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How Mental Rotation Skills Influence Children's Arithmetic Skills

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In this paper, a comprehensive summary of research on the topic will be structured around the concepts of semiotic function and meaning, focusing on recent research in the influence of mental rotation skills over arithmetic skills on children. We search for underlying mechanisms that could explain such influence. As a result of the literature review, we conclude that there exist three possible mechanisms underpinning the relation of mental rotation skills and arithmetic skills: Working Memory, arithmetic-symbol reading capacity, and the concept of composition-decomposition of figures and numbers.
This study represents an effort for organizing and structuring, from an educational point of view, existing research on the relation between spatial and arithmetic skills in order to make it useful for instructional purposes.

Keywords: Mental rotation skills, arithmetic skills, Working Memory, meaning, semiotic function.

The relations between space and math are maybe one of the most studied and well-established findings in cognitive science (Mix & Cheng, 2012). Children and adults who perform better on spatial tasks also perform better on tests of mathematical ability (Cheng & Mix, 2014); for instance, in preschoolers and first graders, spatial skills are a good predictor of math skills (Laski et al., 2013; Rasmussen & Bisanz, 2005; Verdine, Irwin, Golinkoff & Hirsh-Pasek, 2014; Zhang et al., 2014; Zhang & Lin, 2015). However, in spite of this evidence, the mechanisms that could explain such relations have not been well studied yet.

In mathematics education, spatial skills are normally linked to geometry and geometric reasoning in the curriculum (National Council of Teachers of Mathematics, 2000) and to numerical reasoning for some researchers (Godino, Fernández, Gonzato, & Cajaraville, 2012). Nevertheless, the relation between pure spatial skills (e.g., mental rotation, orientation, etc.) and pure math skills (e.g., calculation) has not been well studied (Cheng & Mix, 2014); we argue that research in this direction can help improve math performance in children by making clearer some spatial factors related to mathematics learning, as will be shown in this paper.

Our research question is: what underlying mechanisms serve to explain the observed relations between mental rotation skills and arithmetic skills on children?

Based on some hints arising from our research experience in the subject and supported by a literature review, we propose three possible mechanisms that can potentially explain the specific relation between mental rotation skills and arithmetic skills. First, a theoretical mechanism; Working Memory would be a common 'resource' for both spatial and mathematical skills and could serve as a bridge to connect them. Second, a procedural mechanism; visual-spatial skills could play an important role in the process of identification and interpretation of arithmetic symbols and could therefore help to avoid some educational-related mistakes when reading mathematical expressions. Third, a conceptual mechanism; composition and decomposition of figures would put into play corresponding arithmetic concepts that are useful in the process of addition by distribution and association of numbers (e.g., $9+6 = 9 + (1 + 5) = (9 + 1) + 5 = 10 + 5 = 15$).

The paper is structured as follows: the first section describes a prevalent conception about skills in cognitive science and proposes an alternative semiotic conception, which we consider more well suited for educative purposes. The second section takes a closer look at the concept of skill from a semiotic perspective. In the third section, we analyze the possible mechanisms that can explain the observed relations between mental-rotation and arithmetic skills; we consider cognitive science research findings as well as semiotic interpretations of those findings. The fourth section presents the conclusions of our study and suggests future research guidelines.

Skill Conceptions

Skills as Traits

In cognitive science, most of the studies on skills and their relations are based on the presumption that skills can be measured using validated-test scores applied over statistically significant samples of the target population. Underlying this presumption there is the language and worldview of traits and behavioral psychology (Mislevy, 2008). Skills are considered as features of persons that are relatively stable in different situations and can be measured quantitatively. Similarly, relations between skills are also determined using quantitative methodologies. By its own nature, these findings are useful to elucidate patterns in behaviors and guide general educational policies (Mislevy, 2008). Nevertheless, this worldview is problematic for education scenarios where teaching and learning are considered as evolving processes and cognitive student capacities are considered dynamic facts.

Skills from a Semiotic Perspective

From an educational semiotic point of view (Godino, Wilhelmi & Lurduy, 2011; Sáenz-Ludlow, 2016), we define a *skill* as the understanding of the meaning of a concept. In this perspective, the contributions of cognitive science can be organized into a frame suitable for educational scenarios: firstly, meaning emerges from a *relation* between teacher and student in a specific *context* and implies a *negotiation* process. Secondly, meaning is more a process than a fact; it *evolves*. Thirdly, meaning can be analyzed as a complex process, implying different *components* and *interactions* (Duval, Sáenz-Ludlow, Uribe Vasco, & D'Amore, 2016).

In the next sections, we will delve deeper into the conceptual link between meaning and skills and use the aforementioned characteristics of meaning to organize diverse findings on skills from cognitive science research.

Semiotic Function, Meaning and Skills

We use a semiotic approach to analyze the concept of skill: the Onto-Semiotic Approach to research in mathematics education (OSA). In this theoretical framework, a semiotic function is a relation of correspondence between an antecedent (signifier, expression) and a consequent (signified, meaning) (Godino, 2003). It is established by a person or an institution according to certain criteria or codes; "These codes in mathematical activity can be rules (habits, agreements) that inform the subjects implied about the terms that should be put in correspondence in the fixed circumstances" (Godino, Batanero & Font, 2007, p. 130).

Regarding the meaning of a mathematical object, this can be interpreted as the consequent of a semiotic function whose antecedent is a 'system of practices' realized by a person to solve certain kind of mathematical problems implying the object. For instance, the meaning of the 'median' is given in terms of an abstract entity emerging from a system of practices linked to certain statistical problems (Godino, 2003).

A skill, defined as the understanding of the meaning of a concept, implies both conceptual and procedural understanding, according to the aforementioned definition of meaning.

Emergence of Meaning

In mathematics teaching contexts, meaning emerges normally in classroom situations. Teachers guide the learning process following institutionalized strategies and concepts and students are expected to apprehend the meaning of those concepts and express them in an institutionally acceptable way.

In this sense, skills are defined and evaluated by the institution and correspond, to a certain degree, to related cognitive abilities of the student. The assessment of skills should consider not only the skills to be measured, but also the concurrent skills used in the evaluative process. Sometimes these 'secondary' skills alter the measurement process significantly; for instance, Piaget's experimental results has been criticized for using complicated language and unfamiliar materials and situations in the evaluation of children's mathematical and reasoning skills (Braine, 1962).

Learning context is also a relevant issue (Godino & Batanero, 1994). Meaning emerges from educational situations and is linked, at least in the beginning, to the specificities of those exemplifying situations. A misunderstanding of the concept of triangle in children illustrates this idea: some children do not recognize a triangle when it does not lay horizontal because the examples of triangles they have been given were always horizontally-lying triangles (López, 1990).

Meaning as a Process

Meaning understanding -and thence skills- evolves with the educative process. Not only meaning can become more abstract and generalizable, but also some of its underlying mechanisms can change:

> Functional magnetic resonance imaging (fMRI) investigations have shown that, onto-genetically, frontal functions are predominant earlier in development, and are gradually complemented by recruitment of parietal areas such as bilateral intraparietal sulci. This observation holds both for basic processing of symbolic number (Ansari, Garcia, Lucas, Hamon & Dhital, 2005; Holloway & Ansari, 2010) as well as for mental arithmetic (Kucian, Aster, Loenneker, Dietrich & Martin, 2008; Rivera, Reiss, Eckert & Menon, 2005). Cognitively, this suggests decreased reliance on domain-general resources such as attention and Working Memory during numerical cognition, and, in the case of mental arithmetic, it corresponds to a transfer from methodological (and computationally inefficient) strategies of calculation, to faster and more effortless strategies of memory retrieval (Grabner et al., 2009). (Popescu et al., 2016, p. 256)

In line with this conclusion, Van de Weijer-Bergsma, Kroesbergen and Van Luit (2015) report that the mechanisms that support the domain of arithmetic operations change through the cycle of primary education. Meaning understanding is then a dynamic process that has to be studied as such.

Components and Interactions of Meaning

A single skill can be interpreted as a complex process when looked in detail. For instance, the understanding of counting in children involves, according to Gelman and Gallistel (1986) , three underlying concepts: the one-to-one principle, the stable order principle and the cardinality principle. A more complex skill such as addition proficiency involves several related skills: calculation, estimation, derived fact strategies, memory retrieval, etc. Despite the fact that cognitive science studies have found some statistical relations between some of these skills, particular case studies show that there can be strong discrepancies between almost any couple of skills in specific children (Dowker, 2005). This fact shows that skills and meaning understanding are complex concepts that depend on particularities of each individual. Indeed, the

mathematical expertise of each student is linked to her or his own patterns of use and creation of signs and their related meaning (Sáenz-Ludlow, 2016).
In the next section, taking into account the described characteristics of meaning, we will analyze some research findings on spatial and arithmetic skills.

Mechanisms

We have reviewed scientific literature on the relation between spatial and arithmetic skills, focusing on the influence of mental rotation skills over arithmetic skills on children. Then, we have organized it by taken the concepts of semiotic function and meaning as a framework. In the next subsections we present the possible mechanisms of the relation that, supported by the literature, we have established.

Working Memory

Mechanisms. The concept of Working Memory (WM) is based on the assumption that human cognition shares some of the characteristics of a computer system. WM is a cognitive mechanism in charge of holding and processing temporal information. There are three main components of WM in the model proposed by Baddeley and Hitch: The Phonological Loop (PL), the Visuo-spatial Sketchpad (VSWM) and the Central Executive (CE). PL represents the verbal component of WM, while VSWM represents the non-verbal (visual) component. The CE controls the interactions and functioning of the other two components (Baddeley, 2007).

In spite of the fact that no theory of mathematical cognition gives an important role to WM, this construct has been frequently associated to mathematical knowledge (LeFevre, DeStefano, Coleman, & Shanahan, 2005). Mathematical practice implies the use and orchestration of several concepts and procedures and, as such, involves mechanisms of temporal storage and processing of information. Those mechanisms are also part of the definition of WM. Therefore, mathematical cognition and WM are conceptually related.

Raghubar Barnes and Hecht (2010) review several research on the relation of WM and mathematical skills and conclude that they are indeed related in adults, in children with math difficulties and, typically developing children. Nevertheless, those relations are complex and depend on several factors such as age, presentation format of the problems, type and development degree of the skill, etc.

Different components of Working Memory have different relationships with different mathematical skills on children (Bresgi, Alexander, & Seabi, 2017; Simmons, Willis & Adams, 2012). VSWM predicted magnitude judgment and number writing skills, CE explained addition skills. Similarly, according to the presentation format (vertical or horizontal addition), different WM resources are related to arithmetic skills (Caviola, Mammarella, & Cornoldi, 2012).

A relation between mental rotation skills and WM would imply, transitively, a relation between mental rotation skills and math skills, specifically, arithmetic skills (Caviola et al., 2012). Such relation was reported by Miyake, Friedman, and Rettinger (2001); in fact, mental rotation tasks require spatial storage and mental image processing, two of the main functions of WM.

In an educational context, it is important to determine if mental rotation training can influence arithmetic skills. A necessary condition for the realization of that influence is the malleability of spatial skills and, according to the aforementioned reasoning path, the malleability of WM. Uttal et al. (2013) and Stieff and Uttal (2015) concluded from meta-studies that spatial skills are malleable, Klingberg (2010) and Morrison and Chein (2011) suggest the same for WM.

In conclusion, WM is a possible bridge mechanism to explain the relation and influence of spatial skills over arithmetical skills.

Semiotic analysis. From a semiotic point of view, the meaning of math concepts -and therefore math skills- is bounded to the context from where they emerge. Even if poor VSWM capacity has been shown to be related negatively to math performance in sighted children (Ashkenazi, Rosenberg-Lee, Metcalfe, Swigart, & Menon, 2013), studies with early blind persons show a normal development of arithmetic skills in spite of the absence of vision and -consequently- VSWM. It can be concluded that VSWM is important for arithmetic skills in contexts where visual and non-visual constructs are used to teach arithmetic concepts. If vision is absent, other mechanisms will provide the necessary support and arithmetic skills will be developed differently.

Besides that, when skills are analyzed as dynamic processes instead of stable traits, new relations can be discovered, such as the VSWM mechanism that both spatial and arithmetic skills share.

Diagrammatic Visualization

Mechanisms. A diagrammatic representation differs from a sentential one in two basic facts: First, sentential representations are sequential (linear. e.g., a language sentence) while diagrammatic representations take place on a plane. Second, diagrammatic representation expresses topological and geometrical information explicitly, while sentential information has to express that kind of information implicitly (Larkin & Simon, 1987). This representational difference entails a semantic distinction: some kinds of spatial information are more well suited to be diagrammatically represented, for instance, locate information using maps (Shin, 1994).

The Onto-Semiotic Approach to research in mathematics education assumes that arithmetical expressions presented in the symbolic language of mathematics are sentential information (Godino et al., 2012). Nevertheless, there are some characteristics of arithmetic expressions that make them akin to diagrammatic information: first, according to the algorithms used to solve them, they are presented sometimes horizontally and sometimes vertically, going beyond the classical representation of sentential information. Second, they include symbols and operations that allow the expression to be rearranged, at least mentally; for instance, ‘5 + 3 = 8’ can be restated as ‘5 = 8 – 3’. Third, due to the use of parenthesis and precedence of operations, arithmetic expressions can be spatially grouped and regrouped according to the used solution strategy, revealing a non-negligible spatial component. Even more, Schneider, Maruyama, Dehaene, and Sigman (2012) have found, by examining eye movement sequences during the calculation of arithmetical expressions, that people look the expression as a whole, not sequentially, even in the absence of parentheses. This way of visualizing corresponds more to diagrams than to sentences.

Supporting the precedent reasoning, Cheng and Mix (2014) observed a positive effect on children of mental rotation training over arithmetic calculations of the kind ‘2 + __ = 7’. They hypothesized that maybe children, under the influence of mental rotation training, re-arranged the expression into the more conventional format ‘__ = 7 – 2’ and solved it more easily.

A conceptual link between spatial and arithmetic skills is proposed by van Nes and van Eerde (2010). Mental rotation and assembly of objects implies the processes of composition and de-composition of objects; similarly, the ability to compose and de-compose quantities is important for the development of numerical relations and operations (e.g., 9+6 = 9 + (1 + 5) = (9 + 1) + 5 = 10 + 5 = 15). The authors use this fact to suggest there exists a spatial structuring ability that can influence children’s arithmetical skills.

Semiotic analysis. The previous analysis, seen from a semiotic perspective, can be interpreted in terms of symbolic representation and meaning. Whenever the meaning of a concept is to be presented one must make use of a symbolic representation. The Onto-Semiotic Approach to research in mathematics education refers to this situation as the ‘duality’ content-expression (Godino et al., 2007). It is then important to note that a concept can have multiple expressions and –inversely- an expression can convey more than one concept. Arithmetic word problems, for instance, imply not only arithmetic skills, but also linguist ones.

In the particular case of written arithmetic calculations, the way of presenting arithmetic problems sometimes implies the use of visual skills beyond the typical visual skills used for language reading. Consequently, the result of the evaluation of arithmetic skills can be affected by visuo-spatial skills, even if there is not a conceptual link between both. Jiang, Cooper, and Alibali (2014) reported that spatial features of mathematical equations may influence how people solve and interpret them.

Other concept related to the understanding of meaning is the concept of ‘cognitive optimization’. We use it to describe the strategy by which a person interprets a symbol in a given context using –altogether- essential (conceptual) and non-essential (contextual) information. Braithwaite, Goldstone, and Maas

(2016) found that children rely on symbol distance and operation complexity to interpret arithmetic operators' precedence (for instance, 5+3 * 21 could be miss interpreted as (5*3) + 21). Similarly, children are used to employ 'shortcuts' to identify the required operation in word-problems, avoiding the conceptual understanding of the problem and following –for instance- syntactic hints (Dowker, 2005).

Then, the use of the cognitive-optimization strategy can lead to errors when non-essential information is used to interpret essential facts. Frequently these mistakes are due to failures in the educational process; the illustration of a concept should be varied and include all fundamental characteristics, while avoiding the generalization of non-fundamental ones (López, 1990). In the case of arithmetic, if problems are usually presented in a canonical format, children become prone to error when non-canonical formats are used:

> Research has shown that children's understanding and performance solving exact arithmetic problems depends heavily on the canonical problem format, in which arithmetic operations appear on the left side (e.g., 3 + 4 = 7, A. Baroody & Ginsburg, 1983; Behr, Erlwanger & Nichols, 1980; Seo & Ginsburg, 2003). Children's behavior on a variety of mathematics tasks suggests that children have internalized this canonical "operations on left side" format (McNeil & Alibali, 2004, 2005). For example, when children are asked to check the "correctness" of arithmetic problems written by a child who "attends another school," most mark sentences such as 10 = 6 + 4 as "incorrect" and change them to 6 + 4 = 10, 4 + 6 = 10, or even 10 + 6 = 4 (Baroody & Ginsburg, 1983; Behr et al., 1980; Cobb, 1987; Rittle-Johnson et al., 1999). Similarly, when children are asked to reconstruct a problem such as "3+5=6+ _" after viewing it briefly, many write "3 + 5 + 6 = " (McNeil & Alibali, 2004).
>
> (McNeil, Fuhs, Keultjes, & Gibson, 2011, p. 58)

Cheng and Mix (2014) suggested that mental rotation training improved children performance on arithmetic operations of the kind '2 + __ = 7' because they could re-arrange the expression –by 'rotating' it- into the more conventional format '__ = 7 – 2' and could solve it more easily. We complete this assertion by further suggesting that mental rotation skills improve the quality of the arithmetic symbol reading and interpretation process and help avoiding miss-interpretations due to cognitive-optimization related problems. The relation between spatial skills and arithmetic skills is possible due to some diagrammatic –and visual- characteristics of written arithmetic expressions.

Other Factors

Evidence. Zhang (2016) reports that spatial perception, visuo-spatial skills and executive function relate to the early acquisition of number competence among three-year-old children. Number competence includes number-counting skills, understanding of the cardinality principle and number comparison skills, among others. Those skills are fundamental for arithmetic proficiency. Synthetizing, visuo-spatial skills at a very young age are related to subsequent arithmetic skills.

Similarly, variance in number estimation skills in 6-year-old children can be explained by general intelligence and VSWM competence. Given that number estimation skills have also been related to arithmetic skills in primary school children (Booth & Siegler, 2006) , there is a transitive relation between VSWM and arithmetic.

The fore-mentioned relations can potentially explain the mechanisms connecting spatial and arithmetic skills. Nevertheless, there is not enough research on those topics to make reasonable hypothesis.

Social factors can also play a role in the comprehension of the spatial-arithmetic relations. Casey, Dearing, Dulaney, Heyman, and Springer (2014) underline the importance of maternal support during spatial problem solving for girls' spatial and arithmetic achievement.

Even if social influence can be difficult to analyze from a semiotic perspective, it is possible that social factors could influence cognitive skills and then its influence could be considered from a cognitive point of view. Byrnes and Wasik (2009) considered several factors that could influence math skills in kindergarten and primary-school children. They found that cognitive factors were the most important determinants of math skills, over socio-economical and opportunity factors.

Semiotic analysis. As already said, there is a lack of studies to make stronger statements regarding the connection between VSWM and spatial-arithmetic relations. Ideally, the inclusion of semiotic facts such as meaning emergence, meaning context and particular ways of understanding can offer valuable insights to better comprehend the mechanisms linking space and arithmetic.

Conclusion

We have reviewed a number of research papers on the relation of spatial and arithmetic skills and have found three possible mechanisms that could explain it: First, Working Memory is a common shared resource. Second, diagrammatic reasoning implies a visual capacity that can help interpreting and manipulating arithmetic expressions. Third, composition and decomposition of images can conceptually influence the capacity of association and distribution of numbers.

As part of the reasoning process and suggested by the literature, we have introduced the semiotic concept of 'cognitive optimization' to describe the strategy by which a person interprets a symbol in a given context using –altogether- essential (conceptual) and non-essential (contextual) information. We hypothesize that mental rotation skills can facilitate the reading of arithmetic expressions and help to overcome some difficulties due to cognitive-optimization mistakes.

We believe that a semiotic framework for guiding and analyzing future research on the topic can help to elucidate which are the main causal mechanisms linking spatial and mathematical skills. Such research should take into account the relation between statistical, non-contextual, group results and qualitative, contextual and personal results.

Likewise, a semiotic analysis favors the connection of learning and teaching processes (Daniels, 2003), linking the cognitive aspects studied by cognitive science with the institutional aspects considered in mathematics education.

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