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Personalized Education and the Teaching and Learning of Mathematics: an Australian perspective

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For many educators the idea of personalized learning may seem incompatible with the way they envisage the teaching and learning of mathematics. We argue in this paper that the idea of personalized education in mathematics needs to be taken seriously and that it should be seen as much more than dealing with individual differences or providing for individualized learning experiences. These old models are no longer appropriate for the teaching and learning of mathematics in the 21st century. Some recent ideas from the science of complex systems, especially that of emergent behaviors, may help enrich our concepts of personalized learning in mathematics.

Key words: personalized education; mathematics teaching and learning; emergent thinking

At the final stage of 20th century, Stephen Hawking pointed out that "I think the next century will be the century of complexity." When Einstein was asked what was most helpful to him in developing the theory of relativity, he replied, "Figuring out how to think about the problem." The challenges we face today and those we'll confront in the future require new ways of thinking (Sanders, 2003). Therefore, today we need to develop students in schools who are:

- Knowledge builders;
- Complex, multifaceted and flexible thinkers;
- Creative and innovative problem solvers;
- Effective collaborators and communicators; and
- Optimistic and committed learners

To achieve the above goals, a fundamental shift from traditional pedagogy based on full predictability and certainty to a new, more personalized teaching and learning way is essential. As for the discipline of mathematics, there are two contrasting views about (doing) mathematics: (1) Mathematics is intrinsically impersonal and it depends largely on the learning and reproduction of rules and procedures; and (2) the activity of doing mathematics is a highly personalized activity requiring flexibility, and a capacity for play and improvisation. To large extent, these two views can lead to different types of mathematics teaching and learning and personalized education has now attained more importance in mathematics classrooms as well.

A Common Interpretation of Personalized Mathematics Education

When "Personalized Education" is referred to, people usually and more normally have the understanding of that mathematics should be built on students' interests. Students need to see connections between mathematics and real life and other disciplines.

The Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority, 2010) presents two aspects of numeracy (mathematical literacy). "*One aspect relates to capacities that enhance the lives of individuals by allowing them to see the world in quantitative terms, communicate mathematically and interpret everyday information ...*" (p. 9.) Another earlier document, *Melbourne Declaration on Educational Goals for Young Australians* (Ministerial Council on Education, Employment, Training and Youth Affairs, 2008), emphasizes the importance of educating students to interpret the world mathematically, appreciating the elegance and power of mathematical thinking, experiencing mathematics as an enjoyable experience, and using mathematics to inform predictions and decisions about personal and financial priorities.

According to this interpretation, the teaching and learning of mathematics needs to include contexts where students can see applications of mathematics to real life. Contexts, such as art, music, architecture and sport, should be used to investigate mathematics. As a result, a lot of materials in real life can be used in mathematics classrooms. For example, natural patterns such as fans and hand-made brocades with different shapes and pictures can be used for introduction of teaching of abstract geometric patterns like circle, triangle, quadrilateral, and symmetry. Moreover, students need also to be encouraged to solve real life problems using learned mathematical knowledge. A study of the Pyramid provides a good context to raise problems containing knowledge of geometry and trigonometric function.

In Australian schools, teachers use various kinds of activities to foster students' personalized learning and engagement in mathematics, especially in their elementary and junior secondary classes. Project learning is a common way that is widely utilized in mathematics classrooms. For example, students are given opportunities to begin and complete works connecting mathematics to a school mathematics festival. Among these works, students may raise and then answer a lot of mathematical questions behind making a necktie. Mathematical investigation into building is also students' interest; some students are likely to use statistical knowledge to evaluate the best tennis and/or other sports players of all time.

Second Interpretation of Personalized Mathematics Education

Making connections with real life is essential to help students see the usefulness and many applications of mathematics. But the constraint on this kind of interpretation of personalized mathematics learning is that it leaves out important questions about the kind of mathematics and mathematical thinking that are necessary for young people to learn, adapt, and create and to communicate in the 21st century.

It may seem that improvisation and mathematics are two diametrically opposed ideas, the former being based on emergence and uncertainty and the latter on predictability and certainty (Askew, 2011). However, in this paper we argue that creative and successful mathematics teaching and learning must leave space for personalized thinking and personalized forms of expression, especially in order to foster a rich environment for students to develop and share mathematical thinking and reasoning.

Sawyer (2001) distinguishes between two types of effects: resultant and emergent effects. The first can be fully predicted by studying their constituent parts, like building a house. Emergent effects cannot be fully predicted by studying the constituent parts, like the creation of water by combining hydrogen and oxygen. In artificial intelligence systems, emergence is now a key idea in describing how intelligent behavior can emerge from simple, local rules of interaction, with no requirement for a central leader.

Insights into Personalized Education from the Study of Complex Systems

What is important is to avoid interpreting “personalized” as “individualized”. That interpretation places the focus on the teacher planning tasks for individuals who in turn work as individual elements on these assigned tasks. Instead, we make a strong case for the teacher seeking to promote personalized education by focusing on what different individuals can contribute to effective learning for the whole class. Recent studies of complex systems and their emergent effects can be helpful to draw attention to some key features which might underpin a definition of personalized education. Clearly, the classroom is not a complex system like a weather system or a flock of birds. The role of the teacher is a key difference, both in designing tasks with which students are to engage in, in establishing norms of public discourse by which those tasks can be discussed, orchestrating that discourse towards mathematical goals which regulate that discourse, and in evaluating the products of that discourse.

In complex systems, emergent effects occur that are not pre-determined. Emergent effects are neither good nor bad. Education is never value neutral. In mathematics education, what is being valued is always implicit, as is the case with any definition of personalized education. However, Davis and Sumara (2006) point to five conditions for emergence in the classroom. These five properties transcend individuals: diversity, redundancy, neighbor interactions, decentralized control and enabling constraints. They use this idea of complexity thinking in order "to shift attention from the individual student as the locus of learning to the social collective – the class – as the locus of learning (Newell, 2009, p. 1).

How might these elements inform our definition of personalized education? For diversity, teachers need to welcome the fact that different individuals bring diverse approaches to the thinking about and doing mathematics. For redundancy, the teacher knows that students have some things in common. According to Newell (2009), "in a complex system, however, it is not the existence of diverse talents among its agents, but the appropriate interaction of such talents that gives rise to adaptive behaviors that transcend those of the system's individuals" (p. 9). In this sense, it is argued that the understanding of the class can exceed that of the individuals within it. For redundancy, the teacher knows that individuals have enough in common in order to allow for productive interactions. But redundancy also makes it possible for individuals to compensate for each other's deficits and to work coherently together. Neighbor interactions are a key element of personalized education in so far as the teacher designs tasks not simply with a view to having individual students record their responses in isolation, but, so to speak, to enable students to share their responses with other students and to contribute those responses to a collective understanding. As these responses are shared and tested between individuals and across groups of individuals, some responses may be discarded, some may be recognized as successful without any one response being seen as the response which everyone must adopt. There may be several mathematically acceptable responses. The idea of decentralized control will be evident in the way that tasks are designed to provide an appropriate degree of freedom to explore ideas and solution approaches. Newell (2009) argues that "it is collision of diverse ideas and representations that may lead to a self-organization of the class's knowledge into something that transcends the sum of the students' individual knowledge (p. 11). Enabling constraints are related to the need to set a middle course between everyone working on the same task towards the same goal and everyone doing what they want to do. Enabling constraints are important and are expected to differ between classes and between different occasions for the one class.

Instances of Emergent Thinking in the Teaching of Number and Algebra

"Number and Algebra" is one out of three content strands in "Australian Curriculum: Mathematics" (Australian Curriculum, Assessment and Reporting Authority, 2012): "Number and Algebra are developed together since each enriches the study of the other … They (students) understand the connections between operations. They recognize pattern and … build on their understanding of the number system to describe relationships and formulate generalizations. They recognise equivalence and solve equations and inequalities … and communicate their reasoning (p. 3).” And, students (Year 4) are required to “Use equivalent number sentences involving addition and subtraction to find unknown quantities.” (p. 25). The following two of subtraction number sentences were given to a class of Year 6 Australian students where students were asked to find the missing number and to describe briefly the thinking they had used:

$$39 - 15 = 41 - \square$$

$$104 - 45 = \square - 46$$

The following seven instances of students’ responses show both resultant effects and emergent effects:

Students’ Responses Showing Resultant Effects

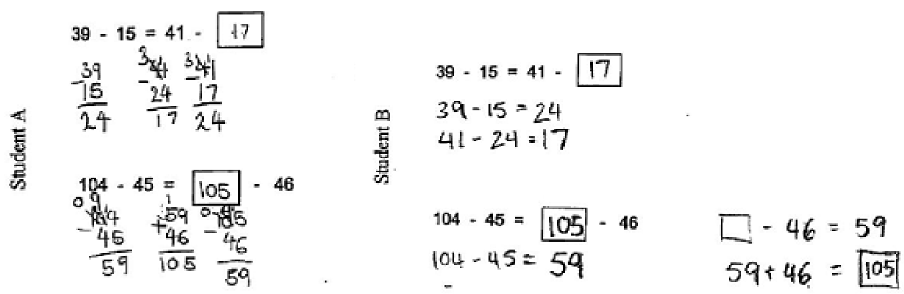

Figure 1. *Type 1 of students’ solutions.*

We can see from Figure 1 that two students, A and B, correctly found the missing number by calculating the result of the subtractions 39 –15, and 104 – 45, and then used these results to calculate the value of the missing numbers on the right hand side. These examples of students’ responses shown in Figure 1 to two subtraction problems may be considered as resultant effects. They show correct answers using standard solution methods that all students have been taught. The next group of examples of students’ work shows more personalized approaches. These students could have solved the missing number sentences using a “standard” computation approach as used above, but somehow chose to adopt alternative ways of seeing and solving the problems:

Students' Responses Showing Personalized Emergent Effects

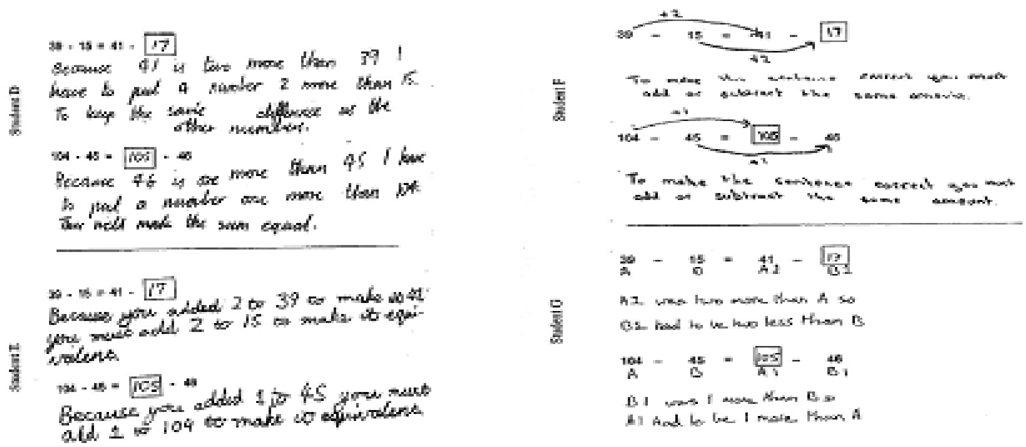

Figure 2. *Type 2 of students' solutions.*

Emergent thinking occurs when students are able to move beyond resultant thinking. This thinking is then shared with the class. Two students, D and E, successfully argued that since 41 is two more than 39 the missing number has to be two more than 15 to keep both sides equivalent. They applied similar reasoning to the second problem. Student F used arrows connecting the two related numbers (e.g. 39 and 41), and also connecting the other number (15) to the unknown number. Above the arrows Student F wrote +2 for the first problem and +1 for the second problem, obtaining correct answers. Finally, student G placed the letters A and A1 beneath 39 and 41, and B and B1 beneath 15 and the unknown number, and found correct values for the unknown numbers using an explanation based on equivalence and compensation. While the answer to the first problem is correct, Student G's written explanation contained a small error. When students are encouraged to write their own solutions, we see more clearly their personalized thinking. Various solution strategies were created by Students D, E, F and G. They could have performed a standard calculation solution. Students A and B need to attend to some features of what these four students have done. Standard calculation methods are important to teach and learn, but they should not become end points for students' thinking. Among these non-computational solutions, offered above, there is no best solution. Each one is different and acceptable.

Errors May Occur In Emergent Thinking

Of course, when students are encouraged to write their own solutions, some errors may emerge. For example, Student C refrained from calculating, attempting to use equivalence, but compensated in the wrong direction to get answers of 13 and 103 respectively (or mistook the operation for + instead of -

). These errors, however, can be very useful for teachers to understand students' thinking and for students to discuss why they occur. Here, for example, the incorrect answers 13 and 103 are not accidental errors. They show the student's incomplete attempt to understand the structure of the two problems.

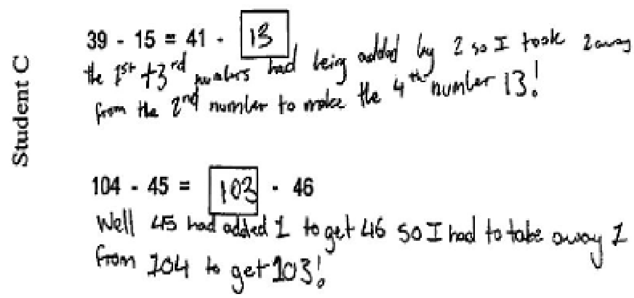

Figure 3: *Type 2 of students' solutions.*

Richly Varied Language and Representations

As we can see in Figure 2, the four solutions show powerful personalized reasoning, where each student expresses their thinking in subtly different ways. These students are able to move beyond computation and can identify important structural features that are generally present in these specific sentences. Moving beyond computation and identifying important structural features is a powerful way of thinking about number operations and equivalence. These ideas are very important for the transition from number to algebra.

Second, personalized thinking can be represented in different ways. This contrast with resultant thinking where everyone is expected to follow the required method. Students need to be encouraged to extend their personalized thinking to other kinds of number sentences and operations. In this way, personalized thinking becomes complex, multifaceted and flexible. How students extend their personalized thinking is not like learning a recipe. Personalized thinking is accompanied by different forms of justification and explanation.

Thirdly, Mathematics is about recognizing relationships, perceiving general properties which are present in specific situations and reasoning on the basis of these identified (general) properties.

Two Contrasting Teachers' Views of Treat Students' Emergent Thinking

We showed the above seven solutions and asked some teachers what they would do with these responses if they occurred in their classroom. In particular, we asked: How would you move students' thinking forward in regard to these types of questions?

Responses came from two contrary directions, one teacher said: "We need to engage students in identifying and using arithmetic relationships ... to solve problems without calculating. I encourage them to use a repertoire of strategies – (using) guess-guess-check (systematic trial and error), logical reasoning and inverse operations to solve a range of number sentences." A completely different view was given by another teacher who said: "The solutions of students who used the regular calculation method need to be energetically popularized (to the whole class). Most students can master them ... The deep thinking of students who use relational thinking deserve praise, *but* it shouldn't be introduced, because it is not very good and some students may be confused by it and cause mistakes."

The second view from teachers may be somewhat frustrating but not unpredictable. There are plenty of reasons that possibly hold teachers back from valuing these alternative methods and ways of thinking:

(1) Calculation methods are very safe for students to get the correct answer and make classroom teaching and management simpler;

(2) The teacher needs to focus on teaching one method well;

(3) Reproduction methods are important for elementary and junior high school students;

(4) Personalized thinking can be left to higher and later stages;

(5) For most exams, correct answers are very important, regardless of students' mathematical thinking;

(6) Pressure from principals and parents to have students succeed in exams doesn't leave teachers enough freedom or time to focus on students' personalized mathematical thinking; etc.

Educational Implementations

There are at least three kinds of values – mathematical values, pedagogical values and human values – which are included in our definition of personalized mathematics education:

First, mathematical values should be evident, such as (1) Mathematical thinking needs to be flexible; (2) mathematical expression can also be flexible; (3) moving beyond seeing specific instances to generalization; (4) moving from generalizations to specific examples and conditions; and (5) making relationships visible, perceiving properties, and reasoning on the basis of identified general properties.

Second, classroom cultural values should be evident, such as: (1) Move beyond seeing answers simply as "correct" or "wrong"; (2) listen carefully to other students' voices; (3) express ideas clearly to friends and classmates; and (4) avoid underestimating classmates' ideas.

Third, human values should be evident, such as: (1) Using previous knowledge and experience is often needed to solve a new problem; (2) when two or more people work together, everyone's thinking can be improved; (3) learning from errors is important; and (4) in order to clarify and understand A, knowing and being able to think about what is non-A is important.

An appreciation of mathematical structure and generalization is essential for a 21st century curriculum (Australian Curriculum, Assessment and Reporting Authority, 2010) and requires personalized emergent thinking. As a result of learning mathematics in school, all students should realize that mathematics is relevant to them personally and in their community. They need not only to acquire mathematical knowledge, but ways of thinking and confidence to use mathematics in order to engage in the mathematical study needed for further education and employment; to possess sufficient command of mathematical expressions, representations and technology to interpret information in which mathematics is used; and to communicate mathematically to a range of audiences (Australian Education Council, 1991, p. 15; OECD, 2004). Australian researchers now strongly advocate that these kinds of thinking and discourse should be valued in the normal mathematics classroom and in all common topics (Askew, 2011; Mulligan, 2010; Warren, Miller, & Cooper, 2011; Mason, Stephens and Watson, 2009). This requires us to read curriculum documents with a new and wider vision. The spectacles of the 19th and 20th are inadequate for this purpose. Insights from complexity science provide a metaphor (not a scientific theory) and a way of seeing the mathematics classroom as a complex adaptive system. In this context, personalized education in mathematics can take on a stronger meaning that allows us to move away from the old discourses of catering to individual interests and individualized learning to a more relevant model of personal learning as a component of collective learning, where the whole may be thought of as greater than the sum of its parts.

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