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A Study on High School Mathematics Teachers' Mathematics Professional Literacy: What Does High Quality Mathematics Teaching Need?

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A new-curriculum reform in mathematics makes it a focus for improving high school mathematics teachers' (HSMT) mathematics professional literacy (MPL). This study was conducted to determine HSMT's views about the level of importance and the level of mastery of MPL needed for high quality mathematics teaching (HQMT). The data was obtained by conducting a questionnaire that was constructed through a literature review and the expert interview of 4 mathematics teacher educators and 4 expert high school mathematics teachers respectively. A MPL framework was derived; teachers' views on the level of importance and the level of mastery were also found. Six suggestions are advanced for teachers and teacher educators.

Key words: high school, mathematics teachers, high quality mathematics teaching (HQMT), mathematics professional literacy (MPL), teacher education

Introduction

As we know, what is a good teacher or good teaching has a clear view and carries it out to the training should be a critical feature of all good training programs or plans. An important amount of recent research in the field of mathematics education revolves around what high quality mathematics teaching (HQMT means good teaching in mathematics education research studies.) needs (Fennema & Franke, 1992; Ball, Bass, Sleep & Thames, 2005; Grossman, Schoenfeld & Lee, 2005; Krainer, 2005; Moreira & David, 2008; Smith & Strahan, 2004; Wilson, Cooney & Stinson, 2005), and what mathematics knowledge for teaching is (Ball, 2003; Cuoco, 2001). Meanwhile, these demands are also concluded in mathematics teacher standards or teaching standards in some countries (DfEE, 2000; INTASC, 1992; KOM task group, 2002NBPTS, 2001; NCTM, 1991;).

This study was carried out in China under particular circumstances. A curriculum reform in basic education, named as new-curriculum reform, has been implemented since the end of the 20th century. It was set forth as the biggest and unprecedented reform since the People's Republic of China established in 1949. In this particular context, *the Standard for Senior High School Mathematics Curriculum (experimental)* (Standard for short in this paper) (MOE of China, 2003) has been put in force progressively throughout the country since 2004. Research studies from experimental areas have brought to light the difficulty of implementing new-curriculum and the immediate necessity of improving the level of teachers' mathematics professional literacy (MPL). MPL is supposed to be a more extensive concept than mathematics knowledge for describing mathematics teachers' mathematics knowledge, mathematical competencies and mathematical affection in this study. It has been identified as one of the key difficult factors that influence new-curriculum implementation (Huang & Li, 2008; Wei, 2001; Wang, 2005; Zhang & Wang, 2005;). How to improve teachers' MPL has been a focus problem in the field of mathematics teacher education and professional development in China. In contrast with other countries, as is the case in America, there is still little research of this subject either in theoretical or in empirical aspect in China. Consequently, the systematic research in the context of high school mathematics is greatly needed. Taking this into account, this study is guided by the question: what MPL is needed for teachers implementing HQMT. We divided the question into four aspects: (1) to identify the specific needs of high school mathematics teachers for HQMT (developing an initial framework to describe the demands of mathematics teachers' MPL); (2) to measure the recognition of the perceived level of importance and the perceived level of mastery of the MPL of each teacher and the relationship between perceived importance and perceived mastery taking by the practicing teachers in Henan province as the examples; (3) to measure the influence of the demographic variables on importance and mastery; (4) to construct a framework describing the demands of mathematics teachers' MPL. It is my hope that this paper will be helpful for understanding the needs of mathematics teachers' MPL and will provide a potential theoretical framework for designing, measuring and evaluating mathematics teachers' MPL. What's more, we hope it provides a common discourse system for teachers and teacher educators to improve their instructional practice.

To solve the research problems, this paper combines qualitative and quantitative approaches together; correspondingly, a questionnaire and expert interview were designed for data collection. The design of this paper is also enlightened by Gustafuson (1984) and Klein et al. (2004a). Initial conceptual framework for quantitative data collection is constructed based on related literatures (more details in Fang's 2009) and expert interview. In this paper, MPL is defined as a synthesis and unified entity that consists of three dimensions in mathematical knowledge (MK), mathematical competencies (MC) and mathematical affects (MA) needed for teachers implementing HQMT. Further, MK involves mathematical contents and ideas implied therein, the mathematical views and mathematical structure; MC involves basic mathematical competencies, posing, analyzing and solving mathematical problem competences, and dealing with and using mathematical language competences; and MA involves the learning inclination to mathematics and the professional-self of mathematics, a kind of mathematics affects deduced from self and professional-self, which is used here to describe mathematics teachers' mental inclination to the feeling, acceptation and affirmation of themselves in mathematics, mathematics learning and mathematics teaching.

Methodology

Participants

Four expert mathematics teacher educators, who are familiar with mathematics education in China, were sampled from four normal universities (entitled "university experts" in this paper) in China and four expert high school mathematics teachers come from four high schools (named as "school experts" in this paper), respectively. The school experts come from the Expert Warehouse of Teaching Material Selection for New-curriculum implementing in Henan province in China. According to their academic background, they all should be familiar with traditional mathematics teaching and the new-curriculum reform for high school.

Six hundred thirty-seven (valid samples, from a total of six hundred seventy-nine) high school mathematics teachers come from Henan province in China participated in this study. They were selected by the whole cluster sampling method from 30 high schools in 15 cities in Henan. Their years of teaching experience varied from one to thirty-six years. The average teachers in this study have taught high school mathematics for approximately 11 years. Almost all the teachers (98.3%) which we surveyed have at least one undergraduate diploma; 7 teachers out of the 637 participants had a master's degree or took post-graduate courses, while no teachers had a doctorate; Two hundred fourteen teachers who participated in this study were female and four hundred twenty-three were male.

Data Collection

In this section, the primary sources of data were two semi-structured interviews (see Fang's 2009, pp.190-191). Each interview was guided by an outline developed by the researcher and was audio-taped and later transcribed. The data was collected in two conferences, each with a different focus. The interviews with four university experts mainly focused on the connotation of MPL and what MPL teachers needed to know for HQMT; interviewing with four school experts mainly focused on what MPL was important for high quality mathematics teaching. The intended purpose of this set was to provide qualitative evidence for understanding the basic concepts, such as, MPL, MK, MC and MA in this study, and guiding the construction of initial framework of MPL.

Instruments

A survey questionnaire was designed to collect data. The first section of the questionnaire collected demographic data concerning educational background and teaching experience. The second part (initial framework) comprised 54 items that described teachers' MPL for good mathematics teaching and asked teachers to determine their perceived level of importance and perceived level of mastery (see Fang (2009, p.192-194)). The level of importance and the level of mastery were recorded from 0-4 and A-E, separately. The third part consisted of open-ended questions with the following instructions: "Please add below additional professional literacy of mathematics that you believe particularly important to mathematics teachers for good teaching." The purpose of the extensive study was threefold: (1) to identify specific professional literacy of mathematics that were particularly important to good mathematics teaching in high school as derived from the literature and perceived by practicing teachers; (2) to determine the relationship between the perceived level of importance and the perceived level of mastery for each teacher literacy; and (3) to determine the influence of the demographic variables of teaching experience and educational background on the perceived level of importance and perceived level of mastery.

There are three main stages to complete the study: (1) according to the conceptual framework, interviews of 4 expert mathematics teacher educators and 4 expert high school mathematics teachers, and a pilot study comprised of, a teachers' questionnaire consisting 54 items, developed to collect data; (2) distributing the instrument to a stratified random sample of 679 high school mathematics teachers in Henan, analyzing the quantitative data by using SPSS15.0 and coding the contents which needed to be revised and added according to teachers' views; (3) identifying the contents and establishing the final framework for describing mathematics teachers' MPL.

Data Analysis

The expert interview data analysis was grounded in an analytical-inductive method in which teacher responses were coded using external and internal codes and then classified according to relevant themes (Knuth, 2002). Coding of the data began using a set of researcher-generated (external) codes that were identified prior to the data collection and that corresponded to and were derived from the conceptual framework. As the data were being examined, emerging themes required the proposal of several new codes. After proposing these data-grounded (internal) codes, the data for each individual expert was then re-examined and re-coded incorporating these new codes.

According to Spradley (1979), domains are categories of meanings that are comprised of smaller categories, the smaller categories being linked to the corresponding domain by a single semantic relationship. Domains selected for this stage of the analysis were informed by the research questions, that is, the issues that were deemed important for this study provided a backdrop against which specific domains were proposed as the data sets were examined. As an example, I used domain analysis techniques to identify the literacy of what the experts seemed to believe constituted MPL mathematics teachers.

The Statistical Package for the Social Sciences 15.0 (SPSS 15.0) was used to analyze the quantitative data (questionnaire survey). The level of importance and the level of mastery as perceived by high school mathematics teachers in Henan were determined for each of the teachers' professional literacy of mathematics. The SPSS 15.0 descriptive statistics procedure called "frequencies" was used. The descriptive statistics variables of mean, median, standard deviation, variance, and range were calculated for each item in the questionnaire.

Results and Discussion

This section reports and discusses the results and related questions mentioned in the introduction of the study. A computer was utilized to rank the 54 teachers' MPL, which were the needed for HQMT, from most important to least important and from most highly mastered to least mastered by using the means from the output of the SPSS 15.0 frequencies procedure. Their rankings are shown in Appendix A.

Perceived Level of Importance and Mastery

Examining all the 54 items in the teacher questionnaire (namely initial framework; see details in Appendix A), you could find that the highest means score among perceived level of importance was 4.392, the lowest was 3.421; meanwhile the perceived level of mastery on each item was 4.106 and 3.046, respectively. According to this, we perceived that teachers who participated in the research approved the importance of all items at a high level. Some additional features can also be discovered from the statistics.

Features of Top and Lowest Ten Items

Features of the top ten items and the lowest ten items ranked according to means of the level of importance were found. Among the items classified as mathematics knowledge (MK), two ranked among the top ten (see item 2 and 3 in Table 1) and their contents were included in Mathematics Syllabus, a traditional curriculum file compared with new curriculum standards; seven ranked among the lowest ten items (see item 7, 16, 15, 8, 17 9 and 18 in Table 2) and their contents were related to knowledge of mathematics views; three were about the knowledge of "mathematics and technology"; four were about knowledge of "the nature of mathematics and the structure of mathematics".

Among the items belonging to mathematical competencies (MC), six ranked among the top ten (see item 19, 21, 20, 44, 43 and 28 in Table 1), and their contents were related to "three basic competencies" (see item 19, 21 and 20 in Appendix A) in the traditional syllabus and "the competency of solving problems" (item 28) were all included; two ranked among the lowest ten items (see item 32, 42 in Table 2); they belonged to the competency of "mathematics representation" and "reasoning and proof ," respectively.

Table 1

The Top Ten Items Ranked According to Means of the Level of Importance

Item No.	Contents of MPL	Importance mean	Rank	Mastery Mean	Rank
19	The competency of spatial visualization.	4.392	1	3.915	**7**
21	The competency of operating and solving problems.	4.389	2	4.035	2
53	The tendency to be full of confidence in teaching mathematics well.	4.386	3	4.028	3
54	The tendency of being ready to be a good mathematics teacher.	4.383	4	4.106	1
2	The knowledge of common core mathematics concepts and ideas and methods implied in the Standards of Mathematics Curriculum for all grades.	4.382	5	3.954	6
20	The competency of abstracting and generalizing.	4.338	6	3.972	5
44	The competency of using mathematics language, notation and symbol.	4.318	7	4.009	4
43	The competency of transferring mathematics contents and thoughts into mathematics language, notation and symbol.	4.285	8	3.890	8
3	The knowledge of common core mathematics concepts and ideas and methods implied in them, and their application in the real environment in the Standards of Mathematics Curriculum for all grades.	4.265	9	3.748	20
28	The competency of applying multiple strategies to solve different kinds of mathematics problems (namely, The competency of solving problems).	4.241	10	3.816	11

Table 2
The Lowest Ten Items Ranked According to Means of the Level of Importance

Item Number	Contents of MPL	Importance		Mastery	
		Mean	Rank	Mean	Rank
7	The knowledge of mathematics nature.	3.421	54	3.219	51
16	The knowledge of how technology has changed the nature of mathematics.	3.443	53	3.046	54
10	The view of mathematics being a dynamic and pending system of connected principles and ideas constructed through exploration and investigation.	3.521	52	3.192	53
15	The knowledge of the relationship of taught mathematics with the overall framework of mathematics.	3.532	51	3.324	47
8	The knowledge of the role of mathematics in culture and society.	3.566	50	3.194	52
17	The knowledge of how technology plays an important role in all kinds of mathematics activities.	3.634	49	3.227	50
9	The knowledge of the relationships of school mathematics to other school subjects, and its applications in society.	3.639	48	3.235	49
18	The knowledge of how to integrate technology with mathematics in appropriate approaches to mathematics instruction.	3.777	47	3.268	48
32	The competency of following and assessing chains of mathematical arguments put forward by others.	3.836	46	3.541	39
42	The competency of modeling with appropriate representation and explaining the phenomena of physics, society and mathematics.	3.843	45	3.447	46

Two items which ranked in the top ten were about mathematical affects (MA); one was “The tendency to being full of confidence in teaching mathematics well” (namely item 53), and the other was “The tendency to be a good mathematics teacher happily” (namely item 54). No items that belonged to mathematical affects (MA) ranked in the lowest ten.

The results show that the items perceived more important by practicing teachers are mainly the items reflected in the Mathematics Syllabus, such as items ranked No. 1 and No. 2, “The competency of spatial visualization” (item 19) and “The competency of operating and solving problems” (item 21) respectively. This is consistent with related research (Wei, 2001).

In addition, items perceived less important by practicing teachers are mainly about the knowledge of mathematics views. These demands are exactly the emphasis of the new-curriculum. For example, “The knowledge of mathematics nature” (item 7) and “The knowledge of how technology has changed the nature of mathematics” (item 16) are ranked respectively as No. 1

and No. 2 counted backwards. Furthermore, the developers of Standards (SGOMCS, 2004) pointed out that traditional teaching has the inclination of neglecting the recognition and understanding of mathematics nature and being over formalized. The claim is supported by findings in this paper.

However, views from experts and practicing teachers are not consistent with each other. Expert Interview in this research showed that experts, regardless of college or high school background, laid stress on “The knowledge of mathematics nature” (item 7) and “The knowledge of the overall structure of mathematics contents” (item 5). This is the reason why they are still enmeshed in the final framework.

Relationship between Importance and Mastery

The top ten items of “the level of mastery” are almost the top ten items of “the level of mastery” perceived by teachers, but they are not one-one correspondences. Furthermore, there is high-positive relationship between the rank of perceived importance and the rank of perceived mastery with a Spearman correlation coefficient .954 and significant at the .01. The result is agreed with the findings of Gustafson (1984). He collected opinions regarding secondary mathematics teacher competencies.

One possible explanation for the finding was that the respondents knew what were important and hold to these views; another equal explanation was that if they didn’t have the literacy mastered, it was not important. If the later is a true case, the different views between the practicing teachers and the experts will forecast that the mastery level of teachers’ knowledge of “The knowledge of mathematics nature” (item 7) and “The knowledge of the overall structure of mathematics contents” (item 5) is as high as it is perceived. Of course, this is just an inference. This could be considered as a limitation of the study in that data regarding perceptions of mastery were collected as opposed to data which actually measured mastery.

In addition, the data also shows that each item has statistically significant at .01 between the means of the level of perceived importance and the means of the level of perceived mastery

So, on the one hand, there is high-positive correlation between “the level of importance” and “the level of mastery”; on the other hand, almost each item showed statistically significant differences between the means of the level of perceived importance and the means of the level of perceived mastery. This apparent incongruence between a high positive number of significant differences can be explained by the fact that in all of the items the level of perceived importance was higher than the level of perceived mastery. There are three possible conclusions which can be formulated from this finding: (1) teachers who responded had levels of mastery that were lower than the levels of importance relative to the literacy and needed to improve their literacy levels; (2) the teachers who participated in the study responded with modesty or lack of confidence and actually there was no difference between their level of mastery and their level of importance for each of the teacher literacies; and/or (3) teacher education programs have not pay enough attention to these literacy items. It was beyond the scope of the study to determine the reasons for the responses. It may need further investigation.

Items Features

The largest statistically significant disparities between the level of perceived importance and the level of perceived mastery (p<.01) occurred on the following five aspects: (1) "The knowledge of mathematics ideas and their use in a real situation" ; (2)"The knowledge of using technology"; (3) "The competency of spatial visualization"; (4) "The competency of mathematics modeling"; and (5)"The competency of applying reasoning to formulate mathematical problems and to guess, to build an example, to construct and access a mathematical argument". Namely, for these items, respondents perceived that they had a lower level of mastery than they should have. One could conclude that these literacy items represent areas of weakness that should be strengthened through pre-service and in-service mathematics teacher education programs. In fact, the difference between mastery and importance ratings in these areas is realistic. These items excluded "The competency of spatial visualization" (item 19), represent areas of importance always focused on by research studies and claimed by New-curriculum in recent years, and they are also areas which teachers who participated in this study showed lack of mastery confidence. Maybe, the lack of pre-service education in these areas contributes to this finding.

Correspondingly, items having the lowest differences occur for "The knowledge of mathematics nature" (item 7) and "The knowledge of the background and overall structure of mathematics contents" (item 5). These items are also ranked lower according to the level of perceived importance; they are No.1, No.4 and No.22, inverse order. Maybe, the reason is that teacher education programs do an outstanding job with respect to the balance between importance and mastery on relating to these items: teachers perceived their importance and really did well, but the reason also could be opposed to it: teachers perceived they had lower levels on these items and perceived them less important. Because these items scored double lower means, this study is inclined to select the latter.

The Effect of Experience Variables

In this study, the variables, such as educational background, teaching experience, teaching high school experience and professional title, were named as experience variables of teacher growth.

It was discovered that teaching experience had almost no effect (significant at the .01 level) on all items when considered as the perceived level of importance. Meanwhile on the perceived level of mastery, experience variables had a massing effect (significant at the .01 level) on the following content: "The knowledge of mathematics concepts, ideas and methods implied in them and the connections among them in the Standards of Senior Middle School Mathematics Curriculum (item 4)；Problem solving, mathematics communication, mathematics connection, and mathematics representation（item 6), having been adjusted to the competency dimension of MPL in final framework）；The competency of spatial visualization (item 19)；The competency of operating and solving problems (item 21)；The competency of distinguishing between different kinds of mathematical statements (item 26)；The competency of applying multiple strategies to solve different kinds of mathematics problems (namely, The competency of solving problems) (item 28)；The competency of representing mathematical thoughts by multiple ways, e.g. orally, writing and visual etc. (item 45)". You could find that all of these items were related to mathematics teacher's basis and common mathematics competencies, namely, no difference in deep meaning. If the significance was selected at the .001 level, no significant effect on any item was found with the experience variables. This finding was also congruent with the research of Gustafson (1984, p.96).

Since the study did not differentiate which sample group had a higher mastery level in these items, it would be inappropriate to conclude from these data: "along with increasing experiences, teachers have a higher mastery level in these items". It can only be concluded: "experience variables have significant effects on the mastery level of these items." At the same time, it also cannot be concluded that "experienced teachers did not have better mastery level than those of inexperienced teachers in most of the items" just because the experience variable had no significant effect on them.

Final Framework

According to the respondents to this research, the importance of the initial framework (IF) for describing high school mathematics teachers' MPL is highly regarded. But data from the suggestions of the respondents shows the necessity of revising, condensing or merging some items in the framework. Finally, the forty-seven item final framework (FF) (see details in Appendix B) is determined as describing high school mathematics teachers' MPL as demanded by HQMT. The procedure of item identification is shown in table 3.

Table 3

The Work Outline of Identifying Items in Final Framework

Item number of IF	Re-construction process on IF (to revise, condense or merge the items in IF)	Item number of FF	The structure of final framework
18	**Decreased by 5 items**	**13**	**Dimension 1 (D1) MK**
6	To merge and revising 2, 3 and 4; to adjust 5 to D13; to delete 6;	3	Dimension 1_1 (D11) Mathematics contents, ideas and methods implied in them
10	To merge and revising 7 and 8; to adjust 9 to D11; to adjust 13 to D32	7	Dimension 1_2 (D12) The concept of mathematics
2	To condense 5	3	Dimension 1_3 (D13) The structure of mathematics
28	**Decreased by 3 items**	**25**	**Dimension 2 (D2) MC**
4	No change	4	Dimension 2_1 (D21) Basic mathematics competencies
15	To merge 24 and 25; to condense 26 and 27; to merge 36 and 37	13	Dimension 2_2 (D22) The competencies of posing, analyzing and solving mathematics problems
9	To condense 39; to merge 42 and 31; to revise 43 and 44; to exchange 45 and 46	8	Dimension 2_3 (D23) The competencies of dealing with and applying mathematical language
8	**Increased by 1item**	**9**	**Dimension 3 (D3) MA**
4	To condense 47; to revise 49	4	Dimension 3_1 (D31) The inclination of mathematics learning
4	To condense 13 and 51	5	Dimension 3_2 (D32) Professional self-concept of mathematics

Notes: The deletion of 6 is because of its content having been integrated into related items of MC; IF represents initial framework; FF represents final framework

Conclusions and Recommendations

There are three basic stages to complete the study: (1) according to a conceptual framework derived from literatures and theory research, interviews of 4 expert mathematics teacher educators and 4 expert high school mathematics teachers, and a pilot study. The instrument, a questionnaire for teachers, is developed to collect data; (2) distribution of the instrument to a stratified random sample of 679 high school mathematics teachers in Henan, analyzing the quantitative data by using SPSS15.0 and coding the contents which needed to be revised and added according to teachers' views; (3) fixing the content and establishing the framework for describing mathematics teachers' MPL.

The research results and conclusions are as followings: (1) A framework is established for describing mathematics teachers' professional literacy of mathematics, including mathematical knowledge, mathematical competencies and mathematical affect with three-dimensions, 8 types and 47 items; (2) By observing the first ten items and the last ten items, which are ranked according to importance and relating to the contents, it is found that the most important items ranked by the teachers are those requirements reflected in the traditional Mathematics Syllabus, such as, "The competency of spatial visualization (item 19, ranked No.1)" and "The competency of operating and solving problems (item 21, ranked No.2)", while the less important items are related to the knowledge of views of mathematics, such as, "The knowledge of mathematics nature (item 7, countdown to the first)" and "The knowledge of how technology has changed the nature of mathematics (item 16, penultimate)"; A strong significant positive relationship (Spearman's rank correlation is .954, Significant at the .01 level) is found between the perceived importance and the perceived mastery; every item has significant higher means on the level of importance than on the level of mastery (at the .01 level); (3) Demographic variables have little or no influence on the ratings of perceived importance, but have some significant influence (total 7 items) on perceived mastery (at the .01 level).

The results throw some light on teacher education and teacher professional development:

1.The MPL of high school mathematics teachers from all three dimensions in this research should be improved. Because almost all scores of perceived level of importance was higher than the perceived level of mastery, and the two have a significant difference at the .01 level.

2.The components of MPL which teachers perceived very important should be strengthened continually. Traditionally, ignoring or missing MPL for good mathematics teaching should be addressed, focusing on teacher education or professional development. However, according to the research hypothesis in this study, MPL items were valued higher by participants (see "features of the top ten items" in this paper) and should have a much more important role in good teaching.

3.The knowledge of mathematical views and mathematical structures should be paid more attention. Expert teachers in this study pay much more attention to these items and that is in accordance with demands of the New-curriculum. In contrast, the practicing teachers (who answered the questionnaire) in this study seem not give enough attention thereon. So, teacher educators should make more contribution to these contents.

4.Teachers who engage in learning knowledge related to mathematics and technology should be encouraged. The results indicate that the perceived level of importance of items related to the knowledge of "mathematics-and-technology" (see item 16,17 and 18 in Appendix A) was less important than other items in our questionnaire according to participants in this study, and participants also didn't have enough confidence in mastering this kind of knowledge. This state is inconsistent with those who advocate a more important role for the knowledge of "mathematics-and-technology" in good mathematics teaching. Thus, those being responsible for the pre-service and in-service education of teachers face the challenge of better preparing and supporting teachers in their efforts to change.

5.The strategies for transforming mathematical knowledge and applying them to solve mathematical competences need to be developed. The results shows that the items of largest statistically significant disparities between the level of importance and the level of mastery are mostly related with the mathematical competences of applying mathematical knowledge to solve problems, for example, "The competencies of mathematical modeling". Perhaps, some effective work should be set down for teachers building their confidence in these areas.

6.The potential functions of experience variables in nurturing teachers' MPL need to be cultivated. As you know, teacher professional development is in a continued changing process from pre-service, induction and for an experienced teacher. Every stage should have its main development goals. The results of this research, however, reveal that experience variables have almost no effect on the perceived importance of all the items in the framework of MPL, and a different effect on the perceived mastery of items related with teachers basic competencies as a mathematics teacher. This is likely to recommend that teacher educators should be productive in elevating teachers' MPL during their in-service teaching process to satisfy the demands of continued educational reform.

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Appendix A (initial framework of MPL)

Means and Rank Orders of Perceived Importance and Perceived Mastery of MPL

Item number	Contents of MPL	Importance		Mastery	
		Mean	Rank	Mean	Rank
19	The competency of spatial visualization.	4.392	1	3.915	**7**
21	The competency of operating and solving problems.	4.389	2	4.035	2
53	The tendency of being full of confidence in teaching mathematics well.	4.386	3	4.028	3
54	The tendency of being ready to be a good mathematics teacher.	4.383	4	4.106	1

2	The knowledge of common core mathematics concepts and ideas and methods implied in them in the Standards of Mathematics Curriculum for all grades.	4.382	5	3.954	6
20	The competency of abstracting and generalizing.	4.338	6	3.972	5
44	The competency of using mathematics language, notation and symbol.	4.318	7	4.009	4
43	The competency of transferring mathematics content and thoughts into mathematics language, notation and symbol.	4.285	8	3.890	8
3	The knowledge of common core mathematics concepts and ideas and methods implied in them, and their application in a real environment in the Standards of Mathematics Curriculum for all grades.	4.265	9	3.748	20
28	The competency of applying multiple strategies to solve different kinds of mathematics problems (namely, The competency of solving problems).	4.241	10	3.816	11
4	The knowledge of mathematics concepts, their thoughts and implied methods and the connections among them in the Standards of Senior Middle School Mathematics Curriculum.	4.224	11	3.876	9
22	The competency of collecting and processing data	4.218	12	3.777	15
51	The willingness to inspect and revise continuously one’s self- perspective on the nature of mathematics, mathematics teaching and learning.	4.195	13	3.763	17
52	The confidence of learning mathematics well and independently.	4.189	14	3.796	12
47	The willingness to having enthusiasm in mathematics, learning new mathematics knowledge and exploring mathematics problems continuously.	4.186	15	3.795	13

45	The competency of representing mathematical thoughts in multiple ways, e.g. orally, writing and visual etc.	4.181	16	3.817	10
50	The willingness to discussing mathematics problems with colleagues and peers.	4.170	17	3.769	16
34	The competency of uncovering the basic ideas in a given proof.	4.131	18	3.745	21
26	The competency of distinguishing between different kinds of mathematical statements, (including conditioned assertions ('if-then'), quantifier laden statements, assumptions, definitions, theorems, conjectures, cases).	4.130	19	3.749	19
49	The habit of working with the guidance of methods.	4.125	20	3.727	22
12	The inclination of encouraging inquiring and solving problems with different methods in mathematics.	4.118	21	3.701	23
35	The competency of designing formal and informal mathematical arguments, and transforming a heuristic into a validated proof.	4.080	22	3.675	25
33	The competency of knowing what a mathematical proof is and how it differs from other forms of mathematical reasoning.	4.069	23	3.663	26
6	The knowledge of problem solving, mathematics communication, mathematics reasoning, mathematics connection and mathematics presentation.	4.066	24	3.691	24
46	The competency of understanding mathematical statements put forward by others, in writing or orally.	4.063	25	3.778	14
27	The competency of identifying, putting forward and designing different kinds of mathematical problems (such as pure, applied, open as well as closed mathematical problems).	4.050	26	3.585	32

48	Having the initial dispositions and abilities to do mathematical research.	4.036	27	3.622	29
37	The competency of applying reasoning to formulate mathematical problems, to build counter examples, to construct and access mathematical arguments.	4.024	28	3.559	37
24	The competency of understanding the scope and limitations of a given concept.	4.022	29	3.638	27
36	The competency of applying reasoning to verify patterns, to generalize, and to form reliable mathematical arguments.	4.019	30	3.615	30
31	The competency of performing active modeling in a given context or appropriate real-world situations.	4.014	31	3.547	38
40	The competency of being able to choose and switch between different representational forms for any given entity or phenomenon, depending on the situation and purpose for solving the problem.	4.005	32	3.608	31
5	The knowledge of the overall structure of mathematics content, namely, the interconnected nature of different branches of mathematics content knowledge.	3.984	33	3.761	18
25	The competency of extending the scope of a concept by abstracting some of its properties, and generalizing results to larger classes of objects.	3.984	34	3.582	33
23	The competency of posing questions with mathematics characteristics knowing the type of the answer.	3.972	35	3.568	34
29	The competency of analyzing foundations and properties of existing models, including assessing their range and validity.	3.947	36	3.499	44
14	The knowledge of understanding the overall structure of mathematics from multiple perspectives.	3.945	37	3.533	42

13	The knowledge of the conception of mathematics.	3.940	38	3.534	41
1	The knowledge of elementary mathematics, advanced mathematics and modern mathematics related to the content in the Standard for Senior High School .Mathematics Curriculum (experimental).	3.932	39	3.631	28
38	The competency of understanding and using the representation for different sorts of mathematical entities and contexts.	3.932	40	3.563	36
41	The competency of creating and applying multiple representations to organize, record and communicate mathematical thoughts.	3.928	41	3.566	35
30	The competency of decoding and interpreting existing mathematical models elements in terms of the real world or situation which they are supposed to model.	3.920	42	3.490	45
11	The view of mathematics being in the process of experiment and exploration.	3.918	43	3.536	40
39	The competency of understanding and using the reciprocal relationship among different representational forms of the same entity (phenomena and contexts), as well as knowing about their relative strengths and limitations.	3.907	44	3.532	43
42	The competency of modeling with appropriate representation and explaining the phenomena of physics, society and mathematics.	3.843	45	3.447	46
32	The competency of following and assessing chains of mathematical arguments, put forth by others.	3.836	46	3.541	39
18	The knowledge of how to integrate technology with mathematics in appropriate approaches to mathematics instruction.	3.777	47	3.268	48
9	The knowledge of the relationships	3.639	48	3.235	49

	of school mathematics to other school subjects, and its applications in society.				
17	The knowledge of how technology plays an important role in all kinds of mathematics activities.	3.634	49	3.227	50
8	The knowledge of the role of mathematics in culture and society	3.566	50	3.194	52
15	The knowledge of the relationship of taught mathematics with the overall framework of mathematics.	3.532	51	3.324	47
10	The view of mathematics as being a dynamic and pending system of connected principles and ideas constructed through exploration and investigation.	3.521	52	3.192	53
16	The knowledge of how technology has changed the nature of mathematics.	3.443	53	3.046	54
7	The knowledge of mathematics nature.	3.421	54	3.219	51

Appendix B

The Final Framework of Mathematics Teachers' MPL

Structure of FF	Contents of MPL
Dimension 1 Mathematical Knowledge (total items 13)	
Dimension 1_1 Mathematics contents, ideas and methods implied in them and (3 items)	·The knowledge of elementary mathematics, advanced mathematics and modern mathematics related to the contents in the Standard for Senior High School Mathematics Curriculum (experimental).
	·The knowledge of common and core mathematics concepts, ideas and methods implied in them for all grades.
	·The knowledge of the connections of school mathematics to other school subjects, and its applications in reality situations.
Dimension 1_2 The concept of mathematics (7 items)	·The knowledge of the history of mathematics.
	·The view of mathematics being a dynamic and pending system of connected principles and ideas constructed through exploration and investigation.

	·The view of doing mathematics being the process of experiment and exploration.
	·The inclination of encouraging to inquire and problems solving with different methods in mathematics.
	·The knowledge of how technology has changed the nature of mathematics.
	·The knowledge of how technology plays an important role in all kinds of mathematics activities.
	·The knowledge of how to integrate technology with mathematics in appropriate approaches to mathematics instruction.
Dimension 1_3 The structure of mathematics (3 items)	·The knowledge of the overall structure of mathematics content.
	·The knowledge of the relationship of taught mathematics with the overall framework of mathematics.
	·The knowledge of understanding the overall structure of mathematics from multiple perspectives.
Dimension 2 Mathematical Competencies (total items 25)	
Dimension 2_1 Basic mathematics competencies (4)	·The competency of spatial visualization.
	·The competency of abstracting and generalizing.
	·The competency of operating and solving problems.
	·The competency of collecting and processing data.
Dimension 2_2 The competencies of posing, analyzing and solving mathematics problems (13 items)	·The competency of posing questions with mathematics characteristic and knowing the answer type to them.
	·The competency of understanding the scope and limitations of a given concept.
	·The competency of distinguishing between different kinds of mathematical statements.
	·The competency of identifying, putting forth and designing different kinds of mathematical problems.
	·The competency of applying multiple strategies to solve different kinds of mathematics problems (namely, The competency of problem solving).
	·The competency of analyzing foundations and properties of existing models, including assessing their range and validity.
	·The competency of decoding and interpreting existing mathematical model elements in terms of the real world or situation which they are supposed to model.
	·The competency of modeling with a given or appropriate real situation.

	·The competency of following and assessing chains of mathematical argument, put forth by others.
	·The competency of knowing what a mathematical proof is and how it differs from other forms of mathematical reasoning.
	·The competency of uncovering the basic ideas in a given proof
	·The competency of designing formal and informal mathematical argument, and transforming a heuristic into a validated proof.
	·The competency of applying reasoning to a series of mathematical activities.
Dimension 2_3 The competencies of dealing with and applying mathematical language (8 items)	·The competency of understanding and using representation for different sorts of mathematical entities and contexts.
	·The competency of understanding and using the relationship among different representational forms of the same entity.
	·The competency of being able to choose, apply, and switch between different representational forms for any given entity or phenomenon, depending on the situation and purpose for solving the problem.
	·The competency of creating and applying multiple representations to organize, record and communicate mathematical thoughts.
	·The competency of understanding symbol and formal language.
	·The competency of dealing with and applying mathematics symbolic statement and expression, consisting of mathematics formula.
	·The competency of understanding mathematical statements put forth by others, in writing, orally or visual.
	·The competency of representing and communicating mathematical thoughts in multiple ways, e.g. orally, writing and visual etc.
Dimension 3 Mathematical Affects (total items 9)	
Dimension 3_1 The inclination of mathematics learning (4 items)	·Having enthusiasm in mathematics, learning new mathematics knowledge continuously.
	·Having the initial dispositions and abilities to do mathematical research.
	·The habit of thinking and dealing with problems through the way of mathematical thinking.

	·The willingness to discussing mathematics problems with colleagues and peers.
Dimension 3_2 Professional self-concept of mathematics (5 items)	·Having the self-concept of mathematics.
	·Examining the concept of mathematics of oneself continuously.
	·The confidence of learning mathematics well independently.
	·Being full of confidence in teaching mathematics well.
	·Being ready to be a good mathematics teacher.

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