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Mastery of Basic Addition and Subtraction Facts: How Much and What Kind of Drill, at What Time is Sensible?

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This case study reports on two Austrian first-grade teachers who centered their teaching of basic addition and subtraction on derived fact strategies. The teachers differed distinctly in their handling of drill. Only one of them made considerable efforts at drilling strategies. In her class, at the end of grade one, there was a high share of fact retrieval even with sums and minuends greater than 10. Conversely, in the other class, more students used derived facts strategies, and some of them repeatedly performed at low speed. While students in both classes demonstrated a sound understanding of strategies, the emphasis on drill in one class might have had negative consequences on one child's motivation. Possible consequences for future research are discussed.

Keywords: *Basic addition and subtraction, derived fact strategies, fact mastery, automaticity, strategy drill.*

In the current literature on mathematics education there is widespread agreement that *eventually* pupils should solve the basic addition and subtraction facts up to 20 without using counting strategies (cf., e.g., Gerster, 2009; Van de Walle, 2004). However, there is fewer consensus when it comes down to defining more differentiated target sets. First, there is the issue of *what precisely* the teaching of basic addition and subtraction should strive. Should *strictly immediate retrieval* of *all* basic facts be the goal, or does *sufficient fact mastery* also include "derived fact strategies" (Dowker, 2014; Steinberg, 1985)? Such strategies also referred to as "reasoning strategies" (Baroody, Purpura, Eiland, Reid, & Paliwal, 2015) build upon *relations* between facts. Using a derived fact strategy (DFS), a child who already knows 6+6=12 could solve, e.g., 6+7 by adding one.

Scholars such as Van de Walle (2004) or Gerster (2009) clearly advocate the consideration of quick ("in about 3 seconds", Van de Walle, 2004, p. 156), effortless and correct use of such strategies as part of fact mastery. However, other scholars insist on the importance of actually *knowing by heart* at least the number combinations up to 10 (Schipper, 2009). The Common Core State Standards of Mathematics (CCSSM) even formulate the clear expectation that "by end of grade 2" pupils should "know *from memory all sums of two one-*

digit numbers" (NCTM, n. d., p. 19). Second, and connected with the question of the desirable *range* of automaticity, there is the question of *time*. Until the end of grade 1, the CCSSM expect only *fluency* (including, e.g., counting on), but *not automaticity,* and fluency only up to 10, not yet 20 (NCTM, n. d.). Conversely, some scholars consider it of paramount importance that *already by the end of grade 1* children know *all* possible number combinations up to 10 actually *by heart* (cf., e.g., Schipper, Dröge, & Ebeling, 2015).

Expectations about what can and should be accomplished in which grade are an important issue since typically, when mathematics educators deal with basic fact mastery, they are highly concerned about the risk of "premature drill" (Van de Walle 2004, p. 158). From a modern mathematics education viewpoint, children's fluency with basic facts is not an end in itself, but rather a means to engage in mathematics as an activity of investigating numerical patterns (Freudenthal, 1973; Wittmann & Müller, 2004). Pushing children to learn basic facts by rote within a short time will not contribute, to say the least, to their comprehension of arithmetic as a meaningful endeavor. Moreover, theoretical approaches (cf. Baroody & Tiilikainen, 2003) as well as empirical findings (cf., e.g., Baroody et al., 2015, Dowker, 2014; Gaidoschik, Fellmann, Guggenbichler, & Thomas, 2017) suggest that concentrating on drill to foster rote learning might in fact be less effective in supporting fluency than mathematical instruction that focuses on DFS.

Of course, being able to retrieve at least some number combinations quickly is a prerequisite for applying DFS, as children need a set of known facts as a basis from which to derive. How do children obtain such a set? According to Van de Walle (2004), simply the "repeated use of [a DFS] will almost certainly render it automatic" (p. 156), however, empirical findings challenge this confidence. Studies indicate that in certain settings a substantial share of students do not shift from DFS to automaticity with basic facts (cf. Clarke & Holmes, 2011; Cumming & Elkins, 1999). Since working memory capacity is restricted, this may cause problems when it comes to solving mathematical tasks of higher complexity (cf. Baroody, Bajwa, & Eiland, 2009).

Of course, Van de Walle (2004) makes a plea for the "drill of strategies" once they are "established" (p. 159). Likewise, Wittmann and Müller, influential authors in German speaking countries who strongly advocate for conceptual understanding, markedly stress the need of practice that aims at automating facts as well as procedures. At the same time, they warn vehemently against rote learning and point at the risk of moving too soon from laying the foundation to automatizing (Wittmann & Müller, 2004, p. 14).

Research Questions

Against the backdrop outlined in the above section, it seems plausible that those teachers who try to follow recommendations of up-to-date mathematics education might become uncertain as to when, to what degree, and in what ways they should integrate drill into their teaching of basic facts. As a starting point for considerations for future research, this paper contributes case studies of two Austrian teachers and their respective classes who, as will be documented, represent two considerably different ways of dealing with drill within a teaching approach focused on conceptual understanding.

The two teachers had attended the same professional teacher development program. A main target of this program had been to convey up-to-date didactical recommendations for teaching arithmetic in grade 1. As a basis for understanding what might actually have happened in the two classrooms, the paper will first give a short outline of the program. Second, the study reports on common features and differences of the instructional measures the two teachers had presumably set throughout the school year. Against this background, the study addresses mainly the following questions:

a) Having received comparable targeted instruction on DFS, yet combined with a distinctly different amount of drill, to what extent do children in two classes at the end of their first school year demonstrate fact mastery on basic addition and subtraction tasks?
b) Are there any indications for qualitative connections between differing classroom practices regarding drill and children's calculation strategies and attitudes towards arithmetic?

Methods

Participating Children

This case study reports on two first-grade teachers and their respective classes from different public primary schools in Carinthia, Austria. Class A, situated in a small village, included 23 pupils. Only 17 of them were regular first-graders (6 to 7 years old), and only 11 of these (8 boys, 3 girls) took part in the study, due to the reservations of some parents who did not want their children interviewed. Twenty-one pupils, 17 of them regular first-graders, attended Class B, an urban class with a mixed catchment area. Parents gave consent to interview 16 pupils (7 boys, 9 girls).

Participating Teachers

Both teachers had participated in a teacher development program that a group of expert teachers had implemented independently from the study author (for details, cf. Gaidoschik et al., 2017). The program had started in 2012/2013 with two three-hour input sessions devoted to first-grade arithmetic, regarded by the program's leader only as a "teaser" to arouse interest in the subject (cf. Gaidoschik et al., 2017). The actual training happened in 2013/2014, while the teachers were teaching Class A and B, respectively. Throughout this year, both teachers were getting support from an expert teacher, who visited their schools for two hours each week. During each visit, these mentors spent one hour with their mentees outside the classroom on the planning and developing of lessons.

Additionally, they would either observe the classroom activities in order to provide their mentee with feedback, or act as team teachers. The expert teachers did not take part in this study, neither as co-researchers nor as interviewees.

In the course of their training, Teachers A and B had received a paper compiled by the program's leaders to serve as a guideline for their teaching. A qualitative content analysis drawn by the author of this paper shows that this guideline as a whole heavily relies on publications by Gaidoschik (e.g., 2007). The guideline recommends taking targeted efforts within the first months to elaborate and consolidate the conception of numbers as composed of other numbers. The paper advises to support children in reaching automaticity already within the first half of the school year with at least those compositions of numbers that contain five as a part, as well as with a limited amount of basic tasks. These are the "one-more facts", the "one-less facts" up to 10 (6+1, 10–1 etc.), the doubles from 2+2 to 5+5, and, finally, the "ten facts" (3+7 etc.). These facts should serve as "helping facts" from which to derive other tasks.

In the following, the paper gives a brief outline of how to use a guided-discovery-learning approach to work out DFS as a smart and convenient way to solve addition and subtraction tasks (cf. Gaidoschik, 2007). What the paper does not reproduce, though, is the further recommendation given by Gaidoschik (2007) to engage children in 5-to-10-minutes daily-training sessions clearly targeted at reaching automaticity in DFS use at their individual pace. In fact, throughout the paper there is no statement at all, on whether teachers should promote automaticity in solving tasks beyond the helping facts.

In order to reconstruct for this study at least main features of the instruction which Teachers A and B actually gave to their pupils, at the end of 2013/2014 the study author visited one arithmetic lesson of each teacher and conducted semi-structured interviews with both teachers. Moreover, he drew a qualitative content analysis of textbooks and other materials used in the classrooms.

The interviews indicated that both teachers had been equally striving to comply with the main recommendations of the guideline as outlined above. Both emphasized the importance of classroom discussions about solution pathways as a means to spread, consolidate and refine any single DFS once a pupil had brought it up. Likewise, both of them pointed to the importance of additional direct instruction for those children who were struggling with adopting DFS on their own. The additional analysis of the learning materials supported the teacher's self-report in all main aspects. The classroom visit added to the credibility of their statements.

At the same time, interviews and analysis of teaching materials revealed differences in both teachers' handling of drill. In her self-report, Teacher A stated that she had strongly advised parents to practice at home exactly those tasks and strategies that had been dealt with in the classroom during a certain time-period. To help guide this homework she organized a total of five parents' evenings over the school year. During the first months, while her class was working on the helping facts, students got lists of the same few tasks they were supposed to practice at home with their parents' assistance. As soon as they themselves felt ready to solve these tasks by quick retrieval, they should volunteer for an oral exam. Later on, following the treatment of a single derivation strategy such as "near doubles" in the classroom, tasks such as 6+7 or 8+7 that could be derived by using that very strategy were put together on yet another list. Again, children should train these tasks at home as well as in daily five-minute classroom exercises.

Finally, in the last months of the school year, Teacher A established a daily individual classroom practice involving a CD-ROM. Her students used a training mode requiring them to solve tasks up to 20 as quickly as they could. Teacher A periodically examined their progress. When they finally had reached the expected speed level, she rewarded them with a diploma.

Teacher A left no doubt that by spending that high amount of time and energy on drill she followed her own conviction that "knowing these tasks by heart in the second grade is very important for [the children]". She declared that she had *not* learnt this from the program.

Teacher B, by contrast, expressed skepticism over drill measures of any kind. She repeatedly emphasized that she did not want pupils to learn by heart anything they had not thoroughly understood: "I think it's more important to know how to figure it out than to answer quickly as a shot." She herself strongly objected to qualifying any of her classroom routines as drill, even if she made clear that her students, too, had spent a substantial part of the first half-year on practicing the helping facts. Unlike Teacher A, however, she had never expected or asked parents to engage heavily in that matter. With regard to fluency, she substantially relied on daily classroom conferences on DFS and having students repeatedly describe their solving pathways. She was confident that students, once they had understood the strategies and acquired routine in using them, eventually would reach automaticity in all the basic facts anyway. Consequently, in Class B, strategy drill played a minor role, if any.

Methods Used to Establish Children's Solution Strategies

The study author interviewed the children in a quiet room separated from the classroom during school time two weeks before the end of the school year. All interviews were videotaped. The children solved 14 addition and subtraction tasks up to ten and 8 tasks with sums and minuends between eleven and eighteen. Tasks were presented one by one both verbally and simultaneously as written on a flash card; children were asked to solve each task mentally in the way they usually would, and state the result verbally. Immediately thereafter, they should explain or show how they had arrived at the solution.

Two additional tasks were used to examine children's conceptual understanding of DFS apart from their computing performance. The child was shown, at the end of each interview, two pairs of terms, each of which printed on a DIN-A7 card, first 7+7 and 7+8, second 9+9 and 18-9. The child was asked not to compute, but to tell whether the first task of each pair could be of help for solving the second one, and if so, in what way.

Evaluation Procedure

Coding of the children's computing strategies was based on their verbal reports, also taking into account gestures, facial expressions and solving time as possible indicators of counting strategies. A fellow researcher double-checked ten percent of the ratings, randomly selected. Disagreements were very few and resolved through discussion.

Out of the various strategies that are described in the research literature to be applied by children of this age group (cf. Gaidoschik, 2012), the pupils of both classes essentially used but two, namely either fact retrieval or a DFS (see Table 1). A pragmatic approach was taken when considering the difficulty to discern whether a child has spontaneously retrieved a known fact from memory or quickly applied a DFS (Verschaffel, Greer, & De Corte, 2007). The main interest of the study is not on fact *retrieval*, but fact *mastery* as defined by Van de Walle (2004) as "giving a quick response (in about 3 seconds) without resorting to non-efficient means such as counting" (p. 156). Obviously, this includes both fact retrieval and DFS.

Therefore, a strategy was rated as "*presumably* retrieved" whenever a child produced a correct answer *spontaneously* stating that he or she "just knew it". This concedes the possibility that the child might in fact have derived the answer very quickly.

Whenever there was an at least short hesitation before the child produced the answer, but no indication of overt or mental counting, *and* the child described a task-fitting DFS as his or her solution path thereafter, the strategy was rated as "*presumably* derived". This leaves open the possibility that the child might in fact have retrieved the answer, yet with some delay, and put forward a reasoning strategy just because s/he believed that s/he was supposed to *justify* the answer. In any case, the *time* needed to produce a solution was taken, allowing for the attribution of "fact mastery" to all cases when a child answered correctly in about 3 seconds or less.

Results

Extent of Fact Mastery in Classes A and B

During the interviews with Class A, there were 242 instances of task solving up to 20 (22 tasks by each of the eleven students). Counting on from larger was observed in one single instance, as was the non-counting use of finger-patterns (cf. Gaidoschik, 2012). Apart from that, all trials were rated as either (presumably) retrieved or derived. Children missed the right answer in only three instances.

The 16 students of Class B did not use counting strategies at all, but fingers were employed without counting in three cases. Class B students either retrieved or derived all other tasks, producing erroneous results in only two cases.

As can be seen in Table 1, there were striking differences regarding the respective shares of tasks that children in Class A and B (presumably; see above) retrieved or derived, as well as regarding the extent of fact mastery in both classes as defined by their solution times.

Table 1

Frequency of presumably retrieved and derived solutions and of fact mastery in Classes A and B

	sums/minuends smaller than 10			sums/minuends greater than 10		
	retrieved	derived	fact mastery	retrieved	derived	fact mastery
Class A (n = 11)	81 %	16 %	95 %	72 %	28 %	91 %
Class B (n = 16)	72 %	27 %	90 %	41 %	56 %	80 %

Table 1 shows that Class B students solved a *higher share of tasks by DFS* than their peers from Class A did. This pertains particularly to tasks with sums and minuends greater than ten (56 % DFS in B compared to 28 % in A). Along with this, the share of *fact mastery was lower* in B (80 % for sums and minuends greater 10) than in A (91 % for these "harder facts"). The joint occurrence of a still relatively high share of fact mastery and a more than 50 % share of DFS in Class B indicates that Class B students were generally rather fast in deriving. However, in addition to Table 1, it is important to note that in Class B 22 out of 352 trials (6.3 %) took ten or more seconds. Six out of 16 children in this class needed that long on more than one task. Conversely, in Class A only two out of 242 solutions (0.8 %) took that long.

There were no such differences between the classes regarding the helping facts. Table 2 shows that the share of automaticity in solving these tasks was (almost) 100% in both classes.

Additional Observations

In both classes, students evidently were experienced and, for the most part, proficient in explaining their solution pathways during the computation part of the interviews. In solving the additional tasks (see above), no child in either class had any problem in verbalizing how 7+8 relates to 7+7 (e.g., "in 7+8 you add one more, so the outcome is one more"). Only about 45 % of both classes alike explained the relation between 18-9 and 9+9 equally adequately (e.g., "18 consists of two nines, so if you take away one nine, the other remains").

Table 2
Frequency of presumably retrieved/derived "helping tasks" and of fact mastery in Classes A and B

Class	2+2, 3+3, 4+4, 5+5 (doubles)			2+5, 3+5 ("power of five") 3+7, 4+6 ("ten facts")		
	retrieved	derived	fact mastery	retrieved	derived	fact mastery
Class A (n = 11)	100 %	0 %	100 %	95 %	5 %	100 %
Class B (n = 16)	100 %	0 %	100 %	95 %	5 %	98 %

Apart from these data, six out of eleven children in Class A showed some kind of attempt in terms of trying to arrive at a solution even before the interviewer had finished with posing the task. Nothing similar occurred in Class B.

One Class A student, who resorted to DFS markedly more often than her classmates, declared, "I don't really like math", adding, "I'm not really good at it." Yet, she demonstrated fact mastery in twelve out of 14 tasks up to ten, and in six of eight tasks up to 20. In Class B, not a single similar occurrence of negative self-image was observed.

Discussion

In comparison with other samples that have been interviewed at the end of grade 1 (c.f., e.g., Gaidoschik, 2012; Henry & Brown, 2008), both classes had reached a remarkably high share of fact mastery. Of course, the present findings are limited by the fact that the children's arithmetic instruction can only by inferred on based upon teacher's self-report, one classroom visit, and analysis of teaching materials. However, these three coincide in all main aspects, indicating that both teachers indeed had been putting a clear focus on the elaboration and practice of DFS throughout the first school year. Against the backdrop of pertaining research (c.f., e.g., Dowker, 2014; Gaidoschik et al., 2017), it seems plausible that the teachers' practices influenced both the *similarities* as well as the *differences* between the performances of their respective classes, as will be argued in the following.

Similarly, all students of both classes solved at least some tasks by using a DFS. In describing their strategies, as well as when directly asked to explain how 7+7 and 7+8 are linked to each other, they demonstrated a sound understanding of the underlying quantitative relationships. Even if some children of both classes did not perform equally well in explaining the relationship between 9+9 and 18-9 (cf. Baroody et al., 2015, on children's difficulties with "subtraction as addition"), both classes must be considered highly proficient in verbalizing operational relations. To ascertain this, we may compare the present sample with the random sample of 139 first graders interviewed on the very same tasks by Gaidoschik (2012). Only some 60 % of that sample provided an elaborate explanation of how to derive 7+8 from 7+7, and only 13 % did so with 18-9 and 9+9.

Another consistency between the two classes, and again in accordance with what both teachers indicated about their teaching, is the particularly high proportion of helping facts that the children presumably solved by immediate retrieval. Both teachers convincingly stated that they had devoted substantial time and attention particularly to this group of tasks. Again, it is instructive to compare their pupils' performance on these tasks with the random sample interviewed by Gaidoschik (2012). Within this sample, only about 35 % solved the "ten fact" 3+7 by fact retrieval, and 41 % did so on 4+6. Of course, those children's instruction had not focused on DFS, nor had their teachers taken targeted measures to consolidate helping facts (Gaidoschik, 2012).

As for the *differences*, it is evident that Teacher A had made much greater efforts at strategy drill. This coincides with the higher share of fact mastery in Class A. Remarkably, six out of eleven students of Class A were eager to put forth the answer almost before the interviewer had finished the question. This might be a side effect of that drill, and presumably a harmless one. Rather worrying, though, is the self-image of being weak at arithmetic that one Class A student obviously had developed, although she showed a sound understanding of DFS and a high share of fact mastery. Teacher A's focus on speed by presenting timed tests might well have contributed to that self-image. It seems plausible that an instructional design like the one implemented in Class A bears the risk to put under stress and even demotivate students who struggle to keep up with the pace of their faster peers.

Furthermore, such a design requires a high level of support from parents, which, in turn, depends not solely on their will but also on their ability and socio-economic backgrounds. Working in a small village, Teacher A was rather successful in obtaining that support, but colleagues who would like to follow her in that respect might be limited by different settings.

Teacher B deliberately did without obliging parents to administer drill practice at home. In her classroom, she seemed to have restricted drill essentially to the helping facts up to 10. However, research calls into question her confidence that all her pupils will eventually reach fluency on all basic facts solely by the repeated use of DFS. By the end of the first school year, six out of sixteen children of her class repeatedly needed ten or more seconds to solve a basic task. It was not possible to interview the same children again in grade 2. Future longitudinal studies could shed a light on the question whether patience and persistence in stimulating children to verbalize their DFS are sufficient to achieve mastery on all the basic facts, but there are severe problems of research ethics connected with such an endeavor.

Conclusion

Properly administered design research studies might offer a solution. In dealing with the issue of “balancing innovation and risk” within a design research project, Edelson (2006, p. 104) points at “ongoing evaluation” as a key feature of design research, calling it the “second way to manage the risk” that an innovative instruction design might indeed prove to be disadvantageous for the participating children. Within the iterative setting of a design research study, constant formative evaluation may lead to modifications of the design in the ongoing teaching experiment and even to its termination. Before that, the design itself, of course, has to be grounded in “research or a sound theory” (Edelson, 2006, p. 103).

As outlined in the introductory section of this paper, there is research as well as sound theory, but at the same time a lack of comprehensive, consistent recommendations for teachers about how to deal with the issue of drill while trying to secure conceptual understanding in the field of basic arithmetic. The issue is relevant, since automaticity of basic facts is becoming increasingly important, or even necessary, when tasks are getting more complex in higher grades. Future design research in this field seems to be an adequate way to expand our knowledge and understanding of arithmetical learning. At the same time, this kind of research offers the chance to generate evidence-based recommendations about whether, how, and to what extent to integrate drill in early grades teaching without compromising other targets such as conceptual understanding and a positive attitude towards mathematics.

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