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Enhancing Students' Geometric Thinking and Achievement in Solid Geometry

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This study aimed to explore if students' geometric thinking and achievement in solid geometry could be enhanced through phase-based instruction using manipulatives and The Geometer's Sketchpad (GSP) based on the van Hiele theory. The researchers employed a case study research design and purposeful sampling to select eight case study participants from a class of mixed-ability Form One students. The results showed that the teaching intervention could enhance the participants' geometric thinking and achievement in solid geometry.

Key words: geometric thinking, achievement, solid geometry, van Hiele theory

Previous studies have shown that phase-based instruction using manipulatives could enhance the van Hiele levels of geometric thinking of middle-grade or secondary students (Fuys, Geddes & Lovett, 1988; Massey, 1993) and the geometry achievement of secondary students (Tay, 2003). However, research findings regarding the effectiveness of phase-based instruction using computer are mixed. Some studies showed that phase-based instruction using the Geometric Supposer could enhance the van Hiele levels of geometric thinking of high school students (Bobango, 1987) and secondary students (Baynes, 1999), as well as the geometry achievement of high school students (Thompson, 1992). But, Bobango (1987) found that it could not enhance the geometry achievement of high school students. Similar mixed findings were reported in the literature regarding the effectiveness of phase-based instruction using GSP. While Choi (1996), using case studies, showed that phase-based instruction using GSP could enhance the van Hiele levels of geometric thinking of secondary students, Moyer (2003), using quasi-experimental design, found that it had no statistically significant effects on the van Hiele levels and geometry achievement of high school students.

Further, previous research on students' learning of solid geometry mainly focused on the ability to translate between 3D solids and their 2D representations, difficulty in enumerating 3D arrays of cubes, and reasoning about geometric solids. Much of the research that investigated the first area focused on drawings of solids (Lehrer, Jenkins & Osana, 1998), drawing nets (Potari & Spiliotopoulou, 1992), recognizing nets (Bourgeois, 1986), describing nets (Lawrie, Pegg & Gutiérrez, 2000), and constructing nets (Despina, Leikin & Silver, 1999). Ben-Chaim, Lappan and Houang (1985) and Battista and Clements (1998) examined students' difficulty in enumerating 3D arrays of cubes in terms of counting strategies and mental models respectively. Studies that investigated students' reasoning about geometric solids focused on the assessment of van Hiele levels (Gutiérrez, Jaime & Fortuny, 1991), assessment of van Hiele levels and spatial abilities (Saads & Davis, 1997), GSP-based instructional environment (McClintock, Jiang & July, 2002), and dynamic transformations of solids using manipulatives and dynamic geometry software (Markopoulos & Potari, 2003).

Therefore, there has been a lack of research that explores if students' geometric thinking and achievement in solid geometry could be enhanced through phase-based instruction using manipulatives and GSP.

Need for the Study

Solid geometry is an important component of the secondary mathematics curriculum in Malaysia. Despite its importance, secondary students still performed poorly on the compulsory solid geometry questions in the public examination (Malaysian Examinations Syndicate, 2006). Further, Malaysian Form Two students (14- or 15-year-old students who are in the second year of secondary school) performed poorly on the geometry item at the Top 10% International Benchmark in TIMSS 1999 and at the High International Benchmark in TIMSS 2003. Their performance on the item was ranked 22nd out of 38 participating countries in TIMSS 1999 (Mullis, Martin, Gonzalez, Gregory, Garden, O' Connor, Chrostowski & Smith, 2000) and 19th out of 49 participating countries in TIMSS 2003 respectively (Mullis, Martin, Gonzalez & Chrostowski, 2004). The rankings in TIMSS 1999 and TIMSS 2003 reflected on the lack of geometric understanding among the Malaysian students. In addressing this concern, it is important that students at the beginning level of secondary school are provided with a strong foundation of solid geometry (van Hiele-Geldof, 1959/1984). Thus, Form One students and the solid geometry chapter were selected for this study.

Purpose of the Study

The purpose of this study was to explore if students' geometric thinking and achievement in solid geometry could be enhanced through phase-based instruction using manipulatives and GSP based on the van Hiele theory. Specifically, this study aimed to answer the following research questions: (1) To what extent did phase-based instruction using manipulatives and GSP enhance students' van Hiele levels of geometric of geometric thinking in solid geometry regarding the concepts of cubes and cuboids? (2) To what extent did phase-based instruction using manipulatives and GSP enhance students' achievement in solid geometry?

Theoretical Framework

The van Hiele theory of geometric of geometric thinking comprises three main components: (1) levels of geometric of geometric thinking, (2) characteristics of the levels, and (3) phases of learning (Crowley, 1987; Usiskin, 1982).

Levels of Geometric Thinking

According to the theory, students progress sequentially through five hierarchical levels of thinking in the process of learning geometry. Two numbering systems have been used for the levels: Levels 0 through 4 and Levels 1 through 5 (Clements & Battista, 1992). In this study, the 1 to 5 numbering system was adopted because students who have not attained Level 1 could be assigned to Level 0 (Senk, 1989). The descriptors of the first three levels (adapted from Mayberry, 1981:143-144) were provided because an analysis of the Form One mathematics syllabus and textbooks showed that the content of the solid geometry chapter is only up to Level 2 and lower secondary students of ages 13-14 years old are highly unlikely to attain Level 4 or 5 in solid geometry (Gutiérrez et al., 1991; Lawrie et al., 2000) and in plane geometry (Noraini Idris, 1998; Tay, 2003).

At Level 1 (recognition), students can recognize and name solids, and distinguish a solid from others on a visual basis. At Level 2 (analysis), students can identify properties of solids. Students at Level 3 (informal deduction), can logically order properties of solids, formulate economical correct definitions for solids and accept different equivalent definitions for the same concept, as well as understand class inclusions and classify solids hierarchically.

Characteristics of the Levels

There are five characteristics of the levels:

(a) Sequential. The levels are sequential. This implies that in order for students to be able to reason at a higher level, they must be given adequate and effective learning experiences at the lower level (van Hiele, 1986).

(b) Intrinsic and extrinsic. Geometric concepts that are implicitly understood at one level become explicitly understood at the next level (Clements & Battista, 1992).

(c) Linguistics. Each level has its own language, set of symbols and network of relations (van Hiele, 1986).
(d) Mismatch. If students are at one level and the teacher, instructional materials and content are at a higher level than the students, the desired learning of the students may not occur because they will not be able to understand the thought processes being used (Crowley, 1987).
(e) Advancement. According to van Hiele (1986:50), "the transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program."

Phases of Learning

To help students progress from one level to the next, the van Hieles propose a sequence of five phases of learning (phase-based instruction).
Phase 1: Information. The teacher engages students in conversations about the topic to be studied, evaluates their responses, and provides them with some awareness of why they are studying the topic so as to set the stage for further study (Hoffer & Hoffer, 1992).
Phase 2: Guided orientation. Students actively explore the topic of study by performing simple tasks designed to elicit specific responses so as to become acquainted with the objects from which geometric ideas are abstracted (Clements & Battista, 1992; Crowley, 1987).
Phase 3: Explicitation. Students learn to express their opinions about the structures observed during discussions in class (van Hiele, 1986). The teacher leads students' discussion of the objects of study using their own words until a consensus is achieved so that they become explicitly aware of the objects of study. Then, the teacher introduces the relevant vocabulary (Clements & Battista, 1992).
Phase 4: Free orientation. Students are challenged with more complex tasks that can be completed in different ways (Crowley, 1987). The teacher encourages students to solve and elaborate on these problems and their solution strategies, and to introduce relevant problem-solving processes as needed (Clements & Battista, 1992).
Phase 5: Integration. Students summarize what they have learned about the objects of study with the goal of forming an overview of the topic (van Hiele, 1986). At the completion of this phase, students should have attained a new level of understanding for the topic studied and are ready to repeat the five phases of learning at the next level (Crowley, 1987).

Methodology

Research design and Sample

The researcher employed a case study research design and purposeful sampling to select the sample since the goal of the case study was not to generalize the results of the study. According to Merriam (1998), in case studies, sample selection occurs at the case level followed by within the case. Also, criteria need to be established to select both the case and the sample within the case.

Based on the purpose of the study, the case selection criteria were: (a) one class of Form One students studying in an academic secondary school that has a computer laboratory equipped with a teacher's laptop, an LCD projector, sufficient computer equipment and facility; (b) the class was of mixed ability in terms of mathematics achievement in the 2005 Primary School Achievement Test (Year 6 public examination); (c) the students were of mixed gender and had not learned the solid geometry chapter in school. To choose the students as case study participants for an in-depth investigation of their levels of geometric thinking, the criteria for selecting the sample within the case were: (a) two low- (Grade D or E), four average- (Grade B or C) and two high-ability (Grade A) students in terms of the aforementioned mathematics achievement so as to form two groups of 4 students with mixed abilities for better student learning (Webb & Palincsar, 1996); and (b) they volunteered as participants and agreed to be audio- and videotaped during the study with their parents' consent. Based on these criteria, 8 students (4 boys and 4 girls) were chosen as the participants (see Table 1).

Table 1
Participants' Mathematics Achievement and Gender

Group	Participants	Mathematics achievement in the 2005 Primary School Achievement Test	Gender
	S1	Low	Female
1	S2	Average	Female
	S3	Average	Female
	S4	High	Female
	S5	Low	Male
2	S6	Average	Male
	S7	Average	Male
	S8	High	Male

Instruments

Interview Instrument

An interview instrument (see Table 2) was developed to assess the participants' van Hiele levels based on Mayberry's (1981:143-144) interview instrument for several reasons. Firstly, Mayberry's instrument was designed to assess students' van Hiele levels regarding specific geometric concepts. Secondly, McClintock et al. (2002) and Choi (1996) also devised their interview instruments based on Mayberry's instrument to assess middle, secondary and high school students' van Hiele levels. Thirdly, the Spatial Geometry Test developed by Gutiérrez et al. (1991) was not designed to assess students' van Hiele levels regarding cubes and cuboids per se. However, only items for the first three levels were devised because the content of the solid geometry chapter is only up to Level 2.

Table 2
Distribution of Items in the Interview Instrument

Concept	Level	Item Type	Item Number
Cube	Level 1 (Basic)	Name	1 (2)
		Discriminate	3a (9a)
	Level 2 (Level I)	Properties	4 (16)
			6a (23a)
		Definition	8 (24)
		Class Inclusions	3b (9b)
	Level 3 (Level II)		10 (25)
		Similarity	11a (42a)
			11b (42d)
		Congruence	12a (44b)
		Implication	6b (23b)
Cuboid	Level 1 (Basic)	Name	2 (2)
		Discriminate	3b (9a)
	Level 2 (Level I)	Properties	5 (16)
			7a (23a)
		Definition	9 (24)
		Class Inclusions	3b (9b)
	Level 3 (Level II)		10 (25)
		Similarity	11b (42a)
			11c (42d)
		Congruence	12b (44b)
		Implication	7b (23b)

Notes: Texts and numbers in parentheses are those used in Mayberry's (1981:128) interview items.

The interview instrument comprised three levels and 12 items. Level 1 (*recognition*)-items assessed student's ability to recognize and name cubes and cuboids, and to discriminate cubes and cuboids from rhomboids and parallelepipeds. Level 2 (*analysis*)-items assessed student's ability to identify properties of cubes and cuboids. Level 3 (*informal deduction*)-items assessed student's understanding of definitions, class inclusions, similarity, congruence and implications regarding cubes and cuboids. The interview instrument required about 30 minutes to complete.

Solid Geometry Achievement Test (SGAT)

The SGAT was constructed based on the Form One mathematics syllabus. It comprised three levels of items: factual, application and problem-solving. Factual items assessed the recall of facts, procedures, and concepts pertaining to solid geometry. Application items tested the comprehension and subsequent application of facts, procedures, and concepts to solve comprehension type problems. Problem-solving items evaluated the synthesis of facts, procedures, and concepts to solve routine and non-routine problems (Lam, 2004).

It consisted of two parts. Part A comprised 15 multiple-choice items whereas Part B comprised 10 short-answer items. These 25 items consisted of 8 factual items (5 multiple-choice and 3 short-answer), 12 application items (10 multiple-choice and 2 short-answer), and 5 short-answer problem-solving items. The allocated time for taking the test was 1 hour and 20 minutes for Part A and 40 minutes for Part B.

Both the table of specifications and the SGAT were submitted to a mathematics teacher educator at a local public university and two experienced Form One mathematics teachers to validate the content of the test. All three field experts judged the test to measure the learning objectives as defined by the table of specifications, with suggested revisions of a few of the items in terms of language and diagrams. Hence, it was judged to have content validity.

The revised SGAT was pilot-tested with two classes of mixed-ability Form Two students (N=56) to determine its reliability. Form Two students were chosen because they would have already learned the topic in Form One. The internal consistency reliability as estimated by Cronbach's alpha for the overall SGAT was .79, that is, .75 for the factual, .73 for the application, and .71 for the problem-solving components. Thus, it was sufficiently reliable for assessing students' achievement in solid geometry for the study.

Phase-based Manipulative and GSP instructional activities

The instructional activities (see Table 3), the accompanying student worksheets, and the lesson plans were designed based on the learning objectives of the solid geometry chapter and the van Hiele theory, and adapted from various sources such as Mayberry (1981), Noraini Idris (1998), Bennett (1999), Bobango (1987) and Choi (1996).

Table 3
Summary of Instructional Activities

Week	Learning Period 1	Lesson	Teaching and Learning Activities
1	Information (10 min) Guided Orientation (10 min) Explicitation (15 min)	1	Activity 1: Name of geometric solids. Activity 2: Geometric properties of cubes and cuboids. Activity 3: Describe geometric properties of cubes and cuboids.
	Guided Orientation (20 min) Explicitation (15 min)	2	Activity 4: Draw cubes and cuboids Activity 5: Explain how to draw cubes and cuboids.
	Free Orientation (20 min) Explicitation (15 min)	3	Activity 6: Make models of cubes and cuboids. Activity 7: Explain how to make models of cubes and cuboids.
	Guided Orientation (20 min) Explicitation (15 min)	4	Activity 8: Volume of cuboids. Activity 9: Explain how to measure volume of cuboids using unit cubes.
	Guided Orientation (20 min) Explicitation (15 min)	5	Activity 10: Develop and find the volume formula of cuboids. Activity 11: Explain how to develop and find the volume formula of cuboids.
2	Guided Orientation (20 min) Explicitation (15 min)	6	Activity 12: Develop and find the volume formula of cubes. Activity 13: Explain how to develop and find the volume formula of cubes.
	Free Orientation (20 min) Explicitation (15 min)	7	Activity 14: Solve problems involving properties and volumes of cubes and cuboids. Activity 15: Explain how to solve

			problems involving properties and volumes of cubes and cuboids.
	Integration (35 min)	8	Activity 16: Summarize what they have learned in Lessons 1-7.
Week	Learning Period 2	Lesson	Teaching and Learning Activities
2	Information (5 min) Guided Orientation (15 min) Explicitation (15 min)	9	Activity 17: Recall properties of cubes and cuboids. Activity 18: Class inclusion between cubes and cuboids. Activity 19: Explain class inclusion between cubes and cuboids.
	Guided Orientation (20 min) Explicitation (15 min)	10	Activity 20: Definitions of cubes and cuboids. Activity 21: Explain definitions of cubes and cuboids.
3	Guided Orientation (20 min) Explicitation (15 min)	11	Activity 22: Similarity between cubes and cuboids. Activity 23: Explain similarity between cubes and cuboids.
	Guided Orientation (20 min) Explicitation (15 min)	12	Activity 24: Congruence between cubes and cuboids. Activity 25: Explain congruence between cubes and cuboids.
	Free Orientation (20 min) Explicitation (15 min)	13	Activity 26: Implications regarding cubes and cuboids. Activity 27: Explain implications regarding cubes and cuboids.
	Free Orientation (20 min) Explicitation (15 min)	14	Activity 28: Solve problems involving volumes of cubes and cuboids, class inclusion and congruence. Activity 29: Explain how to solve the above problems.
	Integration (35 min)	15	Activity 30: Summarize what they have learned about relationships among properties of cubes and cuboids.

During Activity 1 in Learning Period 1, students recall the name and shape of cube, cuboid, cone, cylinder, pyramid and sphere, and learn what direction further study will take. In Activity 2, they investigate geometric properties of cubes and cuboids. In Activity 3, students express and exchange their opinions about the geometric properties of cubes and cuboids that they have discovered in Activity 2, and learn the standard vocabulary for describing the geometric properties of cubes and cuboids. In Activity 4, students draw cubes and cuboids on (a) square grid and (b) blank paper. In Activity 5, they express and exchange their opinions about the conventions for drawing cubes and cuboids that they have found in Activity 4, and learn the conventions for drawing cubes and cuboids using oblique projection. In Activity 6, students make models of cubes and cuboids by (a) joining given faces and (b) folding given layouts of the solids. In Activity 7, they express and exchange their opinions about the layouts of cubes and cuboids that they have found in Activity 6, and learn the correct layouts of cubes and cuboids. In Activity 8, students measure the volume of cuboids using unit cubes. In Activity 9, they express and exchange their opinions about how to measure the volume of cuboids using unit cubes. In Activity 10, students develop and find the volume formula of cuboids. In Activity 11, they express and exchange their opinions about to how to develop and find the volume formula of cuboids. In Activity 12, students develop and find the volume formula of cubes. In Activity 13, they express and exchange their opinions about how to develop and find the volume formula of cubes. In Activity 14, students solve problems involving properties and volumes of cubes and cuboids. In Activity 15, they express and exchange their opinions about to how to solve the problems involving properties and volumes of cubes and cuboids, and learn the correct strategies for solving the problems involving volumes of cubes and cuboids. Finally in Activity 16, students summarize the properties of cubes and cuboids.

During Activity 17 in Learning Period 2, students recall the properties of cubes and cuboids, and learn what direction further study will take. In Activity 18, they investigate class inclusion between cubes and cuboids. In Activity 19, students express and exchange their opinions about the class inclusion between cubes and cuboids that they have discovered in Activity 18, and learn the standard vocabulary for describing class inclusion between cubes and cuboids. In Activity 20, students investigate the definitions of cubes and cuboids. In Activity 21, they express and exchange their opinions about the definitions of cubes and cuboids that they have found in Activity 20, and learn the standard definitions of cubes and cuboids. In Activity 22, students investigate the similarity between cubes and cuboids. In Activity 23, they express and exchange their opinions about the similarity between cubes and cuboids that they have found in Activity 22, and learn the standard vocabulary for describing the similarity between cubes and cuboids. In Activity 24, students investigate congruence between cubes and cuboids. In Activity 25, they express and exchange their opinions about the congruence between cubes and cuboids that they have found in Activity 24, and learn the standard vocabulary for describing the congruence between cubes and cuboids. In Activity 26, students investigate implications regarding cubes and cuboids. In Activity 27, they express and exchange their opinions about the implications regarding cubes and cuboids, and learn the standard vocabulary for describing the implications regarding cubes and cuboids. In Activity 28, students solve problems involving volumes of cubes and cuboids, class inclusion and congruence. In Activity 29, they express and exchange their opinions about how to solve the problems involving volumes of cubes and cuboids, class inclusion and congruence, and learn the correct strategies for solving the problems. Lastly in Activity 30, students summarize what they have learned about the relationships among the properties of cubes and cuboids.

Research Procedure

The study comprised three sessions (see Figure 1). Session 1 consisted of pre-interviews and a pretest. Pre-interviews were conducted individually by the researcher with all the participants using the interview instrument to determine their van Hiele levels prior to the instruction. All the pre-interviews were audio- and videotaped. A pretest was then administered to all the participants using the SGAT to determine their achievement in solid geometry prior to the instruction.

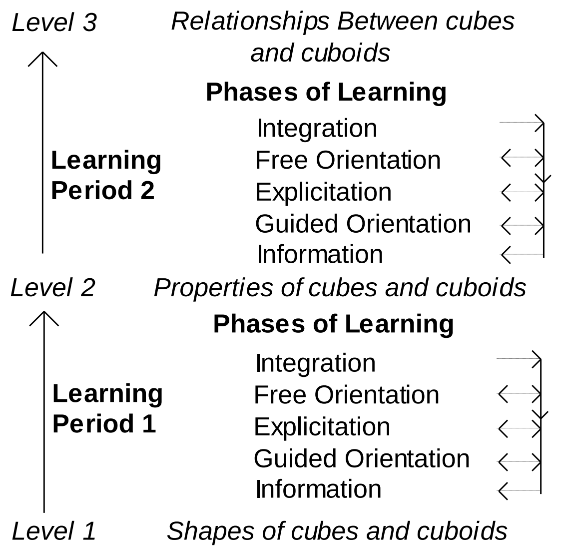
Session 1	**Session 2**	**Session 3**
Pre-interviews Pretest	Phase-Based Instruction using Manipulatives and GSP 	Post-interviews Posttest

Figure 1: *Three sessions of the study.*

Session 2 consisted of phase-based instruction using manipulatives and GSP by the researcher for the whole class. The students formed groups of four based on gender but with different mathematical abilities working together during each phase-based instructional activity. The phase-based instruction comprised two learning periods. Each learning period had five phases of learning: information, guided orientation, explicitation, free orientation and integration. Learning Period 1 was designed for students to learn the properties of cubes and cuboids in order to progress from Level 1 to Level 2. Learning Period 2 was designed for students to learn the relationships between cubes and cuboids in order to progress from Level 2 to Level 3. All in all, the phase-based instruction comprised fifteen 35-minute lessons and 30 phase-based manipulative and GSP instructional activities for the duration of 3 weeks.

During Session 2, data regarding the participants' geometric thinking was collected according to the five phases of learning in Learning Periods 1 and 2. During the Information phases, the whole-class teacher-student conversations were videotaped. During the Guided Orientation phases, the two small-group participant-participant discussions were audiotaped. During the Explicitation phases, the whole-class discussions were videotaped. During the Free Orientation phases, the two small-group participant-participant discussions were audiotaped. Finally, during the Integration phases, the two small-group participant-participant discussions were audiotaped and then the whole-class discussions were videotaped. The student worksheets were collected from the participants and the researcher made field notes of any interesting observations during Learning Periods 1 and 2.

Session 3 consisted of individual post-interviews and a posttest. Post-interviews were conducted individually by the researcher with all the participants using the same interview instrument to determine their van Hiele levels after the instruction. All the post-interviews were audio- and videotaped. A posttest using the same SGAT was then administered to all the participants to determine their achievement in solid geometry after the instruction.

Data Analysis

To answer Research Question 1, audiotapes of the pre- and post-interviews were transcribed verbatim and each participant's verbal transcript was analyzed, scored and assigned a van Hiele level for cubes and cuboids according to Mayberry's (1981:128) scoring criteria. All the videotapes were viewed to provide contextual notes. Finally, the results were presented to the same mathematics teacher educator for validation and verification (Merriam, 1998). Refinement was then carried out based on the comments given by the lecturer.

To answer Research Question 2, the participants' written work in the pretest and posttest were scored according to the SGAT marking scheme. A paired-samples t-test was computed using SPSS version 13 for Windows to ascertain if there was a statistically significant difference in the pretest and posttest scores of the SGAT.

Results

Van Hiele Levels

Table 4 shows the participants' van Hiele levels of geometric thinking of cubes and cuboids before and after the intervention. The results indicated that all the participants were at Level 1 for cubes and cuboids prior to the intervention. This implied that all the participants could recognize and name cubes and cuboids, discriminate the cubes from the cuboids, and discriminate the cuboids from the rhomboids and parallelepipeds before the intervention.

Table 4
Participants' Van Hiele Levels

Participants	Mathematics achievement	Gender	van Hiele levels of geometric thinking: Cubes		Cuboids	
			Pre-interview	Post-interview	Pre-interview	Post-interview
S1	Low	Female	Level 1	Level 2	Level 1	Level 2
S2	Average	Female	Level 1	Level 3	Level 1	Level 3
S3	Average	Female	Level 1	Level 3	Level 1	Level 3
S4	High	Female	Level 1	Level 3	Level 1	Level 3
S5	Low	Male	Level 1	Level 2	Level 1	Level 2
S6	Average	Male	Level 1	Level 3	Level 1	Level 3
S7	Average	Male	Level 1	Level 3	Level 1	Level 3
S8	High	Male	Level 1	Level 3	Level 1	Level 3

After the intervention, the results indicated that six of the eight participants progressed from Level 1 to Level 3 for cubes and cuboids. That is, they could recognize and name cubes and cuboids, discriminate the cubes from the cuboids, and discriminate the cuboids from the rhomboids and parallelepipeds. They could also identify the properties of vertices, edges and faces of cubes and cuboids. Further, they showed understanding of definitions, class inclusions, similarity, congruence and implications regarding cubes and cuboids. They could define cubes and cuboids in terms of six square faces and rectangular faces respectively, understand that all cubes are also cuboids because all cubes are special cuboids that have six square faces, two cubes are always similar because their corresponding edges are always proportional, two cubes with a 10 cm edge are always congruent because their corresponding edges are always equal, a solid that has six flat faces and all the edges are equal is not necessarily a cube because it can be a rhomboid, and that a solid that has six flat faces and in which the opposite edges are equal is not necessarily a cuboid because it can be a cube, rhomboid or parallelepiped.

However, two low-ability students only progressed from Level 1 to Level 2 for both concepts after the instruction. They could only recognize and name cubes and cuboids, discriminate the cubes from the cuboids, discriminate the cuboids from the rhomboids and parallelepipeds, and identify the properties of vertices, edges and faces of cubes and cuboids.

Achievement in Solid Geometry

Table 5 shows the results of the paired-samples t-test. The mean score in the pretest was 57.88 with a standard deviation of 10.68, and the mean score in the posttest was 78.13 with a standard deviation of 10.40. The difference between means was statistically significant, t (7) = 10.18, $p < .05$, with a gain of 20.25 points or 34.99%, indicating that phase-based instruction using manipulatives and GSP enhanced the participants' achievement in solid geometry.

Table 5

Results of the Paired-Samples T-Test

	Pretest		Posttest				
	Mean	Standard Deviation	Mean	Standard Deviation	*t*	df	p
SGAT (Perfect Score=100)	57.88	10.68	78.13	10.40	10.18	7	.00*

Note: * significant at p < 0.05, N = 8

Discussion

All the participants advanced from lower to higher van Hiele levels after the intervention which is consistent with the findings of previous studies on phase-based instruction using manipulatives (Fuys et al., 1988; Massey, 1993; Tay, 2003; van Hiele-Geldof, 1959/1984), and phase-based instruction using static or dynamic geometry computer software packages (Baynes, 1999; Bobango, 1987), and phase-based instruction using GSP (Choi, 1996).

However, Choi (1996) found that not all the participants were on the same level across right, isosceles and equilateral triangles after the phase-based instruction using GSP. Unlike the findings of Choi, this study found that all the participants were on the same level across cubes and cuboids.

All the participants also showed an improvement in their achievement in solid geometry after the intervention, which concurs with the findings of previous studies on phase-based instruction using manipulatives (Tay, 2003) and phase-based instruction using the Geometric Supposer (Thompson, 1992).

There are several possible explanations for the participants' progress. One is the use of instructional activities based on the van Hiele theory. During Learning Period 1, the participants named cubes and cuboids in Information. They identified the properties of cubes and cuboids using the manipulatives in Guided Orientation. Next, they presented the properties to the class using their own words and then learned the standard vocabulary for describing them in Explicitation. In Free Orientation, they solved problems involving properties and volumes of cubes and cuboids using the manipulatives. Finally, in Integration, they summarized the properties. During Learning Period 2, the participants reviewed the properties of cubes and cuboids in Information. Next, they investigated relations among properties of cubes and cuboids in Guided Orientation using the manipulatives and GSP. They presented the relations to the class using their own words and then learned the standard vocabulary for describing them in Explicitation. In Free Orientation, they solved problems involving congruence and volumes of cubes and cuboids using the manipulatives and GSP. Finally, in Integration, they summarized the relations among properties of cubes and cuboids.

Second, the appropriate use of the manipulatives and GSP based on the van Hiele theory provided the participants with opportunities to first investigate visually the shapes of cubes and cuboids, then analyze their properties, and finally develop arguments about relations among their properties. For example, by directly manipulating the physical models of cubes and cuboids, and then the pre-constructed GSP models to generate many examples of cubes and cuboids, and followed by reflection on those actions, the participants were able to recognize their shapes, identify their properties and understand better the relations among properties of cubes and cuboids.

The pre-constructed GSP models were employed to avoid mismatch between van Hiele levels since construction activities are inappropriate at Level 1. Students at Level 1 have difficulty constructing models of cubes and cuboids in GSP because they do not yet know their properties (Level 2) and the logical relations among their properties (Level 3) (de Villiers, 1999). Moreover, the GSP models enabled the Level 1 participants to focus on the properties of cubes and cuboids and then the relations among their properties instead of getting bogged down in constructing the models themselves which is completely inappropriate for their level.

Third, the teacher's guidance might also contribute to the participants' progress. During Information, the teacher engaged the students in conversations to learn what they already knew. During Guided Orientation, he carefully sequenced the instructional activities for the students to investigate. During Explicitation, he encouraged the students to share their findings using their own words and introduced relevant vocabulary when appropriate. During Free Orientation, he carefully chose appropriate problems for the students to solve and introduced relevant problem-solving processes as needed. Finally, he guided the students in summarizing the properties and relationship among properties of cubes and cuboids in Integration.

However, after the intervention, only two low-ability students attained Level 2 for both concepts. The lack of progress into Level 3 might be attributed to several factors. Firstly, they had low achievement in mathematics. Secondly, they seemed to have difficulty remembering geometric terms such as opposite, congruent, parallel, and parallelepiped. This problem seemed to be related to their lack of proficiency in the English Language which reinforces the conclusions made by Fuys et al. (1988) that progress into higher levels was also influenced by instruction and ability, particularly language ability. Finally, the short period of this study did not permit sufficient time needed to revisit the relevant phases of learning so as to understand better the relationship among properties of cubes and cuboids during Learning Period 2.

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