

What Constitutes Good Mathematics Teaching in Mainland China: Perspectives from Nine Junior Middle School Teachers

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This study aims to understand teachers' perception of what constitutes good mathematics teaching in Mainland China. Data collected through semi-structured interviews with nine junior secondary school mathematics teachers suggests that three factors are considered to be very important by teachers for good mathematics teaching in the current social and cultural context of Mainland China: 1) students' active participation, mathematics learning interests and experience; 2) the characteristics and essence of mathematics in classroom teaching and the connection between mathematics knowledge and mathematics thinking; and 3) the development of students' mathematics thinking and mathematics methods.

Key words: mathematics classroom teaching, good mathematics teaching, junior secondary school, teacher perspectives

Introduction

Although it was claimed that “there is no agreement on what constitutes “good mathematics teaching” (Thompson, 1992, p. 127), few would disagree that the nature of good mathematics teaching assumed in one certain context, particularly mathematics teachers' beliefs about what constitutes good mathematics teaching, is an important factor affecting teachers' own teaching and students' mathematics achievement as well. Given this fact, there have been increasing research interests focusing on investigating good mathematics teaching in various contexts, especially those in East Asia, such as South Korea, Singapore, Hong Kong and Mainland China, where students demonstrated outstanding performance in international comparative studies. Those studies have already reported some characteristics of good or excellent mathematics teaching in the high performing systems through the analysis of competent teachers' teaching practices. However, it is still unclear what constitutes good mathematics teaching from the teachers' perspectives in those systems since almost all of the findings were obtained

from the researchers' perspectives rather than the teachers' perspectives. Teachers' teaching, however, is inevitably affected by, if not entirely shaped by, a teachers' belief system (Ernest, 1989). Regarding this, to deeply and completely understand what constitutes good mathematics teaching within a certain context, teachers' perspective should not be ignored.

Expect for Wilson et al.'s (2005) investigation on what constitutes good mathematics teaching in the U.S. from a teacher's perspectives, there were few previous studies investigating teachers' perceptions about good mathematics teaching in other countries and regions,. However, the socio-cultural contexts and educational contexts in the Eastern and Western countries have been found to be different (e.g., Biggs, 1996). In addition, the differences existing in different contexts will further affect mathematics teachers' belief systems (Ernest, 1989), which will in turn affect teachers' consideration of the desirable ways of teaching and learning mathematics (Thompson, 1992). From this perspective, it is reasonable to conjecture that teachers' perception of good mathematics teaching could be different in different social and cultural contexts. Therefore, investigating mathematics teachers' views of what constitutes good mathematics teaching in various educational systems will not only help to establish a deeper and more comprehensive understanding of good mathematics teaching in one society, but also uncover cultural influences of teachers' practice. In view of this, the present study aims to explore what constitutes good mathematics teaching from the junior secondary school mathematics teachers' perspective within the mathematics education context of Mainland China.

Review of Literature

Good Mathematics Teaching in International Context

As mentioned above, recently, there have been increasingly research interests focusing on what constitutes good mathematics teaching or what were the characteristics of good mathematics teaching practices in various contexts. Pang (2009) explored what counts as good mathematics instruction in South Korea through the analysis of three competent mathematics teachers' teaching practices. He found that good mathematics instruction is "teacher-directed and content-oriented". To be more specific, its characteristics include: 1) emphasis on mathematical content; 2) consideration of students' knowledge and ability; 3) emphasis on multiple solution methods and connections; and 4) teacher's reconstruction of textbooks. Kaur (2009) explored the characteristics of good mathematics teaching in Singapore and found that it is "teacher-centered but student focused". The main features of good mathematics lessons in Singapore claimed to mainly include: 1) instructional cycles which have specific instructional objectives; 2) carefully selected teaching examples

which vary in complexity from low to high; 3) teachers actively monitor students' understanding during seatwork, and choose appropriate cases for class review and discussion; and 4) teachers reinforce their students' understanding of knowledge through a detailed review of students' work (including homework). From the findings of those two studies, several common characteristics of good mathematics teaching between South Korea and Singapore could be identified. For example, all teachers interviewed in South Korea and Singapore emphasize mastering mathematics teaching content, students' mathematical understanding, and teacher's reconstructing textbooks. The social and cultural contexts, as stated by these two researchers, have some influence on good mathematics teaching practice. For example, Pang (2009) clearly pointed out that "good teaching practice is nested in deep-rooted cultural values and norms". Therefore, it is impossible to fully understand what constitutes good mathematics teaching without considering its social and cultural context.

In order to understand mathematics teachers' perception of what constitutes good mathematics teaching in America, Wilson et al. (2005) conducted semi-structured interviews with nine high school mathematics teachers. In their study, it was found that the nine mathematics teachers tend to believe that good mathematics teaching: 1) requires prerequisite knowledge of mathematics subject and students; 2) promotes students' mathematical understanding; 3) engages and motivates students; and 4) requires effective management.

From the views of American mathematics teachers about what constitutes good mathematics teaching, Wilson et al. (2005) mentioned that even though teachers feel responsible for making lessons go smoothly, the American teachers placed more emphasis on students' active engagement. In addition, some characteristics of good mathematics teaching practice found in South Korea and Singapore, such as teachers' reconstructing textbooks, problem solving methods, specific instructional objectives, were not found in American mathematics teachers' views. This provides further evidence to hypothesize that mathematics teachers in different contexts will perceive what constitutes good mathematics teaching differently.

Good Teaching in Mainland China

As to the notion of good teaching, there have been some studies focusing on it in Mainland China. For example, Gao and Watkins (2001) found that one teaching concept held by physics teachers in Mainland China was to view "students' examination performance, especially public examinations, as the most important indicator of good teaching and successful schooling" (p. 463). This conception indicates that good teaching should enable students to attain good academic achievement. Chen (2007) explored

the concept of “excellent teaching” held by teachers from different subjects and found that Chinese teachers tend to believe “excellent teaching” as: 1) caring for students, such as creating a comfortable and encouraging classroom atmosphere, establishing positive teacher-student relationship, motivating students through praise, and so on; 2) guiding students’ all-ground development; 3) connecting school knowledge/work to other areas; and 4) planning and preparing structured lessons. Unlike the findings in Gao and Watkins’ study, the emphasis on students’ academic achievement was not found in Chen’s study.

As to mathematics education, although few studies focusing on the notion of good mathematics teaching could be found in Mainland China, there have been several studies on Chinese mathematics teachers’ beliefs about mathematics teaching, especially from a comparative perspective. Cai (2004) found that Chinese mathematics teachers and American mathematics teachers hold different beliefs about learning goals. An (2004) found that compared with American mathematics teachers, Chinese mathematics teachers hold different beliefs about mathematics teaching. The Chinese mathematics teachers tend to believe that “the goals of mathematics teaching are to help students gain mathematical knowledge that can be applied in the real world and to enhance students’ critical thinking and logical reasoning ability” (p. 198).

As mentioned above, the difference in beliefs about mathematics and mathematics teaching and learning will further influence teachers’ consideration of favored ways of teaching and learning mathematics (Thompson, 1992). Actually, in those studies investigating the difference of teachers’ beliefs regarding “effective mathematics teaching”, it was found that teachers from different countries tend to hold different views about effective mathematics teaching (Bryan et al., 2007). As to teachers from Mainland China, they were found to tend to “have more of a teacher-led view of classroom instruction than the teachers from the West, who hold more to a student-centered view” (p. 339). More specifically, after interviewing several experienced primary school mathematics teachers and teaching research officers, Wang and Cai (2007) found that Chinese mathematics teachers tend to believe that effective mathematics teaching should have the following features: coherence, cover sufficient content and practice, appealing language, good questioning and answering processes, comfortable classroom atmosphere, student participation, flexible teaching, use concrete examples, generalize math meaning when using concrete example, and highlight essential points. More recently, Huang and Li (2009) analyzed one exemplary lesson which is viewed as a good mathematics lesson in Mainland China and identified some characteristics of this lesson: instruction coherence, well-developed knowledge, and practical knowledge with problem variations,

exposed mathematical thinking and cultural value, encouraged students' participation and self-exploration.

As reviewed above, even though several studies have investigated what is "excellent teaching", or what is "good science teaching" in Mainland China, there are still few studies which explore what constitutes "good mathematics teaching" from the teachers' perspectives. Moreover, even though there is already a study which explores what is "effective mathematics teaching" in China, the participants in this study were all very experienced primary school teachers and some of them were teaching research officers rather than practicing teachers. Therefore, it is not clear how secondary school mathematics teachers in Mainland China perceive "good mathematics teaching". Moreover, in those studies, the Chinese social and cultural contexts were not highly stressed. So, it is valuable to explore Chinese mathematics teachers' notion of good mathematics teaching with the consideration of contextual influences. This will not only explain the Chinese classroom teaching practice to some extent, but also provide information for those who are interested in Chinese mathematics education in order to understand Chinese mathematics education. More importantly, it will give mathematics teacher educators, especially those from Mainland China, information to help in designing teacher training programs at both the pre- and in-service stages. In view of this, this study aims to investigate what constitutes good mathematics teaching from the Chinese secondary school mathematics teachers' perspective, teachers who work in a very unfavorable teaching environment as compared to the one in Western countries (Biggs, 1996).

Research Methods

Participants

Purposeful sampling was adopted in this study. Nine junior secondary school mathematics teachers (T1, T2, ... T9 will be used to protect personal privacy) from Chongqing, a city located in the western part of Mainland China, were chosen as participants in the present study. These teachers were chosen to reflect a range of schools with different catchment areas and different student socio-economic and academic characteristics. The teachers chosen in this study varied in teaching experience from 7 to 25 years and had qualifications which better reflected the situation of the teacher population (detailed information about them as listed in Table 1).

Table 1
Details of Participants

Code	Sex	Qualification	Years of Teaching	Working School
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			Experience	Background
T1	Female	Intermediate Level 1	7	Key Middle School
T2	Female	Intermediate Level 1	9	Non-key Middle School
T3	Male	Intermediate Level 2	4	Key Middle School
T4	Female	Senior	17	Non-key Middle School
T5	Female	Intermediate Level 1	14	Non-key Middle School
T6	Male	Intermediate Level 2	3	Non-key Middle School
T7	Male	Senior	23	Key Middle School
T8	Female	Intermediate Level 1	10	Non-key Middle School
T9	Male	Senior	27	Key Middle School

Instruments and Procedure

A semi-structured interview was employed in this study to collect data. Every teacher was interviewed twice and all the interviews were audio-taped. Questions used in the first interview were adapted from Wilson et al.'s (2005) study, which include: 1) describe a good mathematics lesson he/she once delivered and tell why this lesson was good; 2) describe a good mathematics lesson of other teachers he/she once observed and tell why this lesson is good; 3) are there any changes he/she would make to this lesson if he/she self taught this topic and why; 4) describe an ideal mathematics lesson and tell why is ideal.

After the first interview, the audio file was transcribed by the interviewer. Then, each transcription was analyzed and some topics which were not elaborated by the interviewees were noted for the second interview. The second interview was conducted after one or two days, depending upon each interviewee's convenience. Before the second interview, the transcript of the first interview was presented to them and they were asked to check it, and any further comments made by them were also allowed and audio-taped. After that, they were further asked to elaborate those topics which were not deeply described by them in the first interview.

Data Analysis

The object of analysis was to identify the categories and relationships of the notions of good mathematics teaching. The general strategy for this analysis was a constant comparative style (Strauss, 1987), which permitted the categories to emerge from the data. The analysis involved three stages. At the very beginning, an analysis was run for each different transcript separately, that is, each utterance about good mathematics teaching in every interview

transcript was identified and labeled.

The second stage of analysis was to group those utterances across different transcripts that had a similar meaning into a common category. After this, a set of categories was established and further, intensive analysis of each particular category was carried out to identify a distinct meaning for them and construct sub-categories, if possible.

The third stage of analysis was to calculate the frequency of each category and sub-category across all teachers and those (sub) categories which had high frequency were thought to be a component of good mathematics teaching. Then, the relationship between categories or sub-categories was further examined. Relevant examples and statements were cited to support further interpretation. Thus, after these three stages, the elements of good mathematics teaching in Mainland China were identified.

Results

After a systematic analysis of the nine teachers' transcripts, seven categories about what constitutes good mathematics teaching were identified. Table 2 summarizes these seven aspects and the composition of each teacher's view.

Table 2
Participants' Views about What Constitutes Good Mathematics Teaching

	T1	T2	T3	T4	T5	T6	T7	T8	T9
Connecting teaching content to real life	+	+	+	+	+	+	+	+	+
Creating favorable teaching atmosphere and cultivating students' interests	+	+	+	+			+	+	
Encouraging students' participation	+	+	+	+	+	+	+	+	+
Respecting students' differences		+	+	+		+	+		+
Emphasizing the essence of mathematics knowledge		+	+	+	+	+	+	+	+
Stressing the integration of knowledge	+	+		+	+	+	+	+	+
Developing students' mathematics thinking	+	+	+	+	+	+	+	+	+

Connecting Teaching Content to Real Life

All these nine teachers emphasized that the teaching content in a good mathematics lesson should be linked to real-life situations and students should have opportunities to discover mathematics knowledge from these situations. According to their responses, though mathematics is essentially abstract, it has its origins in real-life world, especially for junior secondary school mathematics. Therefore, good mathematics teaching should help students to experience the developmental process of mathematics knowledge as well as to understand the origins of mathematics. Through this connecting process, it would not only facilitate students' understanding of relevant mathematics knowledge, but also let them appreciate the fact that mathematics is related to and taken from real. As T1 stated:

In a good mathematics lesson, students should have chances to discover mathematics knowledge in real-life situations. Because most students think that mathematics is abstract and useless, when you introduce new teaching content, you should let them feel that mathematics is taken from real life situations, and mathematics can be used in real life situations. (T1)

According to these nine teachers, there are several ways to connect teaching content to real life, including 1) using real life situations to introduce new knowledge; when commenting on their own or other teachers' good lessons, the teachers all mentioned that examples from real-life or other subjects could be used to present new knowledge; 2) integrating real-life situations into exercises; for a good mathematics lesson, it would be desirable if the questions used for in-class work and homework are posed through integrating information from real life. Through this, the whole teaching process could make students feel that:

Most of the time, mathematics is not so abstract. Some concepts or knowledge can be learned or grasped through analogy with some examples from real life situations, or their origins can be found in the real world. (T2)

Creating a Favorable Teaching Atmosphere

Through the analysis, the present study also found that for good mathematics teaching, it is also very important for teachers to create an enjoyable teaching atmosphere and motivate students' learning interests. Six out of the nine teachers explicitly stated that good mathematics teaching

should have a favorable teaching atmosphere. It was believed that to be able to conduct a good mathematics lesson, the teacher should be humorous and good at creating an enjoyable classroom environment so as to make all students feel at ease rather than to have a stressful environment. Moreover, during teaching, the teacher should care about students' mathematics learning interests and know how to inspire and cultivate their learning interests. They indicated that once students are interested in the teaching, the teaching can be regarded as good. Otherwise, no matter how well a teacher designs his teaching and/or how thoroughly he explains the content, it will not be possible to view the teaching as good. As T3 said:

For a good mathematics lesson, first of all, the teaching atmosphere should be good. It should make students feel that the lesson is very enjoyable rather than just sitting there and feeling uncomfortable. ..., Meanwhile, the teacher should know how to inspire students' mathematics learning interests so as to make them feel that learning mathematics is a very enjoyable thing. (T3)

Encouraging Students' Active Participation

Another aspect of good mathematics teaching is that students should have opportunities to conduct self-exploration and teachers should encourage them to participate in such activities. This point was mentioned by all the interviewees. These interviewees emphasized that students should play the key role in their learning rather than receiving mathematics knowledge passively through teachers' lectures and demonstration. When commenting on their own or other teachers' good lessons, all the interviewees highlighted that in the good lessons, students have opportunities to conduct some exploration on teacher-constructed situations, demonstrate their problem solving methods, make relevant analogies, and be involved in some other activities. Terms, such as "group discussions or activities", "students' activities are carried out very well", and "students' operations", were frequently mentioned by some interviewees. In other words, as T4 stressed, "teachers are organizers and coordinators".

It was generally believed that for good mathematics teaching, students should be given plentiful opportunities and time to discuss their thoughts, to look for problem solving solutions, or to discover mathematical rules through their own explorations. As T5 pointed out:

When teaching, we should let students experience and appreciate how this knowledge point comes out, make them understand the origin of mathematics and really understand it. (T5)

Besides, teachers should also be able to create a learning zone to challenge students, so as to continuously stimulate their curiosity and thirst for knowledge, and make them actively learn and explore. As T4 emphasized:

I think that teachers should fully know students' meta-cognitive level. They can design some challenging tasks on the meta-cognitive level, which provides students a zone for development. This is quite different from a situation in which students easily master the points but there is no development. (T4)

So for a good mathematics lesson, teachers should not explicitly “tell” all the learning points and problem solving methods. Instead, it is necessary that teachers should give students enough time to think and explore by themselves. In some interviewees' opinions, whether or not students can have their own unique understanding or opinions regarding some relevant knowledge or phenomenon was a critical and fundamental feature of good mathematics teaching.

Respecting Students' Differences

Six out of the nine teachers emphasized that it is quite possible that there exist some differences between students in terms of their background, ability, and mathematics achievement. The six teachers remarked that for good mathematics teaching, a teacher should respect and pay attention to these differences. They suggested that teachers need to make necessary changes to the teaching content and present relevant content gradually according to the students' actual situations. T6 pointed out:

In a mathematics classroom, students' levels are different. Therefore, in a good mathematics lesson, you should care about the students: you can use one or two difficult problems in a lesson, that is, advanced students can gain extra knowledge. However, for the other students, you should be able to design some easier questions, because if every question is very difficult, they will lose their [learning] interests, lose their interests in the lesson or even lose their interests in you. Therefore, this lesson definitely becomes an unsuccessful lesson. (T6)

As indicated in the above statement, for a good mathematics lesson, a teacher should be able to design questions with various difficulty levels according to individual student's learning ability, frame a reasonable knowledge structure, and present teaching content gradually, that is, from easy to difficult. In addition, to have good mathematics teaching, during the

teaching process, a teacher should give enough time to students, especially those students with average mathematics ability, to think over or review what has just been learned. As T2 said:

Because usually there are many students in one class, it is difficult to take care of every student sometimes. However, in a good mathematics lesson, it will be much better to give students time during the teaching process so that they can think over or go over the newly learned content. During this time, those who do not understand can seek help from the teacher or their peers. ... This can help to consolidate teaching content over time and will also not place too much a burden to those who are not very good at mathematics. (T2)

T2's statement indicates that for good mathematics teaching, a teacher should regulate his teaching pace and arrange content reasonably according to students' actual learning to cater to individual student's differences. Besides this, it was further mentioned that a teacher should also make good use of those differences. In particular, he can choose the right teaching methods and strategies to help students enlighten each other. T4 further pointed out:

Because students' ability levels are different, the thinking of good students can enlighten the thinking of average students or weak students. In a good lesson, teachers should use students' differences well. (T4)

Emphasizing the Essence of Mathematics Knowledge

Another component of good mathematics teaching is that the essence of mathematics knowledge should be stressed, and this was highly emphasized by eight interviewees. According to these teachers, mathematics as a school subject has its own unique function and characteristics. In a good mathematics lesson, these features of mathematics, particularly the nature of mathematics, cannot be lost or ignored. In a good mathematics lesson, students should have chances to conduct their own explorations. That is, the focus of teaching should not be these activities themselves. Furthermore, students' explorations and participation should not be carried out superficially. The main aim of these activities should be to help students gain experiences of the developmental process of knowledge, facilitate their understanding of the essence of relevant knowledge and/or problems, and obtain a deeper understanding of mathematics knowledge. When T4 described an ideal lesson, she stressed:

I think that a good mathematics lesson, from the perspective of

mathematics, the classroom teaching should not look very active superficially; in contrast, your knowledge and thinking should be very active at a high level. You should stress the essential property of mathematics knowledge, ..., I think as for a good mathematics lesson, you should guide students to start with the essence of problems and make further relevant extensions. You should make your students understand the deeper things of mathematics. (T4)

T4' statement indicates that for good mathematics teaching, the deep and essential meaning of mathematics should be stressed. In addition, the inherent meaning and extensional meaning should also be stressed. This opinion was also echoed by some other teachers. For example, T7 said:

In a good mathematics lesson, a teacher should have his own unique understanding regarding the inherent meaning and extensional meaning of the teaching content. He should point them out during his teaching. (T7)

Stressing the Integration of Knowledge

Another important aspect related to good mathematics teaching was that a teacher should integrate different knowledge together and stress the connections between different knowledge during teaching. This was emphasized by eight interviewees out of the nine. In their opinions, when a teacher plan a good mathematics lesson, he should not only focus on the content of one individual lesson but take into account what the students learned before or will learn in future lessons connected to this topic. During his teaching, he should let students experience the connections between different topics. As T5 said:

As to the content in textbooks, a teacher cannot only consider the content of one individual lesson. Sometimes, this content might be related to other knowledge in this chapter, or in this book, or even at the whole junior secondary school level. During your teaching, you should make your students experience these connections. (T5)

Besides stressing the connections of mathematical knowledge, in a good mathematics lesson the teacher should further stress the connection of different mathematical methods and mathematical thinking. It was described that in a good mathematics lesson, a teacher should integrate different mathematics methods and/or thinking together to make students experience these connections. T8 pointed out:

In a good mathematics lesson, different mathematics thinking should be integrated together. For example, you can integrate several sample problems into one single problem to let students experience the connection of different mathematics methods or mathematics thinking found in a whole chapter or even different grades through solving this one problem. (T8)

Developing Students' Mathematics Thinking

The ability developed students' mathematics thinking, mathematics methods, and their ability to think and reason mathematically were thought of as fundamental components of good mathematics teaching, which was highly emphasized by every teacher. Some teachers pointed out that in a good mathematics lesson, relevant mathematics thinking and methods should be infiltrated into the teaching content. In addition, teachers should encourage students to be involved in mathematics learning intellectually. As mentioned above, students should be given opportunities to participate in some activities. However, in good mathematics teaching, students should not conduct these activities superficially. On the contrary, students should play a very intellectually active role during the process. In other words, they should participate in mathematics thinking. That is to say, during teaching, students' thinking should not be replaced by teacher's thinking. The teacher should use various teaching strategies to inspire students' thinking, guide them to think mathematically and to explore problems solving methods. Problem solving methods were a highly emphasized aspect of this study. It was suggested that, for a good mathematics lesson, a teacher should be able to guide his students to explore different solving methods to a problem so as to extend and broaden students' thoughts. In particular, when guiding students to approach problems, in a good mathematics lesson, the teacher should think about problems from the students' perspective. That is, the teacher should not always think about or demonstrate problem solving solutions at his own cognitive developmental level. As T1 mentioned:

In a good mathematics lesson, you'd better teach them some thinking which they can use in the future when they have to solve some mathematics problems. Because mathematics problems are not invariable, instead, they keep changing, therefore, you should guide them to explore different methods for solving one problem and develop their thinking gradually and slowly. (T1)

Discussion

The main aim of this study is to explore what constitutes good mathematics from the perspective of Chinese junior secondary school mathematics teachers who work in a different context from Western countries. Generally speaking, from the seven themes reported above, it could be seen that there are three main aspects which Chinese mathematics teachers emphasized: students, mathematics knowledge, and mathematics thinking and methods.

Classroom Teaching Atmosphere

The findings of this study indicate that, for a good mathematics lesson, the classroom atmosphere should be comfortable, relaxed and harmonious, an atmosphere in which students can happily learn, and teachers should stimulate students' learning enthusiasm and make them really like mathematics. That is, Chinese teachers think that good mathematics teaching should pay attention to students' affective experience of mathematics learning, their confidence in mathematics learning, etc. For the emphasis on comfortable and harmonious teaching atmosphere, similar findings could also be found in other two studies on Chinese teachers about the topic of "excellent teaching" and "effective mathematics teaching" (Chen, 2007; Wang & Cai, 2007).

Students' Participation and Explosive Learning

According to the above findings, another important aspect of good mathematics teaching is to respect students' differences and their key role in learning, that is, a good mathematics lesson should provide enough learning opportunities and time for students to participate and self-explore. It also indicates that, in the cognition of these teachers, there does not exist the point of view that the teacher should definitely control classroom teaching, a typical view of traditional teaching which has been criticized for a long time. On the contrary, like the opinions held by American teachers reviewed above, good mathematics teaching should avoid directly "telling". Teachers should be aware of and be concerned about students' active role in mathematics learning and teaching, and should encourage them to participate and self-explore. Teachers cannot dominate the whole classroom teaching process. The highlighting of students' participation and self-exploration also implies that in light of the new mathematics curriculum reform in Mainland China, to enrich the diversity of classroom learning ways and improve students learning methods is also a very important indicator of good mathematics teaching. These findings are quite different from the findings or conclusions found in Chinese mathematics classroom practice in previous studies, like teacher dominated teaching, memorization of mathematical facts, minimal students' involvement, harsh classroom climate and so on (e.g., Biggs, 1996).

Factors which caused these differences might be the influence of the new mathematics curriculum reform. Since 2001, there has been a new mathematics curriculum reform in Mainland China. In this curriculum reform, some ideas about mathematics teaching and learning were raised, such as students should have opportunities to practice, explore and communicate. Because this reform had been carried out in Mainland China for almost eight years when this study was conducted, it might be possible that the mathematics teachers have accepted these ideas and carry them out in their teaching practice, at least to some extent. In addition, nowadays, students' participation during the mathematics teaching process is a very important aspect in evaluating whether a lesson is good or not (Zeng, 2003). Therefore, in order to make their lessons look active, mathematics teachers in Mainland China have to organize various activities, like group discussion and students' self exploration in their teaching.

In addition, the large size of class is still popular in Mainland China. Therefore, it is reasonable to think that good mathematics teaching should take into consideration the individual differences of students' knowledge base, mathematics ability and so on. Especially in the new curriculum reform, it is clearly stated out that every individual student should attain relevant improvement in mathematics. (Ministry of Education, 2001). Ideas like this make more sense: that good mathematics teaching should respect the individual students' difference, prepare various classroom tasks to meet the needs of individual students at different ability levels, and present teaching content gradually.

The Essence of Mathematics and the Connection of Multiple Topics

The way to deal with teaching content and present mathematics knowledge was found as another focus in this study. As mentioned by many teachers, for good mathematics teaching, a teacher should deal with teaching content flexibly according to the students' actual situation. In practice, mathematics knowledge should be integrated with real life situations. However, the essence of mathematics knowledge or the inherent meanings of relevant mathematics definitions should be highly emphasized. In the meantime, the connections between different mathematics topics should also be stressed. That is, to teach mathematics well, a teacher may need to reconstruct the textbooks rather than delivering teaching strictly according to the textbooks. Similar findings were also found in the studies exploring good mathematics teaching in South Korea and Singapore (Kaur, 2009; Pang, 2009). In South Korea, reconstruction of teaching materials is considered a component of good mathematics teaching. In good mathematics lessons in Singapore, teachers also design exercises from easy to hard. Although American teachers also emphasized that good teaching should help students

better understand mathematics, the essence of mathematics and the connection of multiple topics is not the focus in their description.

Actually, highlighting mathematics knowledge and the connections between different topics is a very traditional characteristic of Chinese mathematics teaching. In Mainland China, it has been a tradition that mathematics teaching should stress basic knowledge and basic skills, two of which are the so-called “Two Basics” (Zhang, 2006; Zhang et al., 2004). During the teaching, teachers tend to present basic knowledge in detail; and one of the teaching goals is to let students obtain a deep understanding of mathematics knowledge and its systematic and comprehensive structure (Shao & Gu, 2006). In a specific teaching process, as Tu and Song (2005) concluded, Chinese mathematics teachers tend to introduce “new knowledge” based on “old knowledge” and stress a deep understanding of the “new knowledge”. A similar statement was also made by Zheng (2006). In order to help the students grasp mathematics knowledge well, Chinese mathematics teachers “prefer to stress the inner relationship of mathematical knowledge, because in this way, not only can the new subject be introduced much more easily, but the whole course will show clearly the character of coherence as well” (Zheng, 2006, p. 387). Beside influenced by traditional mathematics teaching, such stress on mathematics knowledge and the connections between various topics is also impacted by the mathematics curriculum reform implemented since 2001. For example, in Mathematics Curriculum Standards (Ministry of Education, 2002), it clearly states that the learning content should be realistic, meaningful, and related to the real world. It also stresses integrating knowledge and skills to solve problems and cultivating students’ awareness of mathematics application.

The Development of Students’ Mathematics Thinking and Methods

In this study, what was explicitly stated and highly stressed by all teachers is that good mathematics teaching should be able to develop students’ mathematics thinking, to train students to use the necessary mathematics methods, and finally to make students think mathematically. Good mathematics teaching should integrate different mathematics thinking and methods with the teaching content, should carry out appropriate variations of these contents, should guide students to induce and summarize different problem solving methods and laws so as to promote students to think mathematically. That is, students’ mathematics thinking involvement in the classroom and their mastery of mathematics methods are key factors for teachers to use to evaluate whether a math lesson is good or not. In the interview, several teachers in this study mentioned that there were some lessons in which they even did not finish their planned teaching task or the class structure looked confusing, but they still thought that those lessons are

good because students actively participated in the discussion and raised challenging questions or problem solving solutions. Therefore, it could be seen from the present study that a well structured lesson is not necessary a prerequisite of good mathematics teaching as long as students are involved in mathematics thinking at a deep level. It further reflects that the participants in this study stressed developing students' mathematics thinking and methods. For these teachers, the key of good mathematics teaching is not how many activities are organized, but how much students are involved and are cultivated in their mathematics thinking. The emphases on inducing problem solving methods, the edification of mathematics thinking, and the cultivation of students' intellectual development also echoes some findings of previous studies. For example, Leung (1995) found that teachers in Beijing pay attention to summarizing different question types. An (2004) also found that Chinese teachers think that the goal of mathematics teaching is to cultivate students' abilities and thinking. But finishing a teaching task or having a complete class structure which have been found in previous studies, are not the main concerns for these teachers. For them, the key of good mathematics teaching is to cultivate students' abilities of mathematically thinking and solving problems. However, compared with the findings in America, this was not as explicitly mentioned and stressed by American mathematics teachers (e.g., Wilson et al., 2005).

Actually, cultivating students' mathematics thinking and abilities has always been one of the Chinese traditional mathematics education philosophies. Emphasis on problem solving and its methods and skills is a prominent feature of Chinese mathematics teaching. As Cai and Nie (2007) stated, in the Mainland, Chinese mathematics problem solving has been viewed both as an instructional goal and an instructional approach. During this teaching practice, three different problem solving activities are generally highlighted: one problem with multiple solutions, one problem with multiple changes and multiple problems with one solution (Cai & Nie, 2007). The aim of these activities is to train and develop students' problem solving ability and further help them well grasp relevant mathematics methods and thinking. Besides, as Zheng (2002) pointed out, a fundamental tenet of Chinese mathematics teachers is that (teaching) should make students know how, and, further, make them know why. That is, mathematics teaching does not just hope to increase students' skilled computing capacity, but more their deeper understanding of mathematics. Cultivating students' mathematics thinking is most important. In fact, "mathematics teaching is the teaching of mathematics thinking" has been the consensus of Chinese mathematics teachers (Tao, 2008). What's more, from the aspect of developing students' mathematics thinking, helping students apply mathematics thinking to observing, analyzing and solving problems is also a main part stressed in former national syllabi or curriculum standards. Especially in the newest mathematics curriculum

standard, “basic thinking” becomes one of components of the “Four Basics”. In the teaching process, “Basic thinking” is the main line that integrates the entire contents (Shi & Liu, 2007).

Conclusion

An understanding of teachers’ views about what constitutes good mathematics teaching will not only help to illustrate teachers’ assumption about what should be accomplished during mathematics teaching, and what is the ideal goal of mathematics teaching, but also help us to understand good mathematics teaching practice in a certain context. Through semi-structured interviews with nine junior secondary school mathematics teachers, this study explored what constitutes good mathematics teaching from the perspectives of China’s social and cultural context. The fieldwork was done in Chongqing, a city in southeast part of China. Mathematics teachers interviewed had different teaching experience and qualifications. Their views were quite representative and were shared to some extent by their peers in other parts of China.

The findings of this study also suggest inconsistency with those of previous studies. For example, compared to American teachers’ view on what constitutes good mathematics teaching, Chinese teachers tend place more emphasis on students’ mathematics experience, the essence and characteristics of mathematics presented in the classroom, and the development of students’ mathematics thinking and problem solving methods. Regarding some points raised by American teachers, such as teacher’s knowledge, the application of information technology in the classroom, and the control of the class, none of these is the focus of Chinese teachers. This difference indicates that in different cultures, there are differences regarding what constitutes good mathematics teaching. Thus, in order to understand good mathematics teaching practice, teachers’ views should not be neglected.

According to the findings of this study, mathematics teachers in Mainland China consider the following practices as good mathematics teaching: actions that encourage the students’ active participation and engagement in class; stimulating students’ interests in mathematics; the connection of mathematics knowledge and the essence of mathematics; and the mathematics thinking and mathematics methods. That is, the notion of good mathematics teaching held by the nine secondary school mathematics teachers in Mainland China can be concluded as: “students as the masters of classroom”, “mathematics knowledge as the center of teaching” and “mathematics thinking and methods as the focus of teaching”.

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