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Study on the Inconsistency between a Pre-service Teacher's Mathematics Education Beliefs and Mathematics Teaching Practice

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Using the conversation syllabus based on some researchers' views and the spirits of New Basic Curriculum Reform in China, we studied a Chinese high school pre-service teacher's mathematics education beliefs. At the same time, we observed one mathematics lesson that was taught by this teacher and we found that there were inconsistencies between this teacher's mathematics education beliefs and her mathematics teaching practice. The main reason for these inconsistencies was that the pre-service teacher lacked PCK (pedagogical content knowledge) about mathematics teaching. Thus, teacher educators should help pre-service teachers to realize that mathematics teaching and learning was very complex process further, should teach pre-service teachers more teaching strategies and should instruct pre-service teachers to do self-reflection and so on.

Key words: teacher's beliefs, teaching practice, PCK.

Introduction

The relationship between teachers' mathematics education beliefs and their teaching practice was an important topic in teacher education all-round the word and many studies focused on this topic directly or indirectly.

One of these study findings was that there were inconsistencies between teachers' mathematics education beliefs and their teaching practice. For Example, Hoyles (1992), in her reviews of research on teacher beliefs, noted that these studies threw up evidence of inconsistencies between beliefs and beliefs-in-practice. Fernandes and Vale (1994) found that their two participants revealed very similar conceptions of mathematics and problem solving as pre-service middle school teachers but as beginning teachers their practices differed quite substantially. Beswick (2004) reported on a case study of one secondary school teacher that focused on what specific teacher beliefs were relevant to teachers' classroom practice in various classroom contexts. She found that the teacher held beliefs that were consistent with the aims of the mathematics education reform movement but there significant differences in his practice in regard to the various classes.

Throughout these studies on the relationship between teachers' mathematics education beliefs and their teaching practice, we though two problems were needed to study. One was that most teachers in these studies were in primary school or in middle school. In other words, were there inconsistencies between the beliefs of mathematics teacher in high school and their teaching practice? The other was that correlative studies in China were not noticed fully. That was to say, did these findings consist in Chinese mathematics teachers?

Based on the above, we would study a Chinese high school pre-service mathematics teacher and analyzed the relationship between her mathematics education beliefs and her mathematics teaching practice. In this study, we wanted to know if there were inconsistencies between her mathematics education beliefs and her teaching practice. Furthermore, we would explain the factors that affected these inconsistencies.

Method for Study I

Study 1 was one conversation including two parts between the authors of this paper and a Chinese high school pre-service mathematics teacher, teacher S. The first part of the conversation was carried out based on some researchers' views that was about mathematics beliefs, and the second part of the conversation was carried out based on the spirits of New Basic Curriculum Reform in China that was about mathematics education beliefs.

Tall (1992) wrote, "Advanced mathematics thinking ----as evidenced by publications in research journals ----is characterized by two important components; precise mathematical definitions (including the statement of axioms in axiomatic theories) and logical deductions based upon them." Moreover, Jennifer thought (2003), "the act of producing new mathematics is inherently creative; problem refinement, exploration, and arguments come from within the community, not from an external authority."

Other researchers, such as Yiying Huang found (2002, 2003), that teachers' conception of mathematics influenced their teaching behavior of mathematics classroom. At the same time, teachers' conception of mathematics was one of the most important factors that influenced students' conception of mathematics.

Based on the above, the first part of the conversation syllabus was some questions about mathematics beliefs that asked teacher S to give her opinions.

Question 1: Someone said: "The basic of the development of mathematics was imagination."How did you think about this view?

Question 2: Someone said: "There was no relationship between mathematics and logic."How did you think about this view?

Question 3: Someone said: "Mathematics was the collection of facts, rules, formulas and definitions. When we were solving one mathematics problem, we selected appropriate rules or formulas according to explicit and implicit hints, and applied these rules or formulas to the process of solving mathematics problems."How did you think about this view?

Question 4: Someone said: "Mathematics was one kind of language."How did you think about this view?

Question 5: Someone said: "One of the elements of Mathematics beauty was fantastic."How did you think about this view?

Question 6: Someone said: "There was a close relationship between mathematics and society life."How did you think about this view?

Question 7: Someone said: "Mathematics was an important component of human culture."How did you think about this view?

New Basic Curriculum Reform in China advocated an emphasis on understanding and problem solving in mathematics teaching and learning, and the process of mathematics teaching and learning should be carried out based on students' cognition level, students' current knowledge and students' experience. Teachers shouldn't tell students everything and replace students' learning. The roles of teachers were an organizer, motivator and leader in the process of students learning.

Based on the above, the second part of the conversation syllabus was some questions about mathematics education beliefs that asked teacher S to give her opinions.

Question 1: What the role of a teacher did you think in the process of mathematics teaching and learning?

Question 2: What the role of students did you think in the process of mathematics teaching and learning?

Question 3: Someone said: “Mathematics teaching was to teach students how to prove and compute.”And someone said: “Mathematics teaching was to teach students how to think.”How did you think about these views?

Question 4: Someone said: “Mathematics learning was to memory mathematics concepts and propositions, and then applies them to solve mathematics problems.”How did you think about these views?

Question 5: What was the most important aspect in the process of mathematics learning?

Question 6: Did you think what the characteristic of those students who were good at mathematics learning were?

Question 7: Please said what mathematics teaching and learning was in own words.

Analysis for Study I

The follow was one conversation between the first author of this paper and pre-service high mathematics teacher S.

The First Part:

*(The first Author)*L: Someone say: “The basic of the development of mathematics is imagination.”How do you think about this view?

*(Teacher S)*S: Because reasoning is based on imagination, I agree to this view. For example, when we think about some mathematics questions, suddenly, inspiration visits us. Then, we begin to reason and could get some results.

L: Okay. Someone say: “There is no relationship between mathematics and logic.”How do you think about this view?

S: I don’t agree to this view. There are close relationships between mathematics and logic. That is to say, mathematics is a logical and consistent discipline as opposed to a collection of facts.

L: Someone say: “Mathematics is the collection of facts, rules, formulas and definitions. When we are solving one mathematics problem, we select appropriate rules or formulas according to explicit and implicit hints, and apply these rules or formulas to the process of doing mathematics problems.”How do you think about this view?

S: I don’t agree to this view. The description of doing mathematics is too strictly. Actually, doing mathematics is flexible and it needs abundant imagination.

L: Someone say: “Mathematics is one kind of language.”How do you think about this view?

S: I think this is true. Mathematics plays an important role in our communication.

L: Someone say: “One of the elements of Mathematics beauty is fantastic.”How do you think about this view?

S: Yes, I have the same feeling. After I have done one mathematics problem, the feeling is very wonderful, and I feel mathematics is beautiful and fantastic deeply.

L: Someone say: “There are close relationships between mathematics and society life.”How do you think about this view?

S: I think so. Mathematics is an indispensable tool in our life. Oh, mathematics is really interesting.

L: Do you think mathematics is interesting?

S: Yes, I think mathematics is very interesting. I remember when I studied in the primary school, my mother sent me to learn Olympic mathematics. Since that time, I have been keeping dense interesting into mathematics learning.

L: Yeah. Someone say: “Mathematics is an important component of human culture.”How do you think about this view?

S: I agree to this view. Mathematics has been existed since people began to learn how to count. Mathematics history is very long.

Based upon some researchers’ views, such as Yiying Huang (2002, 2003), we could put mathematics belief into three views, which was the problem-solving view, the Platonist view and the instrumentalist view. The problem-solving view meant that mathematics was dynamic discipline and was developed by problem solving. The Platonist view meant that mathematics was one discipline with precise structure that was organized by logic. The instrumentalist view meant that mathematics was a tool collection of facts, rules, formulas and definitions. People could use mathematics to solve problems.

Teacher S’ above answers showed that her mathematics belief was prone to the problem-solving view. For example, she said mathematics was logical and there were close relationships among its facts, rules, formulas and definitions. Mathematics invention needed both imagination and reasoning. She thought mathematics was interesting, fantastic and beautiful. Specially, her feeling was very good after mathematics problems had been solved.

Teacher S also thought mathematics played an important role in our communication. Mathematics had close relationships with society life, and it was an important component of human culture.

(At the end of the first part of the conversation, the first author of this paper asked teacher S to see the three above views about mathematics belief. After several minutes, the first author of this paper continued to ask teacher S the following questions.)

L: Please see the three views about mathematics belief, which is the problem-solving view, the Platonist view and the instrumentalist view. Which view do you agree to?

S: Let me see. Comparing with the three views, I prefer to select the problem-solving view. I think the Platonist view doesn't reveal the fact that mathematics needs innovation and its development keeps moving. The instrumentalist view only reveals a part of mathematics value. Certainly, mathematics is a useful tool and plays an important role in our life, but this is one mathematics value. Mathematics has much other value, such as science value, humanism value, thinking value and so on.

Obviously, the answer was consistent with her above answers about the conversation syllabus.

The Second Part:

L: What the role of a teacher do you think in the process of mathematics teaching and learning?

S: First of all, a teacher should make students to be interesting in mathematics learning. Then, he/she should be an organizer, motivator and leader in the process of students learning.

L: How about students?

S: Students should be the host in the process of mathematics learning and think about mathematics questions actively.

L: Someone say: "Mathematics teaching is to teach students how to prove and compute."And someone say: "Mathematics teaching is to teach students how to think."How do you think about these views?

S: First, I don't agree to the expression "teach students". In the mathematics learning, students must think about mathematics questions by themselves and teachers only help them to study. Based on this view, I choose the latter view. Certainly, mathematics teaching and learning should teach students how to prove and compute, but the ultimate and essential aim is to teach students how to think.

L: Someone say: "Mathematics learning is to memory mathematics concepts and propositions, and then applies them to solve mathematics problems."How do you think about these views?

S: As to this view, I think we needn't memorize mathematics concepts and propositions mechanically. Mathematics understanding plays an important role in mathematics learning.

L: What is the most important aspect in the process of mathematics learning?

S: Interest. Interest is the most important aspect in mathematics learning. Just as interest, I like mathematics since I learned Olympic mathematics in primary school.

L: Do you think what the characteristics of those students who are good at mathematics learning are?

S: The basic characteristic is that they have many possible solving problems approaches when they encounter one mathematics problems. Then, they should often pose mathematics questions, especially those mathematics questions that could arose mathematics thinking.

L: Please say what mathematics teaching and learning is in own words.

S: Mathematics teaching and learning is students' own mathematics learning that was organized by teachers. Furthermore, if a teacher gives students a little hint, students could get more information according to these hints. That is to say, mathematics teaching isn't the process that a teacher tells students everything, but the process that a teacher should leave sufficient space and time for students exploring.

From the above teacher S' answers, we could see teacher S hold dynamic mathematics education beliefs that a teacher should help students to learn mathematics but not tell students everything. In the process of mathematics teaching and learning, mathematics teacher should pay attention to inspire students learning interest, accelerate students' mathematics understanding, culture students' mathematics learning activity. And she thought students who were good at mathematics were those that could put award good mathematics questions and had better mathematics thinking. Thus, teacher S' mathematics education beliefs were consistent with the spirits of New Basic Curriculum Reform in China.

Method for Study II

In Study 2, we would research one mathematics lesson that was taught by teacher S according to Stein, and Smith (1998) higher-level demands and lower-level demands mathematics tasks.

Lower-Level Demands (Memorization)

- Involve either reproducing previously learned facts, rules, formulas or definitions or committing facts, rules, formulas or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.
- *Lower-level demands (procedures without connections to meaning*):
- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers instead of on developing mathematical understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used.
- *Higher-level demands (procedures with connections to meaning):*
- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulative, symbols, and problem situations. Making connections among multiple representations helps develop meaning.

- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.
- *Higher-level demands (doing mathematics):*
- Require complex and no algorithmic thinking a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Teaching Episode and Analysis for Study 2

This was one mathematics lesson that was taught by pre-service high mathematics teacher S and its contents were about the concept of function.

First, teacher S let students recall some functions that were learned in middle school, such as quadratic function and reverse proportion function. Then, she posed two questions in order to evoke students' cognition conflict:

Question 1: Is $y = 1(x \in R)$ a function?

Question 2: Are the functions of $y = x$ and $y = \frac{x^2}{x}$ the same?

S: (Some students say) Yes. (Some students say)No.

T: S_1, do you think $y = 1(x \in R)$ is a function?

S_1: Yes.

T: Please give your reason.

S_1: ⋯ (Silence)

T: You think $y = 1(x \in R)$ is a function. But you don't know why it is a function, do you? Sit down, please. Today, we will learn the concept of function further based on our previous definition in middle school, and apply it to solve the above two questions.

(*Analysis 1: Using the previous concept of function in middle school, students couldn't solve the two questions posed by teacher S. Thus, students' cognition conflict was evoked. Then, teacher S introduced the learning task that they would learn in this lesson to students. In my opinion, this teaching behavior was feasible, because the role of the two questions was just to evoke student's cognition conflict, and at the end of this lesson, teacher S could lead students to solve the two questions again according to the concept of function that was learned in this lesson. And we would see later, teacher S had such a teaching behavior indeed. Thus, the two questions should be the higher-level demands (procedures with connections to meaning).It is a pity that the higher-level learning tasks were reduced that we would see from the following teaching episode.)*

S: Please open your textbooks and look on page 46. There are three figures on the left (Figure 1). What common features are there among the three figures? (*Students are looking at their books and no student answers her question.*) Please answer my question. (*No student answers her question still*). Look, there is only an arrowhead from the left set to the right set in the first figure, all right?

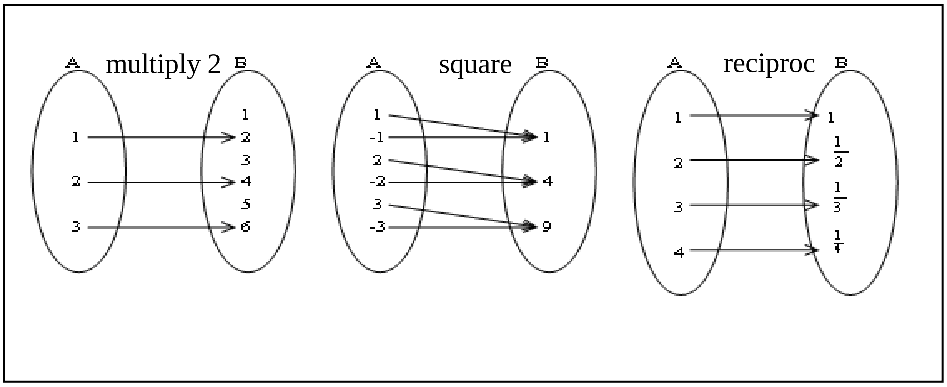

Figure 1. **Figures from the textbook.**

S: Yes.

T: Furthermore, every number on the left set has an arrowhead from the left set to the right set in the first figure. Yes or no?

S: Yes.

T: But some numbers on the right set have no arrowhead from the right set to the left set in the first figure, all right? Now, look at the second figure. Given any a number in the left set, there are two arrowheads from the left set to the right set. These are their differences among the three figures. Next, S_2, please say what common features there are among three figures.

S_2: …… (Silence)

T: As to any a number in set A, there is only a corresponding element in set B, all right? (*No student answers her question.*) So, we get the concept of function. Open your books and look at page 40, the last two rows. Please read the concept of function.

S: (read the concept of function in their books) Regarding A, B are set of numbers that are not empty set, if according to a corresponding relation f , any a number x in set A has only a corresponding element $f(x)$ in set B, we will say f: $A \to B$ is a function from set A to set B.

(*Analysis 2: Teacher S let students find the common features among the three figures on their textbooks. This learning task should be the higher-level demand(doing mathematics), but there were the following teaching behaviors that reduced the level of demand: (1)This learning task wasn't given clearly.(2) Necessary time for students' thinking wasn't given and teacher S gave the differences among the three figures by herself. (3)Teacher S asked student S_2 to give the common features among the three figures, but this student couldn't answer the question. So, Teacher S gave the common features among the three figures by herself. (4) Before students understood these mathematics tasks, teacher S asked students to read the concept of function. Attentively, teacher S didn't let students read the latter of the concept of function: "We record it as $y = f(x), x \in A$. In this mathematics expression, x is called independent variable, its domain is called function domain and its corresponding y is called function value. The set of function value is called function range." Actually, she divided the concept of function into two parts. One was to learn the* corresponding relati*on f ; the other was to learn the function domain and the function range. And we would see later, the two questions that were given at the beginning of this lesson were learning material that just explain or educe the two parts of the concept of function.)*

T: We have read the word expression of the concept of function. Now, let us think how many key words in this concept? That is to say, what do we need notice when we learn the concept of function?

S: No empty.

T: No empty. (Write it on the blackboard) Have any key words in this concept?

S: Any.

T: Any. (Write it on the blackboard) Have any?

S: Only.

T: Only (Write it on the blackboard) Have any?

S: Corresponding.

(*Analysis 3: Teacher S let students find key words that help students understand the concept of function: no empty, any, only, corresponding. From this teaching behavior, we could see that teacher S paid attention to students' mathematics understanding. This learning task should be the higher-level demand (produces with connections to meaning). Furthermore, teacher S posed some questions to lead students to find these key words and the level of the tasks was remained.)*

T: We find four key words. Now, let us see the first question that is posed at the beginning of the lesson:" Is $y = 1(x \in R)$ a function?"

S: Yes.

T: First, let us see if the set of A or the set of B is empty.

S: No.

T: What is the set of A?

S: The set of real number.

T: Corresponding relation?

S_3: 1.

S_4: B.

T: The corresponding relation is $y = 1$. Then, let us see the following question: given a $x \in A$, if y equals to 1.

S: Yes.

T: So, the uniqueness is satisfied. Then, let us consider the set of B.

S: 1.

T: Is it always 1?

S: Yes.

(Analysis 4: Teacher S asked students to think the first question that was posed at the beginning of the lesson: Is $y = 1(x \in R)$ a function? As we said above, this mathematics task was a higher-level demand (produces with connections to meaning), but teacher S replaced students' thinking. Thus, the level of the task was reduced.)

T: Next, let us see the first figure (Figure 1). 1 is an element in the set of B, but it hasn't any corresponding element in the set of A. So, we get the concept of function domain and the concept of function range. x is called independent variable, its domain is called function domain and its corresponding y is called function value. The set of function value is called function range. Attentively, function range is included into the set of B. That is to say, the set of B doesn't always equal to the function range. Thus, function has three essential elements: function domain, corresponding relation and function range. Two functions are the same means their function domains, corresponding relations and function ranges are all the same. Furthermore, if their function domains, corresponding relations are the same, their function ranges are the same. Yes or not?

S: Yes.

(Students' feedback isn't very enthusiastic.)

T: So, we say two functions are the same if their function domains, corresponding relations are the same.

(Analysis 5: Understanding the concept of function must understand the function domain, corresponding relation and the function range. So, teacher S pointed out the three essential elements of the concept of function. This task should be a higher-level demand (produces with connections to meaning), but she didn't give students sufficient time for exploring these questions. Thus, the level of the task was reduced.)

T: Now, let us to see the second question that is gave at the beginning of the lesson: Are $y = x$ and $y = \frac{x^2}{x}$ the same function? First, what is the function domain of $y = x$?

S: R.

T: What is the corresponding relation of $y = x$?

S: $y = x$.

T: What is the range of $y = x$?

S: R.

T: What is the domain of $y = \frac{x^2}{x}$?

S: $x \neq 0$。

T: What is the corresponding relation of $y = \frac{x^2}{x}$?

S: $y = x$.

T: What is the range of $y = \frac{x^2}{x}$?

S: $y \neq 0$。

T: Although their corresponding relations are all $y = x$, their domains are different. Thus, the two functions are different.

(Analysis 6: Teacher S asked students to think the second question that was gave at the beginning of the lesson: Were the function $y = x$ and $y = \frac{x^2}{x}$ the same? This task should be also a higher-level demand (produces with connections to meaning), but teacher S replaced students' thinking. Thus, the level of the task was reduced.)

T: Good! Next, let us to see how to seek a function value. We only replace x with the object independent variable. Please see the following example.

(1) Knowing $f(x)=3x^2-5x+2,$ please seek $f(3), f(-\sqrt{2}), f(a), f(a+1)$.

S: $f(3)=3\times 3^2-5\times 3+2=14$.

T: 14. We replace x with 3. That is ok. $f(-\sqrt{2})$?

S: $f(\sqrt{2})=3\times 2+5\sqrt{2}+2=8+5\sqrt{2}$ 。

T： $f(a)$?

S: $f(a)=3a^2-5a+2$ 。

T: $f(a+1)$?

S: $f(a+1)=3(a+1)^2-5(a+1)+2$ 。

T: Now, I will ask you a question. Is $f(x)$ and $f(a)$ the same mathematics symbol?

S: No.

T: What different between $f(x)$ and $f(a)$?

S: $f(x)$ is a variable, but $f(a)$ is an invariable.

T: $f(x)$ is a function, it changes according to x . But $f(a)$ is an invariable that is given $x=a$ in the expression of $f(x)$. Now, look at another example……

(Analysis 7: Teacher S gave function symbol $y = f(x)$, and told students how to seek function value. Furthermore, she gave an example to explain this method. Based on this example, she posed a question: Is $f(x)$ and $f(a)$ the same? From the question, we could see teacher S emphasized the meaning of the concept. But teacher S didn't let students think how to seek a function value and replaced students' thinking.)

Results

From study 1 and study 2, we could see there were some inconsistencies between teacher S' mathematics education beliefs and her mathematics teaching practice.

As stated earlier in this paper, although teacher S' mathematics belief was the problem-solving view and her mathematics education beliefs were consistent with the spirits of New Basic Curriculum Reform in China, her mathematics teaching practice didn't show her mathematics education beliefs. For example, she didn't give some learning tasks clearly, she didn't give students sufficient time for exploring mathematics questions, she didn't let students think about mathematics question by themselves. All these teaching behaviors weren't consistent with her mathematics education beliefs.

Gill (2004) thought that changing strongly held prior beliefs about academic concepts was difficult when those beliefs conflicted with instruction. Teachers' domain-specific epistemological beliefs, which was, their beliefs about the nature of knowledge and teaching and learning in mathematics, contributed to the difficulties in changing their teaching practices. At the same time, mathematics teachers' general epistemological belief, specifically that knowledge was simple and certain, contributed to their resistance to change.

Throughout our observation and interview, we thought the main reason for these inconsistencies between teacher S' mathematics education beliefs and her mathematics teaching practice was that teacher S lacked pertinent knowledge about mathematics teaching, especially PCK (pedagogical content knowledge).

Shulman (1987) proposed a framework for analyzing teachers' knowledge that distinguished different categories of knowledge: knowledge of content, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge (PCK), knowledge of students, knowledge of educational contexts and knowledge of educational ends, purposes and values. He emphasized PCK as a key aspect to address in the study of teaching.

The analysis for study 2 showed that teacher S lacked PCK because she had little mathematics teaching experience. She didn't know how to lead students to analyze mathematics problems, how to express learning concept. Facing to students' answer, she couldn't correct their understanding errors. She had no choice but to tell students some results. Thus, her mathematics education beliefs weren't consistent with her teaching behavior sometimes.

After this lesson, the authors of this paper asked teacher S to say her feeling about this mathematics lesson. Teacher S said that her feeling was not good. She felt her prearrange teaching project couldn't be carried out in the process of this mathematics lesson and she couldn't help students to understand mathematics. She was very regretful for this mathematics lesson.

Thus, Pre-service teachers should strengthen their learning about teacher knowledge, especially PCK. This needed teacher educators to help them, such as to give more practice chance. Especially, the followings should be noticed:

First, teacher educators should make pre-service teachers realize mathematics teaching and learning was very complex process further. In the process of mathematics teaching and learning, many factors effected mathematics teachers' prearrange, such as students' answer. Teachers shouldn't ignore students' learning response, including their silence. Because their silence meant that they had no interesting into mathematics learning, or they couldn't understand mathematics questions. But interesting and understanding played a very important role in mathematics learning as teacher S said in study 1. Thus, teachers should adjust prearrange according to students' response in the process of their mathematics learning.

Second, teacher educators should teach pre-service teachers more teaching strategies. For example, how to question? Learning how to question is one important PCK. Students were mathematics learning hosts, the role of teachers only help them to study. This needed teachers question students with heuristic way. When a teacher found some students had learning difficulties, he/she should lead them to analyze from known to un-known and help them to learn how to think. At the same time, a teacher must leave sufficient time for students' thinking.

Third, teacher educators should introduce pre-service teachers to do self-reflection. After pre-service teachers had finished one mathematics teaching, they should reflect their teaching process. They should think some questions, such as if teaching aims were achieved, if teaching difficulties were overcome, if students were put into the host of mathematics learning and so on. Furthermore, teacher educators should tell them to write these reflections on their teaching plan, and often see these reflections. Gradually, pre-service teachers could accumulate their teaching practice and their PCK could be enhanced.

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