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The Development of Chinese Students' Understanding of the Concept of Fractions from Fifth to Eighth Grade

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The purpose of the present study was to investigate fifth to eighth grade Chinese students' understanding of the concept of fractions. One hundred and ninety nine students were tested using a questionnaire that required them to make judgments about fraction size and to order a set of given fractions. The participants were also asked to justify their responses. Primary findings revealed that students progressed along three conceptual levels in grasping the concept of fraction: at level 1, fraction is viewed as consisting of two independent numbers; at level 2, fraction is viewed as indicating part-whole relations; and at level 3, fraction is viewed as indicating the ratios between the numerator and the denominator. Results also indicated that, from fifth to eighth grade, Chinese students made significant progress in understanding the concept of fractions.

Key words: fraction concept, misconception, whole number bias

Introduction

As a number concept, fraction indicates proportional relations between part (numerator) and whole (denominator). Mastery of this concept was considered a milestone in students' number development (Ni, 1999), and makes an important effect in understanding the nature of continuity and divisibility of number. However, the concept of fractions was found to be the most complex and abstract concept in the primary grades (Bulgar, 2003; Saxe, Taylor, Mclntosh, & Gearhart, 2005) and was difficult to learn (Hartnett & Gelman, 1998; Mack, 1995; Smith, Solomon, & Carey, 2005). Even adults can not understand the concept well (Gelman, 2006). Recently, researchers generally agreed that fractions can be conceptualized as a set of interrelated meanings: part-whole, ratio, operator, quotient, and measurement (Charalambos & Pitta-Pantazi, 2007). The part-whole meaning emphasizes on the relationship between the part and the whole, and fraction represents a comparison between the numbers of parts of a partitioned unit to the total number of parts in which the unit is equally partitioned. The ratio meaning is defined as a comparison between two quantities, so, it is a comparative index, rather than a number. The operator meaning emphasizes fractions as functions applied to some number, object, or set. Fraction can also be represented as a result of a division situation, and that is the quotient meaning. Furthermore, for the measurement meaning, fraction is considered as a certain number, which can be located on the continuous number line, rather than a combination of two whole numbers.

Children cannot simultaneously understand these five meanings. Compared to others, the part-whole meaning is easier to understand because this meaning can be assimilated with their prior knowledge of natural numbers (Ni, 1999). Moreover, the part-whole meaning is usually emphasized in fraction teaching, and teachers often use the part-whole meaning to introduce the concept. For example, 3/10 means three apples of ten apples. In contrast, the measurement meaning is difficult for children to understand because it requires that children view a fraction as one number (instead of two independent numbers) and as a point on the continuous number line. Children need to undergo a conceptual changing process to accommodate this meaning (Gelman, 2006). Specifically, children must expand their concepts of numbers as a discrete system, which is a characteristic of natural numbers, to a continuous system, which is a characteristic of rational numbers. The differences between natural numbers and fractions often cause students to make errors when carrying out calculations involving fractions.

Therefore, an important question that concerns how students acquire accurate concepts of fractions and to what degree the acquisition process involves conceptual change needs to be addressed. Based on Piaget's cognitive constructivist theory and the concepts of assimilation and accommodation, Vosniadou (1994, 2001, 2003) developed a conceptual change theoretical framework to explain how students develop science concepts. According to the theoretical framework, knowledge acquisition process involves enriching existing conceptual structures and reorganizing existing knowledge. Learning which requires reorganization of existing knowledge is much more difficult than that which entails enrichment. In other words, accommodation is more difficult than assimilation. It is very likely that children create misconceptions in the process of reorganization when they attempt to change existing concepts. This theoretical framework can explain processes that children use to acquire novel knowledge of astronomy, physics, chemistry, and biology (Vosniadou & Vamvakoussi, 2006). In a similar vein, the conceptual change theory can also provide acceptable explanations for comprehending the conceptual journey that students undertake to tackle mathematics concepts such as fractions (Stafylidou & Vosniadou, 2004), negative signs (Vlassis, 2004), and the concept of number (Merenluonto & Lehtinen, 2004).

Specifically with the concept of fraction, Stafylidou and Vosniadou (2004) contended that the reason why children have difficulty in understanding fractions is because before learning fractions, they have established a solid knowledge of natural numbers. It is the existing knowledge of natural numbers that induces a robust tendency in children to use a discrete-unit counting scheme to interpret fractions. Stafylidou and Vosniadou (2004) described a conceptual hierarchy with fraction, which can be divided into three levels (descriptions of these levels are discussed later in the method section). However, whether that conceptual framework is applicable to Chinese children needs to be verified. Cross-cultural studies showed that Chinese students often outperform U.S. and Europe students on international tests in mathematics including fractions concepts (Fang, Tong, & Liu, 1988; Stevenson, Lee, Chen, Lummis, Stigler, & Han, 1999; Wu, Li, & Haffner, 2006). Some scholars believe that differences among different languages in representing and conveying concepts are important factors affecting fraction learning (Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999). The English fraction-naming system differs from East Asian languages. For example, in Korean, Chinese, and Japanese, the notion of fractional parts is explicitly embedded in fraction names. For instance, in the Chinese and Korean languages, one fourth is roughly translated "of four parts, one," which explicitly conveyed the part-whole meaning. In contrast, in English, 1/4 is read as "one fourth" or "a quarter," which does not reflect the part-whole meaning. As supported by research on learning (Miura et al., 1999), the fraction names which reflect transparent part-whole relations facilitate East Asian children's understanding of fractions. Therefore, the part-whole meaning and even other meanings are easy to understand for Chinese students.

There have been various studies in China about fraction learning and teaching including studies on students' understanding of the multiple meanings of fractions (Ni, 1999; Zhang, Liu, & Wang, 1982), the status of fractions teaching (Su, 2007), and the review of the development of the children's concept of fractions (Yang & Liu, 2008). However, seldom has study been done to explore Chinese students' fraction concept with the explanatory framework developed by Stafylidou and Vosniadou (2004). The purposes of the present study include a) investigate whether the explanatory framework applies for the development of Chinese students' fraction concepts, b) examine how fifth to eighth grade students' fraction concepts distribute at different developmental levels of the explanatory framework, and c) identify typical errors and fraction misconceptions for Chinese students and provide corresponding diagnostic suggestions.

Method

Participants

One hundred and ninety-nine students participated in this study: 52 fifth graders (27 boys and 25 girls, mean age 11 years and 3.6 months); 47 sixth graders (33 boys and 14 girls, mean age 12 years and 7.2 months); 46 seventh graders (24 boys and 22 girls, mean age 13 years and 6 months); 54 eighth graders (28 boys and 26 girls, mean age 14 years and 9.6 months). The students came from a private school in north China that serves middle class parents. This school has the primary school, middle school and high school all in one.

Questionnaire

The questionnaire that was designed by Stafylidou and Vosniadou's (2004) was adopted to examine Chinese students' understanding of fractions (see Table 1). This questionnaire consisted of two sets: in the first set, students were asked to write the smallest and biggest fraction they could think of and to justify their answers. In the second set, they were asked to order and compare fractions (e.g., 5/6, 1, 1/7, 4/3), and to give an explanation for their answers.

Codings

We adopted the explanatory framework that was developed by Stafylidou and Vosniadou's (2004). According to the framework, children first think that the numerical value of a fraction was represented by two independent numbers. Specifically, when they order or compare the fractions, they only consider the size of numerators or denominators. This level was called *Explanatory Framework of Fraction as Two Independent Numbers* (Stafylidou & Vosniadou, 2004). Furthermore, there are two different types of performances nested under this level: a) children believe that the value of a fraction increases when the numerator or the denominator increases; b) to the contrary, children consider that the value of a fraction increases when the numerator or the denominator decreases.

Table 1

The Two Sets of the Questionnaire

	Questions
SetI	Question 1: Write the smallest fraction that you can think of, and why do you think this is the smallest fraction?
	Question 2: Write the largest fraction that you can think of, and why do you think this is the largest fraction?
Set II	Order fractions. Please order the numbers 5/6, 1, 1/7, 4/3 from the smallest to the biggest one and write down why did you put them in this order.
	Compare five pairs of fractions. Please choose the biggest fraction from each pair of fractions: 4/5-2/5; 4/15-4/7; 5/8-4/3; 2/7-5/6; and 2/3-4/9. How did you choose these?

Though both types of performances are incorrect, they reflect two types of conceptual understanding. Specifically, children of the first type order fractions influenced by the knowledge of whole numbers, but the kind of mistake children of the second type make reflects informal understanding of the partition. For the fractions questionnaire, there are two response categories in set I. In response category 1, children believe that when the numerator or denominator is larger, the value of the fraction larger. For example, they believe that 1/1 is the smallest fraction, whereas 10000/100000 is the biggest fraction. On the contrary, children who are grouped in response category 2 consider that as the numerator or denominator decrease, the value of a fraction increases. For example, children think the smallest fraction is 10000/100000 and the biggest fraction is 1/1. In set II, there are three response categories according to the performance of children. In response category 1, when the numerator or denominator of a fraction increases, the fraction itself increases, and the unit is the smallest of all fractions. In response category 2, when the numerator or denominator increases, the fraction increases, but children consider the unit as the biggest of all fractions. In response category 3, children also believe that as the numerator or denominator increases, the fraction increases, but they do not order the unit.

At the second level, which was called *A Fraction is a Part of a Whole*, children view the numerator and denominator of a fraction as representing a part-whole of a natural object. Children at this level usually use discrete objects to represent units, and think that a unit is bigger than all fractions. In other words, there is no use of improper fractions (e.g., 11/6). This level is further divided into three subcategories according to Stafylidou and Vosniadou (2004). In subcategory 1: *Naïve Part of a Unit*, children seemed to have grasped some ideas of a fraction as part-whole meaning, but, they still maintained some beliefs from their prior knowledge. For example, some children think that the biggest fraction is 1/100, and the smallest fraction is 1/2, because "when a pie is divided into 100 pieces or 2 pieces, 100 pieces is bigger". In subcategory 2: *Advanced Part of a Unit*, children often use the part "that we take" to represent the numerator, and view the denominator as the parts into which the unit is divided. Therefore, children believe that all fractions are smaller than a unit. For example, some children think that 1/10 is the smallest fraction, and 9/10 is the biggest fraction, because "when the unit is divided into 10 pieces, we take at most 9 pieces". In subcategory 3: *Sophisticated Part of a Unit*, children obtain more sophisticated knowledge such as the concept of improper fractions. Reflected in testing performances for set II, children in this category can correctly order the fractions and the units by using the method of reducing all fractions to a common denominator. For set I, however, they still think that unit is the biggest fraction. A fifth grader represented 11/10 as "11 pieces of a pie with 10 pieces," which indicates that he still tried to use part-whole concept to interpret fractions. Specifically, there are two response categories in set I. In response category 3, the students believe that a fraction is a part of a whole and that the value of a fraction increases as the size of the numerator approximates the size of the denominator. For example, according to a seventh grader, "the smallest fraction is 1/1000, because the unit is divided into 1000 pieces, and I take only one of them." while the biggest fraction is "999/1000, because the unit is divided into 1000 pieces, and I take almost all of them". In response category 4, children for the first time conceptualize the biggest fraction to be the fraction whose numerator is a bigger number than the denominator. In set II, there are two response categories. In response category 4, children consider that as the numerator or denominator decreases, the value of fractions increase, and they put the unit at the end believing that it is bigger than any fraction. Response category 5 includes responses that give the correct ordering of the fractions except for the unit.

The third level was called *Relation between Numerator/Denominator*. Children's concept of fractions is radically different at this level. They now consider a fraction as a relation between the numerator and denominator. Specifically, "they consider a fraction smaller than the unit when its numerator was smaller than its denominator and bigger than the unit when the numerator was bigger than the denominator" (*p*. 513, Stafylidou & Vosniadou, 2004). The third level is divided into two subcategories. Subcategory 1: *Relation of Two Numbers without Infinity*, in which children do not refer to the unbounded infinity at all; in other words, they believe that there is a certain smallest and biggest fraction respectively. Subcategory 2: *Relation of Two Numbers with Infinity*, in which students develop a more complete knowledge of the fraction. These students absolutely understand the quotient and measurement meanings, and they believe that the smallest or the biggest fraction is an unbounded infinite number. There are two response categories in set I. In response category 5, children think that there is a unique smallest fraction but no biggest fraction. Whereas in response category 6, children believe that there is no smallest and biggest fraction. In set II, there is only one response category. Children who are grouped in response category 6 can correctly order the fraction and the unit.

Based on the above explanatory framework, students' performances and answers were grouped into one of the three levels, and further into the six subcategories. A category 7 was created and included all the responses (e.g., "I don't know it is so" and "I think so") that could not be categorized into the identified categories.

Data Analysis

We conducted data analysis with SPSS 16.0. Specific analysis includes a) frequency and proportion calculation for determining level distributions of students' fraction concept among the different categories of the explanatory framework, and b) *Exact Kruskal-Wallis* test for examining level distributions among different grades.

Procedures

All participants completed the questionnaire in a group during their class time, and participants at each grade were divided into two sub-groups so that the number of participants at each sub-group was less than 30. Most students finished the questionnaire in approximately 20 minutes. Two trained graduate students in psychology supervised the testing.

Results

Students Distribution in the Explanatory Frameworks of Fraction

Table 2 presents the frequency/percentage of students distributed in the various categories described above. Generally speaking, across all grades, 10% of students reached the first explanatory framework, 23.6% of students reached the second explanatory framework, and 58.8% of students reached the third explanatory framework. Specifically, the number of lower-grades students in the first explanatory framework is larger than the number of higher-grades students: 19.2% of fifth students were grouped in the first explanatory framework, but only 3.7% of eighth students were grouped in this level. On the contrary, the number of higher-grade students in the third explanatory framework is larger than the number of lower-grade students: 74.1% of eighth graders reached the third explanatory framework, but only 34.6% of fifth graders did so. We performed the *exact Kruskal-Wallis rank* test. The exact K-W test results showed that grade was a statistically significant factor (H = 24.83, p < .001). According to the performance of students, we can see the older students understand the concept of fractions more accurately.

Set I: Smallest / Biggest Fraction

Table 3 presents the frequency/percentage of students distributed in the various categories described above in set I. Students were asked to answer which fraction is smallest or biggest. As demonstrated in table 3, there were 49.7% of students classified in category 6, which is the highest level. In category 6, the percentage of students from fifth grade to eighth grade were 13.5%, 76.6%, 41.3% , and 72.2%, respectively. In contrast, there were only 11% of students classified in the lower categories of 1 and 2.

Set II: Comparison of Fractions

Table 4 shows the frequency/percentage of students distributed in the different categories in set II. For this set, students were asked to compare and order fractions. Overall, 79.4% of students could correctly order the fractions and the unit. The percentage of students who could do so from fifth to eighth grade was 82.7%, 70.2%, 82.6%, and 77.8%, respectively. The results indicated that students can correctly adopt the principle of ordering fractions.

Table 2

Explanatory Frameworks for the Numerical Value of a Fraction

Explanatory frameworks	5th Grade	6th Grade	7th Grade	8th Grade	Total
A. Two independent numbers	10(19.2%)	3(6.4%)	5(10.8%)	2(3.7%)	20(10.0%)
(1) Two independent numbers whose value increases as the numerator (or the denominator) increase	4(7.7%)	2(4.3%)	2(4.3%)	2(3.7%)	10(5.0%)
(2) Two independent numbers whose value increases as the numerator (or the denominator) decrease	6(11.5%)	1(2.1%)	3(6.5%)	0(0%)	10(5.0%)
B. Part of a whole	19(36.6%)	5(10.6%)	14(30.4%)	9(16.7%)	47(23.6%)
(1) Naive part of a unit	0(0%)	0(0%)	2(4.3%)	3(5.6%)	5(2.5%)
(2) Advanced part of a unit	3(5.8%)	5(10.6%)	1(2.2%)	3(5.6%)	12(6.0%)
(3) Sophisticated part of a unit	16(30.8%)	0(0%)	11(23.9%)	3(5.6%)	30(15.1%)
C. Relation between numerator/denominator	18(34.6%)	35(74.5%)	24(52.2%)	40(74.1%)	117(58.8%)
(1) Relation of two numbers without Infinity	11(21.2%)	5(10.6%)	5(10.9%)	2(3.7%)	23(11.6%)
(2) Relation of two numbers with Infinity	7(13.5%)	30(63.8%)	19(41.3%)	38(70.4%)	94(47.2%)
D. Mixed—could not be categorized	5(9.6%)	4(8.5%)	3(6.5%)	3(5.6%)	15(7.5%)
Total	52(100%)	47(100%)	46(100%)	54(100%)	199(100%)

Table 3

Numerical Value of Fraction-Set I: Smallest/ Biggest Fractions Frequency and Percentage of Responses Per Grade

Categories of responses	5th Grade	6th Grade	7th Grade	8th Grade	Total
1.The value of a fraction increases when the numbers that comprise it increase	0 (0%)	1 (2.1%)	2 (4.3%)	2 (3.7%)	8 (4.0%)
2. The value of a fraction increases when the numbers that comprise it decrease	1 (9.6%)	0 (0%)	5 (10.9%)	2 (3.7%)	14 (7.0%)
3. The value of the fraction increases as the size of the numerator approximates the size of the denominator	13 (25.0%)	0 (0%)	2 (4.3%)	2 (3.7%)	17 (8.5%)
4. The value of the fraction increases as the size of the numerator becomes bigger than the size of the denominator	16 (30.8%)	5 (10.6%)	9 (19.6%)	3 (5.6%)	34 (17.1%)
5. There is no biggest fraction (infinity only for big fractions)	1 (1.9%)	2 (4.3%)	2 (4.3%)	2 (3.7%)	7 (3.5%)
6. There is no smallest/biggest fraction (infinity for all fractions)	7 (13.5%)	36 (76.6%)	19 (41.3%)	39 (72.2%)	99 (49.7%)
7. Mixed-could not be categorized	7 (13.5%)	3 (6.4%)	7 (15.2%)	4 (7.4%)	20 (10.1%)
Total	52 (100%)	47 (100%)	46 (100%)	54 (100%)	199 (100%)

Table 4
Numerical value of fraction—Set II: Comparison of fractions frequency and percent of responses per grade

Categories of responses	5th Grade	6th Grade	7th Grade	8th Grade	Total
1. The value of the fraction increases as either the numerator, or the denominator increase—the unit is the smallest of all fractions	0 (0%)	2 (4.3%)	0 (0%)	2 (3.7%)	5 (2.5%)
2. The value of the fraction increases as either the numerator, or the denominator increase—the unit is bigger than all fractions	1 (1.9%)	2 (4.3%)	0 (0%)	1 (1.9%)	5 (2.5%)
3. The value of the fraction increases as either the numerator, or the denominator increase—ordering without the unit	0 (0%)	1 (2.1%)	0 (0%)	0 (0%)	1 (0.5%)
4. The value of the fraction increases as either the numerator, or the denominator decrease—the unit is bigger than all fractions	3 (5.8%)	1 (2.1%)	0 (0%)	0 (0%)	4 (2.0%)
5. Correct ordering only for the fractions	0 (0%)	5 (10.6%)	2 (4.3%)	6 (11.1%)	11 (5.5%)
6. Correct ordering for the fractions and the unit	43 (82.7%)	33 (70.2%)	38 (82.6%)	42 (77.8%)	158 (79.4%)
7. Mixed-could not be categorized	5 (9.6%)	3 (6.4%)	6 (13.1%)	3 (5.5%)	15 (7.5%)
Total	52 (100%)	47 (100%)	46 (100%)	54 (100%)	199 (100%)

The Different Performance between Set I and II

There were differences in children's performances between set I and set II. The number of students (158 students) who correctly ordered fractions in set II was larger than the number of students (99 students) who correctly answered the questions in set I. The difference, however, does not indicate that there is inconformity in our coding between set I and set II. Rather the reasons lie in the different conceptual facets that were tested in set I and set II. Specifically, the questions in set I emphasized concepts such as infinity; whereas the questions in set II emphasized operational fluency. To further pursue this matter, we first chose the students who had correct answers in set I, then explored their performance in set II. Results show that from fifth to eighth grade, the percentage of students who correctly ordered the fractions was 85.7%, 75%, 89.5%, and 87.2%, respectively. Then, we chose the students who could correctly order the fractions in set II, then explored their performance in set I. From fifth to eighth grade, the percentage of students who had correct answer in set I was 14%, 81.8%, 42.1%, and 81%, respectively. From the different patterns of data, we can see that although some students could order and compare fractions fluently, they still had difficulty in mastering the concept of fractions.

Discussion

Developmental Levels of Explanatory Framework of Fraction

Findings from our study confirmed the three developmental levels of explanatory framework of fractions (Stafylidou & Vosniadou, 2004) with Chinese students. As did their Greece counterparts, Chinese students in the initial explanatory framework *Fraction as Two Independent Natural Numbers* demonstrated two almost contradictory beliefs. One is the belief that "the value of the fraction increases as the value of the numerator (or denominator) increases" and the other is the belief that "the value of the fraction increases as the value of the numerator (or the denominator) decreases". While the former belief is consistent with the laws of natural numbers (the larger the number, the bigger the value it represents), the latter belief is a misconception. For example, a student reasoned that "the bigger the numbers a fraction has, the smaller it is," or "the smaller the numbers a fraction has, the bigger it is." Those reasonings indicated children's incomplete understanding of the knowledge of "the more the number of partitions, the less each partition has."

The second explanatory framework Fraction as *Part of a Unit* seems to be a transitional phase between the initial framework and the ultimate framework. The concept that a fraction is a part of a whole is not adequate enough to enable students to solve all of the problems, especially the improper fraction problems. However, when children begin to learn the concept of fractions, Chinese teachers usually teach the part-whole meaning, and primarily use discrete objects for modeling fractions which may hinder students' further development to the ultimate stage.

The ultimate framework is to interpret fractions as a quotient or ratio that can be compared, added, subtracted, multiplied or divided. In our sample, 58.8% of the students used this framework to solve problems. This proportion is much larger than that of the initial and secondary framework (20% and 23.6%).

Misconceptions of Fractions

All children's conceptual frameworks develop from their experiences and are built upon accumulated naïve ideas. However, frequently, their intuitive understanding of the world around them does not agree with the scientific explanation. During the process of learning mathematical concepts, the generation of misconception is a noteworthy problem. Scholars and teachers can't focus only on whether the answer is correct or wrong, but must also pay attention to the logic of the misconception and how children change their conceptions from naïve ideas to the scientific concept. Chinn and Brewer (1993) discussed the situation when children acquired new knowledge. They found that children may have three reactions to new knowledge: a) children may ignore, reject, or exclude the new knowledge, therefore the misconception was maintained; b) they may also reinterpret the new scientific concepts according to their prior framework, or change their own theory surfacely, but actually they do not change their misconception; c) children may change their existing knowledge and form the new schema. In our study, these three conditions were embodied in three conditions. The first condition: children only focused on the numerator or denominator of fractions when they ordered or compared the fractions. It seemed that they ignored the information that teachers taught to them. This misconception derived from the qualitative differences between the knowledge of fractions and the prior knowledge of natural numbers. The second condition: although children regard the fractions as part-whole meaning, they still use the principles of natural numbers to order the fractions, which is called whole number bias (Ni & Zhou, 2005). Whole number bias is defined such that when beginning to learn the fraction concept, children often use previously formed single-unit counting schema to assimilate it. The third condition: children who acquired the multi-concepts of fractions can solve the problems about fractions correctly.

Moreover, we also found that some students had the misunderstanding of unit 1. In the concept of fractions, 1 represents not only a certain number, but also represents a unit. For the former meaning, 1 is the smallest natural number, but the largest number of the proper fraction; for the latter meaning, 1 is not a number, it is a unit or a whole. Because of this complicated meaning, children may make mistakes easily. For example, a fifth grader said “1 is the smallest fraction and the largest fraction”. In addition to this, some students did not know the value of the unit, and therefore they could not justify which fraction was larger. In particular, when the unit of 4/5 is 1 (natural number) and the unit of 2/5 is 2, 4/5 is equal to 2/5. Therefore, students had difficulty in understanding the concept of the unit. The difficulty is ubiquity in learning the concept of fractions (Chinn & Brewer, 1993). Many students blurred the boundaries between 1 as a whole number and 1 as a unit of fraction, ignoring that 1 is an absolute value and thereby made mistakes in solving problems. In sum, the unit 1 is the key concept in learning fractions, and it is the bridge between the natural number and the rational number.

Performance on Ordering and Comparison Tasks

The result also showed that Chinese students were more familiar with the calculation and comparison rules than the concept of fractional numbers: those who got the wrong answer in set I were able to correctly answer the ordering problems in set II; the proportion of 85% right answer rate was much higher than the proportion of students reaching the ultimate framework of understanding, which was 47.2%. Our study results were consistent with those shown by Moss and Case (1999) and Su (2007) in that students were better at calculation than at the understanding of the fraction concept. Moss and Case (1999) suggested that the difficulty with learning fractions was due to the fact that instruction paid more attention as to how to correctly solve the problem rather than understand the meaning deeply. Su (2007) also believed that students grasped the rules of operation more expertly than understanding the meaning of the fraction.

The number of 1 is re-mentioned in this part as it is commonly acknowledged as the key number in the learning of fractions. When making fraction comparisons, 1 can be seen as a reference point. However, only those with framework in the third level were able to use this strategy to save time and energy. The reason is that they truly understood that fractional numbers are in fact numerical values.

Educational Implications

The present study generates three specific implications for teaching of fractions.

First, teachers should provide multiple representation models and help students to form accurate and solid conceptual understanding of fractions. For the current teaching practice, the part-whole meaning of fractions is over-emphasized but the other meanings (e. g., ratio, operator, quotient), especially the measurement meaning, are largely neglected. Although it is logical to introduce fraction with its part-whole meaning by using region, line segment, discrete object models, it is necessary to further develop students' conceptual understandings by exposing them to the other representation models. For instance, the number line model is an ideal instrument representing the measurement meaning (Ni, 1999). Zhang and colleagues (Zhang, Sun, Huang, Huang, & Tang, 2009) have found that the number line model integrates whole numbers, fractions, and decimal fractions into one system and therefore has great potential in promoting children's understanding of rational numbers, and therefore should be emphasized in teaching.

Second, teaching fraction calculation should build upon students' conceptual understanding but not rote formula application. As advocated by National Council of Teachers of Mathematics (2006), sixth grade students should use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. In China, the primary school mathematics curriculum standard (Ministry of Education of China, 2001) also suggests that teachers create meaningful problem solving situations to cultivate students' understanding of the rationales behind arithmetic operations.

Lastly, teachers need to help students understand and differentiate the concept of 1 as a number and the concept of 1 as a unit. In order to construct the correct concept of fractions and solve fraction problems correctly, students need to understand that the unit 1 may be composed of one object or multiple objects.

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