

# An Investigation of Mathematics Learning Efficiency for High School Students in China

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*Learning efficiency is an important issue in mathematics education. In this quantitative research, authors developed a questionnaire from four dimensions: consciousness of mathematics learning, mathematics cognition, mathematics meta-cognition, and affection of mathematics learning, in order to explore the factors that affect mathematics learning efficiency and find the relationship between mathematics learning efficiency and mathematics academic performance. Results showed that affection of mathematics learning is the most important factor that influences learning efficiency. Mathematics learning efficiency is highly correlated with the advance of mathematics academic performance.*

**Key words:** *High-efficiency, consciousness, cognition, meta-cognition, affection.*

In the digital age, there has been an explosion in new knowledge and information. A qualified citizen in the future should be able to explore new knowledge, identify valuable information, accumulate necessary skills and integrate them to solve real world problems. Dealing with more knowledge and information in less time is a quality requisite to the new generation. Thus, even middle school or high school students should focus on not only how much knowledge they gain or how strong their abilities are, but also whether they can get them in a high-efficient way.

Research interest in mathematics learning efficiency in China has an extensive and diverse empirical history. Literature from 1993 to 2012 showed that more and more researchers examine mathematics learning efficiency. Generally, researchers explore the ways to interpret (Barrie, 1997; Brines, 2007; Wang, 2005), measure (Chen & Tang, 2009; Zhang, 2010), and improve (Karady, 2003) mathematics learning efficiency from two perspectives: psychology and pedagogy. From the perspective of psychology, researchers tend to adopt the cognitive load theory as a foundation to define learning efficiency and set the research framework (Clark, 2006). From the perspective of pedagogy, researchers typically focus on five topics: (1) How to construct

the theory framework of mathematics learning efficiency? (Tang & Wang, 2005; Wang, 2005; Wang & Wei, 2006) (2) What factors contribute to high-efficient mathematics learning? (Shu, 2006; Tang, 2004; Wang, 2006; Yang & Zhou, 1999) (3) What behavioral traits do high-efficient mathematics learning students have? (Diao, 2009; Shi, 2006; Sun, 2005) (4) What are the differences between low and high-efficient mathematics learning? (Ma, 2005; Ma, Lian & Feng, 2009) and (5) How to set a standard or framework to evaluate mathematics learning efficiency? (Chen & Tang, 2009; Pan, 2001; Wang, 2005; Zhang, 2010)

In terms of the factors of high-efficient mathematics learning, Yang and Zhou (1999) adopted a questionnaire to examine factors which affect students' mathematics learning efficiency. They designed the questionnaire from four dimensions: cognition, ability, non-cognition, and instructional process. Results indicated that interests, following by teachers in the class, confidence, calculation ability and logical thinking are the top five factors that play an important role in mathematics learning efficiency. Tang (2004) detected factors from six aspects: cognition, gender, instructional method, learning motivation, teachers' feature and teachers' expectation. She concluded that interests, focusing on importance, following teachers' thinking, logical thinking, calculation ability, doing homework independently and learning attitude are core factors for mathematics learning efficiency. Li (2006) examined elementary school students and summarized that learning method and affection have a significant contribution to leaning efficiency. There's an, especially, interesting relationship between anxiety and learning efficiency. Both high and low anxious levels have a negative effect on mathematics learning. Only an appropriate anxious level can arouse students' learning motivation; further, enhancing the learning efficiency.

As mentioned above, many researchers have conducted meaningful explorations in this area, however, there are still some problems: (1) Lack of theoretical framework. Few studies illustrated their research systematically based on the theory of mathematics learning, mathematics psychology or other theories; (2) Most studies concluded only based on descriptive statistics (e.g. means, standard deviation, frequency or percentage). (3) Lack of interpretation of reliability and validity. Few studies reported the reliability and validity of the questionnaire or framework.

The purpose of this study is to provide a further test of factors which contribute to mathematics learning efficiency. It is a quantitative research. We developed a questionnaire based on the theory of "four principal functional systems of the brain" (Tang & Huang, 2003) and some current research results to investigate factors of high-efficient mathematics learning for high school students. At this stage in the research, high-efficient mathematics learning will be generally defined as a kind of mathematics learning state in which students make the best of their time in order to obtain mathematics knowledge; improve mathematics cognitive structure; enhance learning ability and mathematics

literacy; and obtain a satisfactory academic achievement. Different from the definition in economics, efficiency is not simply the ratio of inputs to outputs because the object of education is the human being instead of production. We highly emphasized the learning process, in other words, how to learn mathematics efficiently. High-efficient mathematics learning is a relative concept. For the same mathematics learning outcome, if you spend less time or less energy, your learning efficiency will be relatively high, and vice versa.

In the present study, we focus on two questions: (1) What factors contribute to mathematics leaning efficiency? (2) What's the relationship between mathematics learning efficiency and mathematics academic achievement?

### **Theoretical framework**

Luria (1973) put forward the theory of "Three Principal Functional Units of the Brain". These units are necessary for any type of mental activity. Specially, the first functional unit includes the brain stem and the old cortex. It is responsible for regulating cortical tone or waking. The second functional unit is primarily responsible for the reception, analysis, and storage of information. The third block is responsible for programming, regulating, and verifying conscious activity.

Tang and Huang (2003) developed Luria's theory and put forward "Four Principal Functional Systems of the Brain". They claimed that there are four relatively independent and connected systems in the brain. The first functional system administers and regulates the tensity and wakefulness. The second functional system plays a significant role in accessing, coding and storing information. The third system refers to planning the procedure and regulating the mental activity. The fourth system refers to generating emotional experiences. All of the human behavior and mental activities result from the interaction of four functional systems (Figure 2).

For mathematics learning, four functional systems of the brain affect mathematics learning efficiency. The first functional system, administrating and regulating the tensity, wakefulness, and consciousness, leads people to adopt positive action and contribute an active impact on business (Huang, Zhang, & Jin, 2005). Students who have the consciousness of efficiency will learn more efficiently than those who don't have it. The second functional system, accessing, coding and storing information, is responsible for accessing, processing and understanding mathematics knowledge and skills, which relates to cognition in mathematics learning. Students with a strong capability in this field tend to have insight for mathematics knowledge and skills. Thus, they can learn and understand mathematics efficiently. Strong capability in mathematics cognition is a stable basis for high-efficient mathematics learning. The third functional system, planning the procedure and regulating the mental activity, is

related to mathematical meta-cognition. In mathematics learning, a high-efficient learning student should have the ability of scheduling their mathematics learning progress after class (reviewing notes, doing homework, reviewing chapter, and so on), thereby regulating their learning plan in accordance with specific conditions on time, monitoring their mathematics learning performance, and evaluating their mathematics learning achievements. These abilities are not only needed for mathematics learning, but also needed for the process of mathematics problem solving. The fourth functional system, generating emotion experiences, is related to the affection of mathematics learning.

Consequently, we designed the questionnaire from four dimensions to examine mathematics learning efficiency; (1) Consciousness of Mathematics Learning (CML); (2) Mathematics Cognition (MC); (3) Mathematics Meta-cognition (MM); (4) Affection of Mathematics Learning (AML). For each dimension, we chose some factors based on related theory (e.g., learning theory, meta-cognitive theory) and current research results.

### **Consciousness of Mathematics Learning**

Consciousness has been defined as: sentience, awareness, subjectivity, the ability to experience or to feel, wakefulness, having a sense of selfhood, and the executive control system of the mind (Farthing, 1992). More and more researchers have begun to pay attention to the role of consciousness in high-efficient mathematics learning (Sun, 2005; Wang, 2006; Yang, 2007; Yu, Wang, & Yang, 2006;). There are two factors that current researchers mentioned most frequently: consciousness of efficiency and consciousness of critique. Wang (2006) and Sun (2005) pointed that a student who has the consciousness of efficiency will focus on both learning quality (output) and learning time (input), so that they can try to enhance efficiency when they are learning. The other factor is consciousness of critique. Critical thinking shows a kind of state of thirst for knowledge, which represents the spirit of exploration that can nurse creativity. Mathematics is not a finished product, and its results remain open to revision (Ernest, 1989). In order to learning mathematics in a right and efficient way, one cannot absolutely accept what the mathematics teacher told you or what is in the textbook without doubting.

### **Mathematics Cognition**

Shu's (2006) research showed that mathematics gifted students are more knowledgeable in mathematics than most students. They, especially, know how to connect one concept to another. Rich mathematics knowledge, various problem solving skills and good cognitive structure make them competent to solve mathematics tasks quickly and learn mathematics in less time. Shi (2006) and Sun (2005) verified that well-structured mathematics

cognition is a significant factor in high-efficient mathematics learning. What's more, mathematical understanding (Sun, 2005; Wang, 2005), mathematics thinking and method (Shi, 2006; Wang, 2005), mathematics ability (Shi, 2006) are mentioned in current research. In addition, we believe classroom learning is the core for mathematics learning. Thus, students' engagement in mathematics class should also be identified as a factor for learning efficiency. In all, we adopted five factors to investigate students' mathematics cognition: cognitive structure, mathematical understanding, mathematics thinking and method, mathematics ability, and engagement in mathematics class.

### **Mathematics Meta-Cognition and Affection**

Metacognition was put forward by Flavell, J. H. in 1976. Based on this theory, we chose four factors for this dimension: planning, regulating, reflecting, and self-evaluating (Chen, 1997; Flavell, 1976). Generally, affection of mathematics learning refers to all the psychological elements except cognitive elements. In this study, we adopted Mao and Mao (1992) and Zheng and Zhang's (2003) definition that affection of mathematics learning includes psychological disposition, motivation, attitude, emotion and willpower towards mathematics learning. Motivation includes both internal motivation and external motivation. Internal motivation is the key for high efficient learning. If the learning content is what a student wants to learn, he/she will learn it in less time. What's more, as we all know, "Interest is the best teacher". Interest in mathematics is a source of internal motivation which can exert positive impact on learning efficiency. Additionally, willpower is another factor for high-efficient mathematics learning. This factor examines whether a student will easily give up when they face a difficult mathematics task, which is also important for mathematics learning efficiency. Thus, motivation, interest, and willpower were chosen for this dimension.

## **Method**

### **Participants**

Seventy-eight high school students from two high schools served as participants. Given that the investigation is about high-efficient mathematics learning, two sample schools we adopted are ranked 1 and 2 respectively in our district. Then we chose 37 students and 41 students who were enrolled in a mathematics course of their fourth semester at random from each sample school. All participants completed a questionnaire designed to assess their mathematics learning efficiency at the end of the semester.

### **Instrument**

Given that a review of the literature did not find one suitable questionnaire to assess high school students' mathematics learning efficiency, a two part questionnaire was developed as an assessment instrument. In the first part, participants were asked some personal items including self-evaluation of mathematics learning efficiency (10 point scale, from low to high), midterm score, and final score. In the second part, we designed statements based on the framework mentioned above, two or three statements for each factor. Totally, 37 statements ( $\alpha = .852$ ) were adopted. For example, the statement that "I can regulate mathematics learning independently after class and try to learn more in less time." for the factor "consciousness of efficiency"; the statement that "Engagement in mathematics class is very important, or I'll spend more time to gain what was taught in the class" for the factor "engagement in mathematics class". A four point Likert-type scale was used to encourage thoughtful responses to statements, rather than "undecided": Strongly Disagree=1, Disagree=2, Agree=3, Strongly Agree=4.

Participants were asked to respond to the questionnaire from the point of view of their mathematics learning experience during four semesters. The reliability, means, and standard deviations of each dimension in the questionnaire are shown in table 1.

Table 1  
*Reliability, Means, and Standard Deviations of the Questionnaire*

	consisted of n statements	$\alpha$	M	SD
Consciousness of Math Learning (CML)	7	.54	18.94	2.34
Mathematics Cognition (MC)	13	.74	34.09	5.67
Mathematics Meta-cognition (MM)	9	.66	23.70	3.91
Affection of Mathematics Learning (AML)	8	.81	21.77	4.73
TOTAL	37	.85	98.50	13.25

*Note.*  $N=78$ .

### **Procedures and Data Analysis**

Independent sample *t*-test was used to answer the first research question "What factors contribute to mathematics learning efficiency?" First, all the participants were asked to judge their mathematics learning efficiency in a 10 point scale in which 1 point meant extremely low efficiency and 10 point meant extremely high efficiency. It was called self-evaluation. Then they completed the second part of the questionnaire which measured their mathematics learning efficiency from specific dimensions (CML, MC, MM, AML) mentioned above.

Participants were assigned into two groups: relatively low-efficient group (1, 2, 3, 4, or 5 point; N=43) and relatively high-efficient group (6, 7, 8, 9, or 10 point; N=35) based on the result of their self-evaluation. We thought that if the dimension indeed contributed to mathematics learning efficiency, there would be a statistically significant difference in this dimension between low-efficient group and high-efficient group. Thus, independent sample *t*-test was applied to explore if there were significant differences between the two groups in terms of CML, MC, MM, AML, Total (the sum of four dimensions), Midterm score, and Final score. Effect sizes, calculated in terms of the difference between low-efficient group and high-efficient group means divided by the pooled standard deviation, were applied to determine if the difference were practically meaningful (Cohen, 1988).

For the second question “What’s the relationship between mathematics learning efficiency and mathematics academic achievement,” correlation analysis was implemented.

## Results

### Factors for High-Efficient Mathematics Learning

Independent sample *t*-test results indicated that both mathematics learning efficiency (Total) and mathematics learning achievement (Midterm and Final) of the relatively low-efficient group were significantly different from the relatively high-efficient group.

With respect to mathematics learning efficiency, we detected five variables: Consciousness of Math Learning, Mathematics Cognition, Mathematics Meta-cognition, Affection of Math Learning, and Total. Four of them were significantly different. The greatest difference was in Affection of Mathematics Learning, followed, in consecutive order, by Mathematics Cognition, Total, and Mathematics Meta-cognition. The effect sizes of these four scales ranged from 0.61 to 1.19, suggesting that the magnitudes of differences were large ( $d > 0.60$ ), and therefore indicated that the difference between the relatively low-efficient group and the relatively high-efficient group in these dimensions were practically meaningful (Cohen, 1988). We deduced that Affection of Mathematics Learning, Mathematics Cognition, and Mathematics Meta-cognition were the factors which influenced mathematics learning efficiency. It is noteworthy that there was no significant difference in terms of Consciousness of Math Learning between the two groups. Table 2 reports the result of *t*-test.

Similarly, there also was a significant effect in terms of Midterm and Final,  $t(76) = -3.07$ ,  $p < .001$  and  $t(76) = -3.50$ ,  $p < .001$  respectively, with high-efficient students receiving higher scores than low-efficient students. This

result indicated that mathematics learning efficiency exerts some impact on students' mathematics academic performance.

Table 2  
*Sample Descriptives for Equality of Means*

	Group 1		Group 2		T-Test	
	M	SD	M	SD	<i>t</i>	Effect Size
Consciousness of Math Learning	18.74	2.17	19.09	2.48	-.66	
Mathematics Cognition	30.94	4.80	36.65	5.03	-5.01 <sup>***</sup>	1.16
Mathematics Meta-cognition	22.43	3.97	24.74	3.57	-2.71 <sup>**</sup>	0.61
Affection of Math Learning	18.89	4.05	24.12	3.90	-5.79 <sup>***</sup>	1.32
Total	91.00	10.86	104.6	11.88	-5.23 <sup>***</sup>	1.19
Midterm	52.43	16.88	62.95	13.40	-3.07 <sup>***</sup>	0.69
Final	56.17	20.37	70.02	14.47	-3.50 <sup>***</sup>	0.78

### Correlates of Learning Efficiency with Academic Performance

Generally, simple correlations show that there were significant correlations between mathematics learning efficiency and mathematics learning achievement. With respect to the specific dimension of learning efficiency, Affection of Mathematics Learning was strongly correlated with both Midterm score and Final score, with  $r(76) = .613, p < .01$  and  $r(76) = .635, p < .01$  relatively, followed by Mathematics Cognition and Mathematics Meta-cognition. The correlation between Consciousness of Math Learning and mathematics academic performance was also statistically significant, but it was not as strong as other dimensions. Especially, all the dimensions of mathematics learning efficiency had stronger correlations with the difference between Final and Midterm, instead of Midterm and Final singularly.

Table 3  
*Correlations Between Measures*

	Midterm	Final	Difference
Consciousness of Math Learning (CML)	.281 <sup>*</sup>	.297 <sup>**</sup>	.304 <sup>**</sup>
Mathematics Cognition (MC)	.540 <sup>**</sup>	.674 <sup>**</sup>	.741 <sup>**</sup>
Mathematics Meta-cognition (MM)	.352 <sup>**</sup>	.532 <sup>**</sup>	.646 <sup>**</sup>
Affection of Mathematics Learning	.613 <sup>**</sup>	.635 <sup>**</sup>	.793 <sup>**</sup>



(AML)

Total	.586**	724**	.819**
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Note. 1. \* $p < .05$ , \*\* $p < .01$ ; 2. Total means the sum of CML, MC, MM, and AML; 3. Difference = Final Score – Midterm Score.

### Discussion

Mathematics educators, researchers, or teachers, especially those in China, typically believe that mathematics cognition is the most important aspect for mathematics learning and mathematics academic performance. Thus, lots of mathematics teachers focus on imparting knowledge and skills to students in their daily teaching practice. As a result, Chinese students usually demonstrate good performance in international mathematics competitions or tests (PISA, TIMSS); however, some researchers argue whether students really like learning mathematics, or do they feel confident enough to learn mathematics? The answer is not optimistic. Most students don't want to be mathematicians and use mathematics in the future, even for those who have been the winners of international mathematics competition when they were young. Our research verified this problem. Independent sample *t*-test result showed that the greatest statistically significant difference between relatively low-efficient group and high-efficient group was in Affection of Mathematics Learning instead of the Mathematics Cognition. High-efficient mathematics learning is a topic on how to learn mathematics, specifically, on how to learn more in relatively less time. Stress or preaching from teachers or parents does not allow students to learn fast. Affection of mathematics learning mainly includes interests and motivation. Practically, mathematics educators and teachers should not hold onto the hypothesis that "every student wants and likes to learn mathematics and that all teachers need to do is just to impart knowledge and skills to students". Contrarily, students' interests and affection for mathematics need to be aroused by teachers though different teaching strategies or different approaches, such as a Graphing Calculator or mathematics games.

For other factors affecting mathematics learning efficiency, it is interesting that lots of previous researchers mentioned the importance of consciousness of high-efficient learning and regarded it as an important dimension that influences mathematics learning efficiency, however, our study didn't support this view. We attempted to interpret this problem in two aspects. On the one hand, we double checked previous research and found that they were lacking reliability reports of their questionnaires and that most conclusions were made based on descriptive statistics. On the other hand, we believed there is more or less influence that consciousness of high-efficiency exerts on mathematics learning efficiency, but were at a less as to why there's no significant difference between two groups? Given participants in this study were second year high school students, they can be aware of the importance of

learning efficiency, but sometimes for some reason, such as lack of self-control ability, they may not practice it well.

In addition to the specific dimensions, mathematics learning efficiency highly correlated with mathematics academic performance, especially with the advance of the academic performance. It indicated that high-efficient learning exerts positive influence on the advance of mathematics academic performance.

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