Journal of Mathematics Education
July 2017, Vol. 10, No. 1, pp. 1-16

DOI: https://doi.org/10.26711/007577152790001

"Zahlenblickschulung" as Approach to Develop Flexibility in Mental Calculation in all Students

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Flexibility in mental calculation is a highly pursued goal in elementary math education, and the presented qualitative study focuses on its development in 20 first grade students from different instructional contexts who have difficulties in learning arithmetic. The crucial question was how the approach "Zahlenblickschulung" supports the development of flexibility in mental calculation especially in this group of students. The paper is structured in two parts: In the first part, the term "Zahlenblick" is defined based on the consideration of the different notions of number sense and structure sense. Furthermore, the approach "Zahlenblickschulung" is explicitly described and illustrated by specific examples. The second part of the paper is dedicated to present the methods of data collection and data analyses, as well as the essential results of the study. In this study, the students from different German classrooms have been interviewed three times during first grade and one time at the beginning of second grade. The interviews focused on students' solution strategies and argumentation strategies, and data was qualitatively analyzed with the same focus. The results suggest "Zahlenblickschulung" as an instructional approach that supports recognition of number patterns and numerical relationships. Furthermore, they indicate that this recognition is a crucial requirement not only for developing flexible and adaptive expertise, but also for learning to calculate itself (beyond counting).

Keywords: flexibility, mental calculation, less advanced students.

In the last two decades, flexible mental calculation has been increasingly considered as an important goal in elementary math education (Lorenz, 1997; Selter, 2000; Verschaffel, Luwel, Torbeyns, & Dooren, 2009). Moreover, various research with different aims have been conducted in this field. Distinct studies revealed that "Zahlenblickschulung"[1] supports the development of flexible mental calculation. Those studies predominantly

[1] We keep the German term "Zahlenblick" because the concept that is connected with that term gets lost in translation and cannot be expressed with the linguistically related term "number sense".

focused on students from second and third grade (Grüßing, Schwabe, Heinze, & Lipowsky, 2013; Heinze, Marschik, & Lipowski, 2009; Heinze, Schwabe, Grüßing, & Lipowsky, 2015; Rathgeb-Schnierer, 2006). There is hardly any research regarding the development of flexible and adaptive expertise in first grade students. Additionally, there is still no satisfactory answer to the question at what time flexibility in mental calculation should be supported and developed: Is it after students have developed basic abilities in mental calculation or is it with the beginning of first grade (Threlfall, 2009; Verschaffel, Torbeyns, De Smedt, Luwel, & van Dooren, 2007)? In addition, there is still research needed to answer the question whether less advanced students are able to develop flexible mental calculation or not (Geary, 2003; Peltenburg, van den Heuvel-Panhuizen, & Robitzsch, 2011). With its aim and methodological approach, the presented project tries to bridge the described gap.

Theoretical Framework

The recognition of problem characteristics and numerical relationships is a crucial prerequisite for flexible and adaptive expertise (Rathgeb-Schnierer, 2006; Threlfall, 2009). "Zahlenblickschulung" is a special approach for mathematics education that puts the emphasis on number patterns and numerical relationships. Within the theoretical framework, this approach is described in detail as well as the notion of flexibility that underlies this study.

"Zahlenblickschulung"

The approach "Zahlenblickschulung" pursues the aim to develop "Zahlenblick". To clarify what "Zahlenblickschulung" means, we describe the meaning of "Zahlenblick" and its related concepts like number sense and structure sense.

Number sense, structure sense and "Zahlenblick". The concept *number sense* is defined in two distinguished ways; as result either of experienced based development or as inherent skill.

> "With respect to its origins, some consider number sense to be part of our genetic endowment, whereas others regard it as an acquired skill set that develops with experience." (Berch, 2005, p. 333)

Regarding the construct *structure sense,* the notions are quite similar. Lüken's definition (2010) of early structure sense is reminiscent of inherent competence, whereas Linchevski and Livneh (1999) point out the necessity of its development.

"Zahlenblick" is a result of development, and means the competence to recognize problem characteristics, number patterns and numerical relationships immediately, and to use them for solving a problem (Schütte, 2004a). Comparing number sense, structure sense and "Zahlenblick", it is obvious that the meaning of number and structure sense as acquired skill is quite similar to

whole-concept, getting insight in various partitions, and learning to reason about their thinking.

Figure 1. *Visual number pattern.*

(2) *Sorting the problems:* This activity emphasizes on sorting addition and subtraction problems up to 20 in categories like “I need to count it”, “I know it” and “I know a trick to solve it” (Rechtsteiner-Merz, 2011). When introducing the activity, students were encouraged to sort each single problem into a suitable category without solving it. Therefore, they need to carefully look at each problem and focus on problem characteristics as well as numerical relations. All three categories are subjective, and lead to different sorting results among the students. Nevertheless, the students can talk about their sorting, and compare their problems in each category. While discussing with other students and the teacher and comparing their sorting, they might discover new patterns, new relations or new strategic means. This activity aims to reflect about problem characteristics on a more general – algebraic – level, for instance: What is the reason for assigning several different problems to one category? Do all problems in one category have similarities? Are there some problems in the category “I need to count it” which are similar to those in the category “I already know a trick”? Is it possible to use problems already known by heart to facilitate the solution of problems from the other categories?

(3) *“Problem-family”*: A “problem-family” is described as a set of structurally related problems (see Figure 2). This activity intends to put the emphasis on structural relations between a set of problems. (Rechtsteiner-Merz, 2011). The students start with one problem (e.g. 5+5=10) and arrange many cards with related problems (e.g. 5 + 6, 6 + 6, 4 + 6 etc.) around this first one with the aim to make the relations visible (Figure 2). Subsequently, the students were encouraged to describe their arrangements, and give reasons for their decisions. This activity does not focus on solving problems, but on recognizing problem features and relationships. Within this activity, students discover relations between problems; this is a relevant prerequisite for going beyond counting and using derived fact strategies.

beginning of second grade, contained addition problems up to 100 (Table 1). In Germany, numbers up to 100 are introduced in second grade, and at the beginning of second grade, students are not familiar with two-digit addition problems.

Table 1

Addition Problems for Interview 1 – 4 (Rechtsteiner-Merz, 2013, p. 186)

In all interviews, students were first asked to sort problems into the categories “easy” and “hard” and to reason about their sorting. In the second step, they were encouraged to solve the problems categorized as “easy”, and some of those that were judged as “hard”. After each solution process, the students were requested to explain their solution method. Since the standard

solve problems up to twenty predominantly by calculating. Therefore, they always exhibited the same solution procedure without referring to any problem characteristics. They argued for instance: “I do it always like this” or “like always I go up to ten and then the rest”. Students who exhibited *partly basic facts with relational expertise* used different strategic means and relied on problem characteristics. They were able to describe the solution process and gave reasons for their strategic means in an elaborate way like in the following example: “These problems are easy (*points to* 8+5 *and* 4+9), because here it’s one less and here it’s one more (*points to* 4 *and* 9)”. Students who depicted *basic facts extended with relational expertise* relied on basic facts with addition problems up to twenty. Additionally, they were able to solve two digit problems with numbers higher than twenty based on recognized characteristics and numerical relationships (even if this is not a topic in first grade). For example, Lars argued in the last interview regarding the problem 91 + 4: “90 is a high number but if this here is small number (*points at 1 and 4*) I can find a solution”. This type is unique for first grade since all addition problems up to twenty can be memorized by heart.

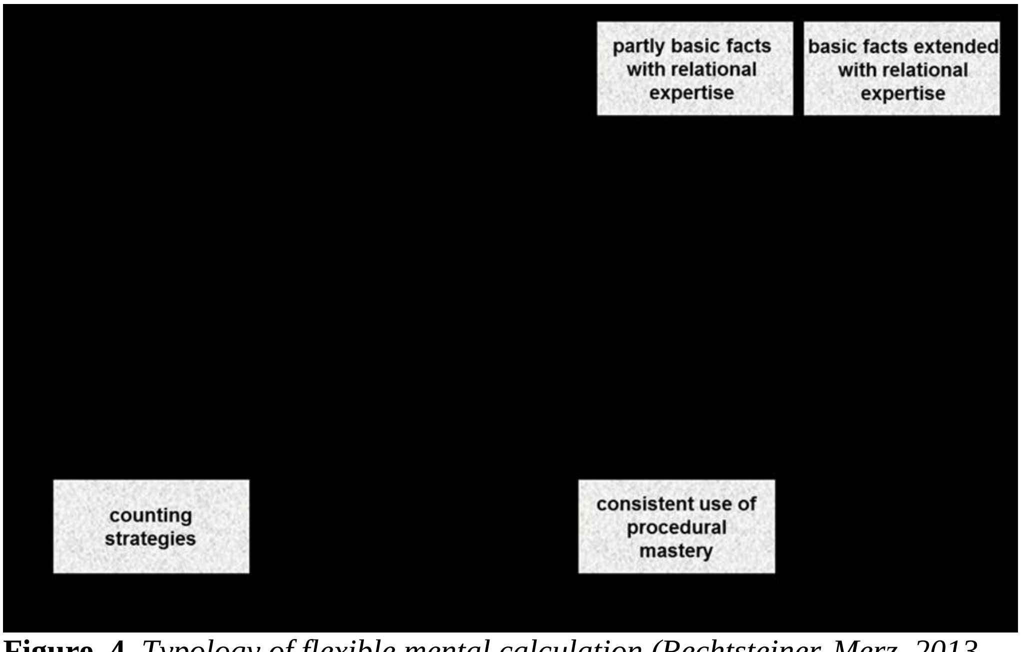

Figure. 4. *Typology of flexible mental calculation (Rechtsteiner-Merz, 2013, p. 243)*

Table 2 shows students’ distribution between the different types over all interviews. Each column represents one single type (e.g., CUPM represents the consistent use of procedural mastery, TT 1 stands for temporary type 1), and each line one of the four interview sessions. The numbers in the cells represent the amount of students who were assigned to the specific type. The first number refers to students without “Zahlenblickschulung”, the second number to

students when solving problems on addition and subtraction of three-digit numbers]. *Journal für Mathematik-Didaktik*, 21 (3/4), 227–258.

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