to ideas such as intellectual quality, but his lack of ability to implement them. While visiting his classroom, we noticed he spent so much of his time trying to get students to talk, work cooperatively, and engage in the activity that much of the mathematics became trivial.

In all, three findings emerge from the data. First, teachers' beliefs are related to their instructional practices. Second, beliefs about Construction of Knowledge tend to be the best indicator of intellectual practices, followed by beliefs about learning, and pedagogy. And third, teachers' beliefs tend to be more intellectually guided than their instructional practices.

Conclusions

Research, recent policy decisions in the U.S., and increases in accountability around mathematics reform have focused on learning mathematics with understanding (Burkhardt et al., 2012; NGA, 2010; NMAP, 2008). The move from traditional to more reform-oriented instructional methods is extremely difficult (Cady, Meier, & Lubinski, 2006; Kennedy, 2004). By focusing on teachers' beliefs about learning, teaching, and mathematics as related to construction of knowledge and depth of knowledge, teacher educators know more about teachers' thinking. This knowledge will allow them to focus their attention on critical aspects that are related to implementing changes in pedagogical practices. Teachers must become aware of their views of learning, teaching, and mathematics and how these beliefs affect their instructional practices and ultimately student outcomes. This study was motivated by wondering why intellectual pedagogy (or teaching for understanding) is so rarely found among high-school mathematics classrooms, especially when research has shown it to be related to improved student achievement (Newmann & Associates, 1996). In this study, only one teacher's instructional practices rated intellectual on the instruction scales while the other seven remained near or below the midpoint of the scales. With a motivated group of teachers, this result speaks toward the difficulty of teaching intellectually.

This research leads to a number of additional research questions to be pursued within mathematics education. First, as teachers are forced to grapple with their own beliefs and the ramifications of these beliefs on student learning, will their pedagogical practices become more intellectual? The implication here is that teacher-preparation and in-service programs need to carefully examine the balance between providing "math methods" and providing opportunities for teachers to reflect on their belief structures.

Second, how do teachers content knowledge influence beliefs and practice? High school mathematics teachers may have high levels of specific content knowledge in mathematics topics and procedures. However, they may not have a broader content knowledge understanding of applying these topics to real-life situations, such as the understanding of physics topics displayed by Lenny. The new standards and underlying tenets of intellectual quality may be asking teachers to have subject matter knowledge well beyond the content addressed in typical mathematics education preparation programs. A lack of knowledge around intellectual mathematics applications may be, in conjunction with beliefs, an additional barrier to reforming mathematics education.

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