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Semantics and Syntax: A Theoretical Model for How Students May Build Mathematical Mis-Understandings

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In this manuscript we examine the intersection of reading and mathematics skills to theorize a model that may account for student understandings, whether they be correct or incorrect. The theoretical model relies on research from reading, special education, psychology, mathematics, and mathematics education in the formation of the theoretical framework. By no means is this manuscript intended to draw conclusions but to propose a synergistic and interdependent idea to focus researchers from across several disciplines and to issue a challenge to rigorously explore the model in various contexts with broad levels of implementation.

Key words: intersection of reading and mathematics, synergistic and interdependent ideas, teaching and learning.

Introduction

The need to improve mathematics performance in the U.S. is paramount (Mullis, Martin, & Foy, 2008; Schmidt, Houang, & Cogan, 2002). There have been many attempts to move the U.S. along the continuum toward greater success. These innovations have included research based textbooks (Grant, Kline, & Weinhold, 2002; Li, Ding, Capraro, & Capraro, 2008; McNeil et al., 2006), teacher professional development (Darling-Hammond & McLaughlin, 1995; Schneider, Krajcik, Marx, & Soloway, 2002), and even accountability testing (No Child Left Behind Act, 2002). These by no means cover all the broad spectrum attempts to help students build mathematical proficiency but may encompass the ones receiving the greatest attention whether that yardstick be funding or representation in the extant literature. However, as the US struggles with greater and greater language diversity in the classroom and with the impact of texting (English) and e-mail (English) being acquired as legitimate communication forms, it is paramount to examine the co-dependent relationship between general communication language skills and mathematical language skills. Further, arithmetic computation has long been abandoned in assessments and replaced by highly contextualized problems surrounded by text and it is in this setting in which U.S. progress has been flat (Mullis et al.).

For the purposes of this paper we use mis-understandings as tongue in cheek term, because whether or not an understanding is correct or incorrect, it is still an understanding and it is only during the overt externalization of it that someone (the person or the evaluator) can make a determination as to the quality of the understanding (internal representations) being one that is likely or unlikely to facilitate admissible solutions. Of course, just being able to recognize and say the words (represented by letters) may be considered by some as reading, but without comprehension the effort is futile. Therefore, those exhibiting good comprehension are active readers. They relate ideas in text to their prior knowledge, construct images, and generate summaries or analogies. They monitor their awareness during reading which affects how they process the text. This highly specialized focused on the here-and-now metacognition in the form of awareness is always being generated as the good reader reads. Strategies that serve these purposes in solving mathematics problems are: (1) Solve a simpler problem by transforming a difficult or complicated word problem into a simpler problem with similar steps and operations; then, this is transferred to conceptualizing the complicated word problems. (2) Guess and check enables the approximation of a solution for a word problem and activates prior knowledge for the elaboration of the process to be applied to the original problem and to construct a model for solution. (3) Logical reasoning is used to determine the key concept carrying words in a mathematic word problem and what essential events are listed.

Semantics of Reading

Research has informed us that semantics, word identification, and vocabulary (e.g., repeated readings, rhymes), as shown in Figure 1, are essential cognitive features in word problem solutions (Capraro, Capraro, & Cifarelli, 2007) just as they are in reading comprehension and understanding (Pressley, 2002; Smagorinsky, Cook, & Reed, 2005). Littlefield and Rieser's (1993) semantic features model of discriminating information advanced the credibility and paramount importance of these features, and presented evidence that the model fits the discrimination performance of students who are and who are not successful with mathematics word problems. Their model demonstrated how successful students analyze the problem text and questions into semantic units, including the actions, agents (persons/things carrying out the action), objects acted upon (these typically correspond to the units of measure), as well as the time and place of actions. Successful students identify relevant information by searching the problem text for information; trying to match the values of semantic features requested in the question with those in the problem text. In contrast, less successful mathematics students are much more likely to base their discriminations on surface level aspects of the text such as the *position of information* within the problem statement (e.g., consistently selecting as relevant the first and last numbers). This strategy is almost identical to those employed by readers who have good word recognition skills but lack a cognitive understanding that the purpose of reading is comprehension. These types of readers are dealing with the surface or text level features and failing to connect with their prior knowledge to build meaning (Afflerbach, Pearson, & Paris, 2008; Jetton, Rupley, & Willson, 1995).

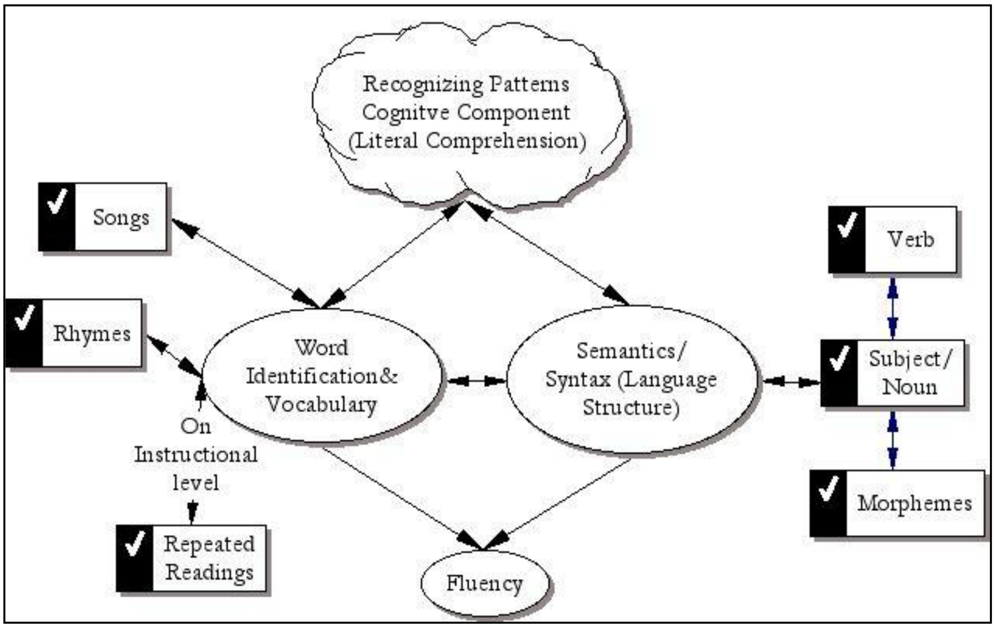

Figure 1. **Essential cognitive features in word problem solutions.**

Conceptual mathematics understanding and conceptual mathematics knowledge refers to the understanding of ideas and generalizations that connect mathematical constructs (Ashlock, 2001) and is rich in relationships (Hiebert & Lefevre, 1986). Conceptual understanding is similar to comprehension in reading with reading comprehension defined as making sense of what is read (Blachowicz & Ogle, 2001; Rupley & Willson, 1997). Specifically, in developing this model conceptual understanding was defined the meaning and sense making of linear equations.

Procedural mathematics understanding/knowledge is focused “on skills and step-by-step procedures without explicit reference to mathematical ideas” (Ashlock, 2001, p. 8). Mere procedural skills often fail to provide readily applicable methods for solving problems. As children learn mathematics, they must also learn the meaning of new words that are not part of their oral vocabulary or with completely different meanings from what they already know. For example, the phrase “*more than”* seems to mean “to add” to students, but it is often used to indicate subtraction. Variable is usually understood as something liable to change, especially suddenly and unpredictably; however, when students encounter variables in algebraic situations, variables may have restricted solution sets because they are introduced in simple equations (X+7=9).

Syntax of Reading

Research has informed us that syntax (e.g., verb, sentence structure, subject/noun agreement) is paramount to comprehension (Bensoussan & Golan, 1981). The underlying assumption governing research on language acquisition for reading comprehension is sentence processing (Guthrie, 1977; Muth, 1982). Sentence processing is mediated by (1) the person’s real world experiences, (2) the context or setting of the sentence, (3) the role of the sentence in the passage (4) the understanding of linguistic structures. It is the last point that forms the basis for syntactic importance. Syntax has two meanings, the first deals with the observable and underlying structures of a sentence and the other is the scientific study of grammatical systems. When taken as a whole, these two common definitions are similar with mathematics (Manzo & Sherk, 1975).

The observable and underlying structures are commonly related to how words are organized and the order in which the reader develops word and sentence meanings. This is often governed by grammar (Bechervaise, 1992; Senechal, Pagan, Lever, & Ouellette, 2008). Grammar functions as a roadmap for organizing the words into comprehensible chucks that form sentences that convey meaning. In the written English language, words are intended to be read in order from left to right building meaning as one progresses across the sentence; in mathematics, the syntax can require a great deal more flexibility (Linville, 1976). The nexus of semantics and syntax is evident in complex representations such as (3*((2+5)/(3+(2/5)))).

Semantics of Mathematics

Semantic cues enable readers to better understand and comprehend what they are reading in mathematics. These cues categories are essential for conceptualization in mathematics (Mayer & Hegarty,1996**).** Johnson and Pearson (1984) classified the major kinds of semantic clues available to readers. A modified listing applicable to mathematics follows:

- Signal words in mathemathics texts. Words such as *is, are, and combined* are often used to alert the reader about equivalencies or operations.
- Synonyms and antonyms. When students encounter unknown mathematical words they can use either synonyms or antonmyns (e.g., subtraction is an antonmyn of addition) with their zone of proximal development to support their problem solving development.
- Summary statements. Based on connected mathematics story information, there may be multiple solutions, which are defensibile based upon cognitive reasoning.

We have found in our own research in mathematics (Capraro & Capraro, 2006; Kulm, Capraro, & Capraro, 2007) and science (Rupley & Slough, 2008) that students' understandings of mathematics and science concepts are inextricably bound to their identifying words; understanding vocabulary; and knowing the text structure (semantics and syntax) (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008) used to define, represent, and communicate concepts. Cognitive confusion results when students try to apply their general language meanings, leading to inhibited reasoning that obscures the content due to multiple meanings of familiar words being applied to an understanding of the preciseness of scientific usage.

Beyond grade four is where degrees of meaning have to begin to transition through literal and inferential stages toward conceptual learning. Literal meaning is composed of the who, what, why, where, when, which and how of a text. In order to become conceptual learners, students are required to move into inferential learning where they infer about ideas before or earlier than the context of their word problem, the cause and effect of events within word problems, and possible approaches to problem solutions (Capraro, & Capraro, 2006; Rupley, 2006). Reading in mathematics necessitates that one understand the meaning of the words. As children learn mathematics, it is essential they learn the meaning of new words that are not part of their oral vocabulary or have wholly different meanings from what they already know (Capraro & Joffrion, 2006).

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