

MUSIC-MATH

Integrated Activities

For Elementary and Middle School Students



Song A. An

Mary M. Capraro

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Mary M. Capraro**

Texas A&M University



Education For All

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Song A. An Dedicate This Book To His

Ph.D. advisor, Professor at Texas A&M University

Gerald Kulm

Abecedarian of music, Professor at Nanjing Arts Institute

Guoliang Wen

Former colleagues and former students

(Class of 2010)

in Nanjing Jiangdongmen Elementary School

Grandparents

Preface

Are there really a large number of relationships between mathematics and music? Until I started my former academic study as a musicology major in the Nanjing Arts Institute as a college student, my answer was always “No”. Although I started to learn music when I was very young, I did not really understand music until I was a senior student. When I actually figured out what music was—I could compose music and design musical instruments—I realized that music and mathematics are inextricably tied to each other.

I was a suona (a Chinese traditional instrument) player when I was an elementary school student in the Children’s Traditional Orchestra of my province—Jiangsu for three years, and then, because I was extremely motivated by Kenney G, I started to play saxophone. Perhaps Kenney G is not very famous in the United States, but he is a super star in China—his saxophone works are played everywhere from schools to restaurants to department stores. In middle school and high school, I was a saxophone player in the school’s band. As I look back now, my orchestra and band experiences were very helpful in my life because they increased my confidence and attitude toward life; however, I misunderstood “music” for a tremendously long time. Because both the instruments—suona and soprano saxophone—that I played were high-pitch, solo-orientated instruments, I paid limited attention to the other orchestra members. I had a long time misunderstanding of the function of the low pitch, harmonic oriented instruments. It took me more than a decade to understand that music was not just the “main melody”. Until I learned to improvise while playing piano and composing music later, I found music was not a unidimensional art but was a multidimensional art that combined pitch, duration, dynamic, tempo, timbre, and harmonic.

I learned to play piano later. I started to practice piano from my freshman year because piano playing was a compulsory course for all the students at the conservatory. I respected pianists and felt an awesome respect toward my classmates who could play and sing the latest popular songs. After years of practicing and exploring, I finally discovered the “secret” among the hundreds of pop songs. The secret of music was that there exists a hidden line with mathematical patterns behind all musical works—if one can manipulate the hidden line of the music directly, then music playing will be a fun process. In this book, I am going to share my mathematical secrets of music with you through interesting activities as well as share teaching experiences and research results on how to teach mathematical lessons with music. The two most important things that I want to share what I have learned in my academic musical experiences are (1) the pattern of structure that different kinds of musical instruments make, and (2) the core principles of music composition.

At an early age, I decided my goal in life was to be a teacher. Throughout my high school years, with an ambitious goal of being a student in one of the best normal universities in China, I never imaged that in the future I would have a career related to music. However, all of my family members did not like my career choice and all of them hoped that I would become an engineer or a businessman in the future. In order to balance the “conflicts” within all of my family members, in my last semester of high school, I finally compromised with my parents and uncles—I proposed a middle-path-solution—and I made my decision to go to a conservatory of music. I chose musicology as my major because when I read the major description described in the university program book, it was one in which students with a major in “musicology” would have opportunities to take various music theory courses including courses on musical instrument designing and manufacturing and courses on musical instrument marketing. Additionally, students could have different specific emphases and thus may have different career choices: music teacher in a K-12 school, engineer for a musical instrument company, or businessman in the music industry.

After painful struggles over my career goal decisions, and as one of the best students in my college, I had offers to work for several world famous musical instrument industry companies. I decided to continue my graduate studies in education. Throughout semesters of playing, designing, and investigating musical instruments and being lonely in workshops, I increasingly realized that I love people more than things. My decision of pursuing a Ph.D degree in mathematics education has developed gradually from various interactions with and inspired from lots of educators and educational researchers.

In the summer of 2004, I was lucky to be recommended by my aunt as an interpreter for Dr. Gerald Kulm (my future Ph.D advisor) when he attended several conferences in China. With Dr. Kulm, I attended several presentations and workshops, and I learned that a revolutionary reform was taking place in mathematics education. The traditional system of teaching and learning was being replaced by a whole new one. I admired Dr. Kulm and his colleagues who were working at the forefront of this great revolution, and I eagerly hoped that I could become one of them working within and for this exciting reform. Soon I decided to walk into this exciting area. Thus, when I finished my undergraduate studies I hoped to provide my unique contributions to the field of education with my distinctive music background.

My initial decision to become a mathematics educator and the original idea of writing this activity book was also inspired by Dr. Marina Krause as well as her book, *Multicultural Mathematics Materials*. Krause, professor at California State University-Long Beach. During my sophomore year, my father was invited by the Beijing Normal University Press to work on the Chinese translation of Krause's *Multicultural Mathematics Materials (2nd edition)*. I worked as a translating assistant to proofread and amend the translation from the title page to colophon of the book. The whole process of the translation provided me a great opportunity to look into how interesting games can be suitably integrated into mathematics lessons. Deeply influenced by the work on Krause's book, I started to think about how I could also write a mathematics activity book that integrated interesting music content.

This book was written based on my own music learning and performing experience as well as my experience as an elementary mathematics teacher. The initial inspiration for this book was in 2007 during my first year of PhD study on mathematics education. With increasing knowledge in education practice and theory, more and more mathematics activities that connect with music were developed, and were used in classroom research. Some research was published in peer-reviewed journals and presented at national level conferences. Based on the results of a series of research projects as well as the feedback from students, teachers and educational researchers, some activities were revised and more activities were inspired. This book is my earnest work for students and teachers. I hope students can benefit from these music-math activities while enjoying, practicing and understanding mathematics.

Song A. An
Texas A&M University
2010 Summer

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Song A. An
Texas A&M University
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Introduction

Should Arts Be Integrated Throughout the Curriculum?

Using interdisciplinary curriculum in school education is not a new concept. Since the end of the nineteenth century, Progressivism educators such as Francis Parker and John Dewey had already tried to construct a connected comprehensive approach in school education. Dewey conducted a series of exploratory studies on curriculum integration and reported his findings concerning students' developments from these curricula in various books. By the 1930s, Dewey and other educators proposed three progressive approaches including the project method, the experience curriculum, and activity movement. From Dewey's perspective, schools should pay more attention to the aesthetic experience, and arts educators are encouraged to explore interrelations with other subject areas (Bresler, 2003). In his view, the separation of knowledge into isolated compartments as independent curriculum formed a learning environment that was unfamiliar to students' everyday experiences. This proved harmful for education (Beane, 1997; Dewey, 1938). The popularity of this curriculum integration was impeded because of the launching of Sputnik in 1957. During the post-Sputnik era until the end of the Cold War, integrated curriculum approaches were mainly forgotten by educators. During this time the educational field was plunged into a rigid, discipline-based approach paying special attention to mathematics and science education. By the late 1980s, with the publication of *Caught in the Middle* (California State Department of Education, 1987) and *Turning Points* (Carnegie Council on Adolescent Development, 1989), a new call for curriculum reform was raised. Educators proposed that schools should be relevant for students, and the integrated curriculum—using arts to connect all subjects - was once again a hot topic among teachers and researchers. In the past two decades, because of Gardner's (1983) multiple intelligences theory, more and more research-based evidence for maintaining the multi-faces of human intelligence further prompted the popularity of arts integration among school curricula.

Against Arts Integrated Curriculum

Researchers with concerns about the arts-integrated curriculum have expressed their views about integrated curriculum implementation. One of the key arguments they have made is that there was not enough evidence based on empirical research findings to support an arts-integrated curriculum. There were no statistically significant improvements in student academic achievement when compared with students who used traditional curriculum (Hetland & Winner, 2001). Other concerns about the advantage and effectiveness of arts-integrated curriculum were in teachers' complaints that the integrated curriculum required more work on the part of the teachers who were already overloaded. There was also a lack of support for teachers to teach based on integrated curriculum from researchers and teacher educators (Horowitz, 2004). Artists also expressed their warnings for integrating the arts. Their worries were that integrating the arts across the curriculum may undermine and cause misunderstandings of essential disciplinary arts knowledge, and this kind of curriculum may cause arts specialists to leave schools (Veblen & Elliott, 2000).

The finding of investigations conducted by Ellis and Fouts (2001) based on examination of the benefits and drawbacks of music integration suggested that interdisciplinary education with no scientific, research-based standards should really be called nondisciplinary. In further argumentation, they claimed that superior teachers will always find and use connections across different school subjects during the teaching process while preserving the separate subjects approach and that an integrated curriculum will remove the benefits that exist in traditional independent discipline delivery. Beside the argument of interdisciplinary curriculum meaning nondisciplinary curriculum, more weaknesses were identified by various researchers including the concerns of support, research, time, money, patience, fear, training, and integrity (Mallery, 2000; Wood, 2001).

One of scholars who expressed concerns about the current arts-integrated curriculum, Eisner (2002) claimed that integrated curriculum, in reality, was not as effective as many researchers argued. He found

that although numerous research studies claimed the arts can improve students' academic achievement or that the arts courses "strengthen" academic performance, the research did not provide enough data to support the argument. Hetland and Winner (2000) also proposed that in many areas between academic achievement and the arts, researchers failed to find reliable causal links. In Hetland and Winner's view, some academic improvement issues of the arts-learning-students could be explained by non-causal mechanisms such as high achieving students (no matter their ethnic or racial group, or social class) may choose or be guided to study the arts. They suggested that the differences between achievement scores were probably caused by the students who take arts courses also being high-achieving, high test-scoring students. They provided more evidence for their argument in 4 published reports showing no relationship between studying arts and verbal creativity test measures and 15 published reports showing there existed a small relationship between music/visual arts and reading/math. However, due to the limitation of these studies, these relationships could not be generalized to new studies.

For Arts Integrated Curriculum

Supporters of the integration of arts into curriculum generally maintain that the arts provide students and teachers with learning experiences that can intellectually and emotionally motivate understanding (Chrysostomou, 2004; Deasey, 2002; Mansilla, 2005). Recent research has reported beneficial results not only for students with special characteristics, but for all students learning mathematics along with other subjects when integrating topics with the arts: (a) significant enhancement in students' attitude and beliefs towards learning mathematics (An, Kulm, & Ma, 2008); (b) effective motivation in students' engagement in mathematics (Shilling, 2002); (c) remarkable improvement in understanding mathematics (Autin, 2007; Catterall 2005; Peterson, 2005); (d) development of cognitive abilities (Peterson, 2005); (e) improvement in critical thinking and problem solving skills (Wolf, 1999); (e) development of ability to work collaboratively in groups (MacDonald, 1992; Wolf); (f) enhancement in students' self confidence (MacDonald); (g) improvement of empathy and tolerance in class (Hanna, 2000); (h) development of the imagination (Greene, 2001); (i) improvement of motivation for learning (Fiske, 1999); (j) increase in students' creativity (Marshall, 2005), and (k) decrease in dropout rates (Catterall).

Researchers who favored arts integration also argued that this integrated curriculum can assist students in developing holistic thinking skills; therefore, students' knowledge may be developed through interdisciplinary connections (Mason, 1996). With the development of cognitive science showing that evidence of learning is a situated, socially-constructed, and a culturally intervening procedure, educational researchers further argued that integrated curriculum can facilitate students in increasing their creativity because arts integration provides opportunities for learners to fulfill these requirements (Marshall, 2005). Another key reason supporters argue is that arts-integrated curriculum provides students with a learning environment that enables them to develop better social relationships with their everyday experiences because the interaction of breaking down disciplinary boundaries engages students through a reflective inquiry process (Parsons, 2004). Ellis and Fouts (2001) advocated that interdisciplinary education not only improved students' higher-order thinking skills and motivation for learning but also provided opportunities to understand knowledge from multiple perspectives as well as transfer learning (Erickson, 1998; Scripp, 2002). Hargreaves and Moore (2000) also noted that the integrated curriculum provides teachers chances to address important issues which may have difficult to investigated in individual subjects, and also allows students to develop a wider view of curriculum reducing redundancy of content.

Our Perspectives

The literature on both sides of the issue of arts integration discussed indicates that interdisciplinary education—integrating the arts through other core school curriculum—such as mathematics and language arts may accomplish the goal of educating well-rounded students. There is far more literature supporting the claim for using arts-integrated curriculum which beneficial for developing students’ cognitive growth than opposing literature. The studies conducted by various researchers from multiple perspectives provided evidence showing that an integrated curriculum offers a positive impact on student achievement and meeting student needs (Wineberg & Grossman, 2000; Wood, 2001). However, the interdisciplinary qualities of arts-integrated education are not fully understood or capitalized on by teachers and researchers.

In spite of the many advantages of arts integration in the studies and programs, previous studies contain a number of concerns that we cannot ignore. One of the key concerns is that teaching arts with other subject areas may weaken the understanding of the concepts in each subject area. The pedagogical perspective of this argument is that cognitive operation demands involve different subjects that are essentially different and detached. One perspective of this argument is that the combination of all the art forms together may undermine the place of each particular art in the school curriculum and may cause students to form misconceptions about the arts. The other concern is from artists who fear that to integrate the arts across the curriculum may damage disciplinary understandings and experiences of an art, and could cause the disappearance of art specialists in schools.

However, even with all these limitations as well as concerns about the arts-integrated curriculum in school, interdisciplinary curriculum provides an opportunity to help students find important connections between the arts and other school subjects. I do not suggest that the integration of arts into other school subjects such as language arts and mathematics is a prototype for all classroom activities related to students’ learning processes. I argue that the development of a higher-level intellectual understanding, beliefs, and attitudes of mathematics should not emanate from a single curriculum but should permeate the curricula with content other than mathematics, such as music and visual arts. I am not suggesting that teachers teach all subjects areas with connections to music or the visual arts. However, teachers should understand that all subjects are interconnected and can be taught through the integration of content. Thus, teachers’ pedagogical content knowledge of how to teach students mathematics with sense making, especially linked with arts might provide an alternate way to design and teach an effective lesson. I suggest that teacher education programs familiarize teachers with various connections— those within and out of school curriculum.

The goal of education is not to train test-takers but to teach students how to acquire knowledge and seek truth in real life. School curriculum should have enough flexibility to facilitate all students with various characteristics to learn through different approaches. Schools should not be a place to mold all students into the same shape but should be a place to identify students’ unique characteristics and strengths and then develop these characteristics in order to help students have enough ability to acquire and analyze knowledge in the future. To teach students “wisdom” or “skills” to pass a test in a certain subject area is a growing concern not only for educators but also for the all of society. Teaching should not be a labor that is repeated to mold all students, no matter the differences among various students, into the same shape. The teacher should be a “root carving artist”— identifying each student’s own unique characteristics first, and then developing these unique characteristics in order to build their own intellectual foundation.

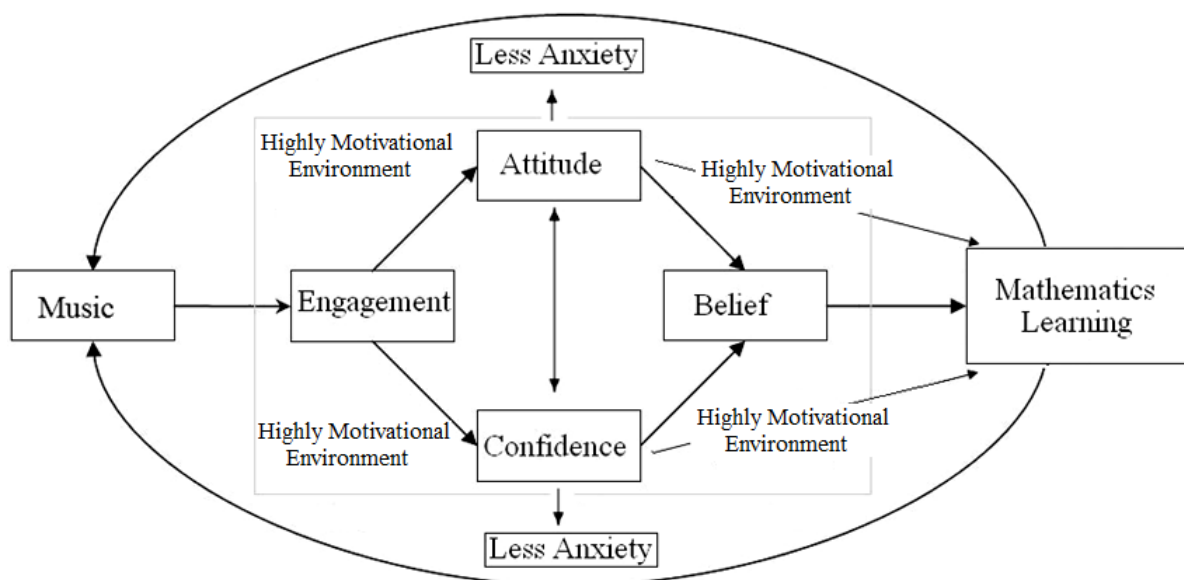
We believe that we need to find a balance between single subject curriculum and interdisciplinary curriculum. We should admit the importance of each individual subject as an important goal of education. If a curriculum cannot provide learners’ enough rigor, and knowledge of each individual subject that they can use immediately after they finished their study for their career path, the curriculum cannot be labeled as an effective curriculum. Also, from an intellectual and theoretical knowledge perspective, for example, arts, with no connection to a single subject curriculum cannot help learners become wise thinkers. In order

to solve problems in the school curriculum and in the real world, we need to develop and adopt more flexible interdisciplinary curriculum that can provide enough opportunities for students to have intellectual communication and enhance their basic intellectual and theoretical knowledge of each individual subject.

Basic Ideas and Suggestions for Teachers Using This Book

Frankly, the mathematics learning process is not always pleasurable. Because of the abstract property of mathematics, many concepts are very difficult for students especially in making connections with things in their life experiences that they are familiar with. As a result of failure in making connections with existing knowledge, students’ understanding as well as memorization will be limited. For example, the concepts and the computations of fractions, proportions and algebra are difficult to connect to life experience and previous knowledge. According to learning theorists’ perspectives, when students feel troubled about making sense of new knowledge and making connection with existing knowledge, they may not complete the learning task. When this happens, students’ engagement will be reduced, also their attitude and confidence may be negatively influenced, which may impede students’ learning of mathematics.

Emotion is essential in students’ learning, because positive emotions may lead to a higher level of motivation to facilitate students focusing attention on learning (Sylwester, 1995). In applying motivation theories, Miller and Mitchell (1994) suggested teachers should create a highly motivational environment for learning, free from tension and other possible causes of embarrassment or humiliation. Music, with its aesthetical features, has the potential of creating this type of environment for students, in which they can discover and think about mathematical concepts in various ways and build fundamental understandings and appreciation for both mathematics and the arts (Lawrence & Yamagata, 2007). What is more, music can provide students a highly motivational environment with less prejudice and violence, helping them becomes better risk takers and communicators (Trusty & Oliva, 1994). As previous research has shown with implications for motivation theory, teaching mathematics integrated with music can effectively engage students in learning mathematics. By creating a highly motivational environment for students learning of mathematics (see figure below), music will be used as a sugarcoating for learning particular concepts in mathematics. Students’ attitude and confidence toward mathematics can be increased while their anxiety toward mathematics can be reduced (An, Kulm, & Ma, 2008; An, Ma, & Capraro, 2011).



Framework for Creating a Highly Motivational Environment through Music

Far more than just sugarcoating mathematics, the goal of the music-math activities listed in this book is to provide a comprehensive connection between mathematics and world-wide cultures. The culture-related content also provides students with opportunities to understand the world from a new perspective. The music related content within the activities in this book, are used as the medium to connect mathematics to different cultures around the world. Additionally these activities can be used as a catalyst for students' understand of mathematics through an alternative approach. For example, in the first part of the book, students will not only learn the mathematics behind the construction of each instrument, but also will learn the origin of each instrument throughout the world. In the second part of the book, students will have chance to learn some nontraditional musical knowledge such as the Chinese pentatonic scale and the Arabic double harmonic scale, as well as instrument arrangement for a Chinese orchestra. These activities provide students an opportunity to glimpse into how people in different countries throughout history have applied mathematics to the development of musical instruments. Students will also identify and use mathematical patterns from music to create different types of music.

In each activity, we provide a *Background* section which details concise historical or cultural information about the origin, development or the characteristics of the musical instruments as well as music composition theory. Teachers should share this background information with students as an introduction to each activity, in order to provide a more concrete and tangible idea of how each musical instrument works. We have also provided two great masters' musical pieces and one contemporary musician for each instrument in the *Recommend Music and Musicians* section. Teachers can put the musical works and the musicians we have recommended on a video-sharing website. Additionally, teachers can simply place the instrument names on a video-sharing website and find suitable pieces for students to view. The *Mathematical Standard* and *Mathematical Concepts* section details the mathematics content covered in the activity matching the corresponding (National Council of Teachers of Mathematics) [NCTM, 2000] standards. The *Activity & Questions* section allows students to engage in each activity as well as discuss questions allowing students to share their ideas in groups. Depending on grade levels, teachers can follow or redesign their own mathematics activities based on designing different musical instruments.

When using the music composition activities, we encourage mathematics teachers to partner with a music teacher in their school. If one is not available, teachers can try to find a community partner who is musically talented to share their musical abilities. We encourage capable students to play their own music. Students can translate their music pieces from graphic notation to a music staff, and then play the melody by using their own musical instruments. Especially, for lower elementary students, we strongly recommend that teachers use handbells for students to play their own music. A set of handbells usually contains eight bells each of a different color and tone, with the colors and tones corresponding to the graphic notation rules which we have designed throughout the book. For the last unit of advanced music composition, "My New Age Music" and "My Pop Song", students can work in four-member groups to play their own works by using handbells or using other instruments (e.g. recorder, violin, piano)—one student can play the melody while three other students play the preset chords as an accompaniment. We suggest that teachers use music-mathematics activities as an integrated part of their regular lessons based on their own weekly pacing. We also encourage mathematics teachers to plan cooperatively and collegially with music teachers in teaching the activities when there are too many music requirements. We suggest that teachers "wisely adapt" all of the activities to meet individual students' grade level and prior knowledge. In order to change the difficulty levels, for example, teachers can change the more advanced decimals or fractions into simple whole numbers for students to conduct computations. Moreover, teachers can provide hints or mathematical formulas to students when necessary.