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Estimation Skills of Chinese and Polish Grade 6 Students on Pure Fraction Tasks

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On the base of Lewin's Field Theory and Bronfenbrenner's Bio-ecological Systems Theory, the authors examined cognitive, motivational and environmental factors associated with estimation skills of Chinese and Polish Grade 6 students through estimation tests, questionnaires and interviews. The unique features of this research were the focus on pure fractional tasks and the newly developed scoring system. Results indicated that significant cross-national differences existed on estimation performance, that being good mathematicians did not provide guarantee for being good estimators, and that estimation skills were related to some cognitive, motivational, environmental factors and the interactions among these factors. Finally, implications were discussed.

Key words: computational estimation pure fractions, Chinese and Polish students, estimation strategy and process, new scoring system, person and environmental factors

The interdependence between the person and environment should be considered in studying human behavior and development (Lewin, 1951, Bronfenbrenner, 1979, 2005). In reality, basic education in China and several other East Asian countries has been doing a better job than that of the other nations in the world in terms of academic achievement in mathematics and science. But what it is like in computational estimation?

Computational estimation refers to making reasonable guesses as to the approximate answers to arithmetic problems. Based on previous research, from the factors related to "the person", Sowder and Wheeler (1989, p.132) found computational estimation involving several components: conceptual components, skill components, related concepts and skills, and affective components. Furthermore, Estimation skills had developmental features. Studies (LeFevre, Greenham, & Waheed, 1993, Reys, 1982, Sowder, 1984, Sowder & Wheeler, 1989) indicated that estimation performance improved with age in that older children showed more understanding of the role of approximate numbers than younger ones, and were more likely to agree that there were multiple routes to an estimate.

Furthermore, there were great differences in strategy use between groups, between individuals and within individuals due to age, task features, relative strategy performance and other strategy characteristics. For example, Baroody (1989, 1992) and Dowker (1989) studied young children's estimates about additional tasks and found that they did use some limited and even inappropriate estimation strategies. Some adolescents and adults, who had not received explicit instruction about estimation, showed considerable skills in estimation, (Dowker, 1992, Hiebert, 1984, Levine, 1982, Reys et al., 1982). Levine's (1982) study, with non-mathematics-majors, indicated that there were differences in the frequency with which the strategy types were used. Dowker (1992) found that the mathematicians tended to use strategies involving the understanding of arithmetic properties and relationships. Dowker´s (1996) further research demonstrated that the mathematicians and accountants used significantly larger numbers of appropriate strategies than the other groups, and that there was no significant effect of mathematics educational level or general education level on their estimation score and on the number of strategy types used per subject. It was found that, at all ages, the most common strategies used were rounding, truncation, and compensation (Lemaire, Lecacheur, & Farioli, 2000, Lemaire & Lecacheur, 2002). However, Lemaire, Arnaud and Lecacheur (2004) found that older adults provided less accurate estimates, took more time to estimate, and chose estimation strategies less adaptively than young adults. Liu (2009) found that Chinese students performed better when tasks were presented visually than orally, and that third graders tend to use rounding based strategies while fifth graders tend to use written algorithm based strategies.

Meanwhile, research revealed related abilities and concepts to estimation skills. For third graders, estimation skills were predicted by cognitive factors, such as arithmetic number combination skill, nonverbal reasoning, concept formation, working memory, and inattentive behavior (Seethaler & Fuchs, 2006).

Estimation processes were found to be varied by age, test format, types of numbers, foil characteristics within format, mathematical operations, and/or grade level of students (LeFevre et al., 1993, Schoen, Blume & Hoover, 1990, Sowder & Wheeler, 1989). Moreover, estimation processes were not only cognitive processes, but also meta-cognitive processes and beliefs (Sowder, 1989, Sowder & Wheeler, 1989). As Sowder (1992) found successful estimators were usually characterized as flexible, self-confident, tolerant of errors in estimates, and disposed toward making sense of the mathematics being undertaken and seeking reasonableness in results. Evidence indicated that favorable attitude toward estimation (Bestgen et al., 1980), confidence in estimation (Reys et al., 1982), the degree of valuing estimation and the frequency of estimation use in daily lives (LeFevre, Greenham, & Waheed, 1993) led to better estimation performance.

It has been long recognized that East Asian students outperform their counterparts in other parts of the world in traditional classroom mathematics (Arbeiter, 1984, Lapointe, Mead, & Philips, 1989, Ramist & Arbeiter, 1984a, 1984b, Song & Ginsburg, 1987, Stanley, Huang, & Zu, 1986, Stevenson, Stigler, Lee, & Lucker, 1985). There were studies (e.g., Reys et al., 1982, 1991) compared the computational estimation performance and strategies used by East Asian students and students from the West, but comparisons between East Asian students and students from East Europe were rare.

Estimation tests, questionnaires and interviews were frequently used to measure estimation skills. Generally, many estimation tests were developed to assess estimation skills through manipulating the test format, numbers type and complexity of the numbers, context, arithmetic operations, and time constraint. In estimation literature, mixed results were revealed regarding the impact of context (Rubenstein, 1985, Sowder, 1992). Estimation tasks with whole numbers were found easier than those with decimals and fractions (LeFevre et al., 1993, Rubenstein, 1985, Tsao & Pan, 2011). Moreover, Reys and Bestgen (1981) suggested that the numbers in the estimation questions should be complex enough to encourage and reward estimation. In terms of operation, Rubenstein (1985) found that students made fewer errors on addition and subtraction than on multiplication and division tasks. Furthermore, whether time constraint was applied on computational estimation tests as a whole only (Schoen, et al., 1990) or on each item of the test (Reys & Bestgen, 1981, Reys et al., 1991) was worth considering. Some researchers were for time constraint because they think if too much time was allowed, mental calculation might occur, other researchers feared that if too little time was allowed, only wild guessing might happen. However, most researchers applied time constraint (Levine, 1982, Reys & Bestgen, 1981, Reys et al., 1991, Schoen, et al., 1990), and only a few researchers applied no time constraint (Dowker, 1992, Dowker et al., 1996).

In summary, although research has provided a good deal of information on estimation, environmental factors were mostly ignored. For example, macro-environment, such as different countries or regions, and teacher support for estimation use, etc. Were not considered. East European nations were ignored in this line of research as well. Pure fraction tasks were not investigated until now. Another limitation was that the assessment process in previous estimation literature was inappropriate because it concentrated on the absolute accuracy level of the estimates.

Research Goal and Hypothesis

Theoretically, the interdependence between the person and environment should be considered in studying estimation skills. In the research field, the most striking developmental changes were found in the conceptual knowledge used to perform estimation tasks (LeFevre et al., 1993): from Grade 6, students seemed to understand the role of the simplification principles in estimation and reduced complex problems through rounding and prior compensation sufficiently to produce reasonable estimates. Therefore, Grade 6 would be a good starting point to make a cross-national comparison regarding students' computational estimation skills. The present study was to examine some text-structure-related cognitive factors, motivational factors in mathematics and estimation, and environmental factors related to estimation skills. To fulfill this research purpose, the following hypothesis was formulated:

Hypothesis: Estimation test structure, motivation factors in mathematics and estimation, environmental factors and interactions among them would have an impact on students' computational estimation skills.

Methods

Participants

Participants were Chinese (n = 83, M = 12.31 years old, SD = .42) and Polish (n = 101, M = 12.43 years old, SD = .53) Grade 6 students from 12 primary schools ranging from high, middle and low level in terms of school quality respectively in China and Poland.

Measures: Estimation Test, Questionnaire and Interviews

The *Estimation Test* included 16-item pure fractional open-ended tasks and the task format followed that of Vermeer and his colleagues (2000): context-free and context-imbedded tasks paralleled as pairs and each pair of tasks were to be solved by the same arithmetic operation (addition or multiplication) at the same level of difficulty. Around each operation (addition or multiplication), there were eight items of tasks, which included 4 context-free and 4 context-imbedded items. The structure of each task was determined by four factors: operation, context-imbedded or context free, complexity of fractions (8 simple-operand and 8 complex-operand tasks), and numbers of operands used in one task (8 two-operand and 8 three-operand tasks). Specifically, simple-operand tasks consisted of only operands of fractions whose numerators were “1”, while complex-operand tasks consisted of only operands of fractions whose numerators were more than “1”. The denominators of both simple-operand and complex-operand were either one-digit or two-digit whole numbers whose value is more than “1”, but less than “20”. When all the operands of fractions used in one task were simple operands, the task was called “simple-operand task” （e.g., “1/4 + 1/15”, “1/6 × 1/19” , “1/4 + 1/15 + 1/8”）. When all the operands of fractions used in one task were complex operands, the task was called “complex-operand task” （e.g., “3/8 + 7/17”, “5/6 × 9/15 ×6/8”, “3/4 + 8/13 + 5/6”）. If there were 2 operands of fractions in one task, it was named a “two-operand task” (e.g., “1/4 + 1/15”, “3/8 + 7/17”, “5/6 × 6/19”), whereas, if there were 3 operands of fractions in one task, it was named a “three-operand task” (e.g., “1/4 + 1/15 + 1/8”, “3/4 + 8/13 + 5/6”, “1/6 × 1/19 ×1/4”, “5/6 × 9/15 ×6/8”). For the Estimation Test, the coefficient alpha is .75 in the present study.

After noting down an estimate to each task in the answer sheet, students were asked to respond 3 questions respectively addressing their confidence level (I have great confidence in the estimate for this task), memory load (Doing this task requires a great memory load) and level of guessing (I guess a lot to find the estimate). Answer choices were based on a 5-point Likert scale, from “1” standing for “not at all true of me” to “5” standing for “totally true of me”.

The first part of the *questionnaire* was adapted from Pintrich and De Groot’s (1990) Motivated Strategies for Learning Questionnaire (22 items), which was designed to find some information about the students’ self-efficacy (9 items, α = .75), intrinsic value (9 items, α = .73), and test anxiety (4 items, α = .85) in classroom mathematics. The answer-choices were designed by using a 7-point Likert scale: from “1” standing for “not at all true of me” to “7” standing for “very true of me”.

And the second part of the *questionnaire* consisted of Attitude toward Estimation and Teacher Use of Estimaiton Sub-Scales, which were designed to examine students’ attitudes toward estimation (5 items, α =.62), an example item was: “I really dislike to work under such time pressure to get an estimate”, good estimation strategy tendency (5 items, α = .71), an example item was: “I will first change some numbers of the task into simple ones”, and frequency of estimation use of the students themselves (5 items, α = .70), an example was “In mathematics, I sometimes estimate a result before calculating it”, and frequency of estimation use of their mathematics teachers (5 items, α = .75), an example was “In mathematics, such tasks are never used by the teacher”. The answer-choices were designed by using a 5-point Likert scale, from “1” for “not at all true of me”, to “5” for “very true of me”.

Interviews were designed to investigate how good and poor mathematicians estimate. Among the participants, 18 students from each country took part in the interviews. Half of them were identified as good mathematicians and the other half as poor mathematicians by their mathematics teachers. The content of the interviews were: Generally, how did you find the estimates？Did you find that different types of tasks had different level of difficulty and influence the way you found your solutions? Could you please state your strategies by using 2 tasks as examples? Does your mathematics teacher use estimation frequently in the classroom and do you think he or she would support estimation use in mathematics?

Procedure

The instrument was originally designed in English. The equivalence of the complete questionnaire was reached through back-translation. After obtaining parent, headmaster and teacher consent and student assent, the estimation-test answer sheets and questionnaires were distributed and administered in a 45-minute session. The administrating procedure followed this order: estimation test, questionnaire, and then the interview. Specifically, before the test, 5-minute instructions were given to explain what estimation was by solving two estimation tasks, but without mentioning the specific strategies. It was also emphasized that paper-and-pencil calculation was not allowed. Each task was presented orally. About five seconds were allowed for noting down each estimate and then students were asked to cross out the corresponding answers to their confidence level, memory load, and level of guessing. Immediately after the test, subjects were asked to answer the questionnaire. It took about 15 minutes. Finally the good mathematicians and poor mathematicians were asked to take part in the interviews.

Results and Findings

General Accuracy Performance under New Scoring System

A new scoring system based on relative accuracy standards in the present study was created. First, students' estimates and the exact answers to tasks were converted from fractions into decimals. Then the absolute difference between each estimate and corresponding exact answer was worked out. Meanwhile, the median for this absolute difference was calculated as well. If the absolute difference was within the range between 0 and the median, an estimation score of "1" was given to the estimator. Otherwise an estimation score of "0" was given to the estimator. Namely, the total best score for each student would be 16, and the worst score would be 0.

Generally, Chinese and Polish students' performance on estimation was not good. Almost 75% of students got a total estimation score equal or less than 10 (62.5% of the total best score). And among them, about half of them got a score less than 7 (43.8% of the total best score). Cross-nationally, Chinese students (M = 10.40, SD = 3.00) performed significantly better than Polish students (M = 6.11, SD = 3.83) on all tasks (t = 9.94, ρ < .001).

Group Differences on Memory-Load, Confidence Level and Guess Level

On the base of their total estimation scores, subjects were grouped as poor, middle-level and good estimator groups. MANOVA was conducted with memory-load, confidence level and guess level on all the tasks as dependent variables, with the estimation group as an independent variable, and with Roy's root as the criterion. Very significant (ρ <.001) differences existed between poor, middle-level and good estimator groups (see Table 1) on the memory-load, the confidence level and the guess level. Apparently, good estimators had less memory-load, higher confidence levels and less guess levels on all tasks.

Table 1

Memory-Load, Confidence Level and Guess Level group differences

Variables	Estimation score groups: Low=1, n=65, Mid. =2, n=53, High =3, n=65.	*M*	*SD*	*F*
Memory-load on all tasks	1	3.35	.87	21.38***
	2	3.20	.94	
	3	2.39	.87	
	Total	2.97	.99	
Guess level on all tasks	1	3.50	.87	16.41***
	2	3.30	.85	
	3	2.67	.85	
	Total	3.15	.93	
Confidence level on all tasks	1	2.70	.91	13.60***
	2	2.66	.90	
	3	3.38	.80	
	Total	2.93	.93	

Note: N =183, * ρ < .05,** ρ< .01, ***ρ< .001

Predictors of Estimation Performance

A multiple hierarchical regression analysis was run with total estimation score as the dependent variable and with the following 4 blocks of variables as
independent variables: the estimation test structure variables (1^{st} block), motivational variables in mathematics and attitude toward estimation variables (2^{nd} block), environmental factors including country (Polish student group as reference group) and teacher use of estimation (3^{rd} block), and

interaction variables between teacher use of estimation and motivational variables in mathematics and estimation. In the regression model (see Table 2), among the test structure variables in the 1st step (ΔR^2 = .16, F (5, 177) = 6.56, ρ <.001), complex tasks (*ß* = -.20, ρ < .05), numerical tasks (*ß* = -.42, ρ < .05), and 3-term tasks (*ß* = -.21, ρ < .05) acted as significant predictors of estimation performance, in step 2 (ΔR^2 = .30, F (8, 169) = 11.35, ρ <.001), intrinsic value (*ß* = .26, ρ <.01), test anxiety (*ß* = -.18, ρ <.01), positive attitude (*ß* = .21, ρ < .01), good strategy tendency (*ß* = .16, ρ < .05), and poor strategy tendency (*ß* = -.12, ρ < .05) were significant predictors, in the third step (ΔR^2 = .11, F (2, 167) = 20.34, ρ <.001), significant predictors were frequency of teacher use of estimation (ß = .15, ρ < .01) and country (*ß* = .41, ρ < .001), in the fourth step (ΔR^2 = .04, F (8, 159) = 2.21, ρ <.05), significant predictor was "teacher estimation use × self efficacy" (*ß* = .99, ρ < .01), and the overall model was significant as well (R^2 = .61, F (23, 182) = 10.49, ρ < .001).

Table 2

Regression of Estimation Performance on Relevant Variables

Predictor	Estimation Performance											
	Step 1			Step 2			Step 3			Step 4		
	B	*SE B*	*ß*	*B*	*SE* B	*ß*	*B*	*SE* B	ß	*B*	*SE* B	*ß*
Complex operand	-.50	.22	-.20*	-.63	.19	-.25**	-.67	.17	-.27***	-.67	.17	-.27***
Context-free	-1.23	.59	-.42*	-1.17	.50	-.40*	-.57	.46	-.19	-.34	.46	-.12
Three-operand	-1.16	.46	-.21*	-.45	.39	-.08	-.49	.36	-.09	-.34	.36	-.06
Self-efficacy in maths				-.04	.16	-.02	-.03	.14	-.01	-2.0	.74	-.87**
Intrinsic value in maths				.70	.20	.26**	.31	.19	.12	.15	.85	.06
Test anxiety in maths				-.37	.13	-.18**	-.27	.12	-.14*	-.18	.54	-.09
favorable attitude toward estimation				.65	.22	.21**	.20	.21	.06	.24	.90	.08
Good estimation strategy tendency				.56	.24	.16*	.33	.22	.10	1.71	.97	.49
Poor estimation strategy tendency				-.48	.24	-.12*	-.28	.22	-.07	.13	.95	.03
Teacher support for estimation use							.64	.24	.15**	-2.36	1.56	-.54
country							2.95	.53	.41***	2.83	.53	.39***
Teacher support for estimation use × self efficacy in maths										.58	.22	.99**
ΔR^2			.16***			.30***			.11***			.04*
R^2						.46			.57			.61

Note: *N*=183. * ρ < .05,** ρ<.01, ***ρ<.001, reference group was Polish student group.

Estimation Process and Strategy Use

Interview results indicated that children used three processes involving rounding, truncation, and compensation. In an adaptive way, specific strategies were chosen and differed in frequency and effectiveness due to task structure differences. Among the task-structure factors, both good and poor mathematicians found that complex-operand tasks, context-free tasks and 3-operand tasks were more difficult to estimate. In contrast, in the case of computation, context-free tasks normally would be easier than context-embedded tasks. This implied that students might use more estimation in real-problem solving, but less in classroom mathematics.

Estimation strategies used by Chinese children were mental computation, rounding, pie analogue, and benchmarks such as 1, ½, and ¼ etc. The estimation strategies applied by Polish children were: pure guessing, mental calculation, rounding, using benchmarks, and using nicer numbers (e.g., when estimating tasks “1/4 + 1/15” and “3/8 + 7/17”, by using nicer numbers, the task could be transferred into “4/16 + 1/16” and “6/16 + 7/16”). Distinct from previous literature, a new estimation strategy was found by Chinese children: using a pie analogue, which implied that they understood abstract fractions in a meaningful and concrete way. Furthermore, good mathematicians admitted that they used more algorithms-based estimation strategies and saw estimation as a supplement in classroom mathematics when doing paper-and-pencil calculations or as the last option when calculations were not available. However, poor mathematicians used more rounding and compensation strategies and had more open view toward estimation use. Different from good mathematicians, poor mathematicians treated estimation as an independent tool either in real life situations or in classroom mathematics.

Conceptually, Polish students did not understand estimation as well as their Chinese counterparts because they thought there was no need to check their estimates even if they have time to do it, because they had already knew their answers were only estimates, hence not correct.

Although both Polish and Chinese children admitted the usefulness of estimation in mathematics, they did not think that their mathematics teachers would support the use of estimation in classroom mathematics.

Discussion and Implications

Conclusion

In the present study, it was found that for all the Polish and Chinese samples, the general performance on estimation test was not good, which might be a reflection of an ignorance of computational estimation in both countries' classroom mathematics. Cross-nationally, Chinese students performed better than their Polish counterparts as far as the accuracy level of the estimates were concerned. Different estimation groups (good, middle-level and poor estimator groups) had further differences on the memory-load, confidence level and guess level when working with the estimation tasks. In terms of significant predictors, 61% of the total variance in estimation performance was explained by Cognitive factors such as complex-operand, three-operand and context-free, by motivational factors in maths and estimation such as self-efficacy, intrinsic value, test anxiety, favorable attitude toward estimation, good and poor strategy tendencies, by environmental factors such as teacher use of estimation and country variable, and by interaction between teacher use of estimation and self-efficacy. But when taking interview contents into account, it seemed that being good mathematicians could not guarantee being good estimators either in terms of estimation strategies or in terms of attitudes toward estimation.

Implications

Practically, present study signified that:

(a) If an estimation test was designed, task structure should be given more weight.

(b) Not only classroom mathematics was a related ability to computational estimation as previous studies proved, the motivational factors, such as intrinsic value and test anxiety in classroom mathematics also had great impact on estimation performance. This suggested that classroom mathematics really had much to do with estimation.

(c) Self-efficacy in classroom mathematics was very important, but when test anxiety was too high, the positive impacts of self-efficacy could be reduced to zero or to negative impacts.

(d) To mathematics education reform, computational estimation could make some contribution as well. Reys and Nohda (1994) noted that computational estimation was, thus, a balance approach to paper-and-pencil computation.

(e) With the present research design, it was encouraging that 61% of the variance in estimation performance was explained, but the rest of the 39% variance was not explained yet. For future estimation research, specific suggestions might be offered. For example, other related abilities might be considered, such as mathematics achievement, real-problem solving ability, more environmental factors could be investigated, such as school and educational systems orientations in mathematics and use of mathematics, more interactions between environment and the person, between environments, and between factors belonging to the person might be examined as well.

Theoretically, an computational model should integrate environmental factors, such as frequency of teacher use of estimation and macro-environment, the country and regional factors, because these environments bring norms, rules, traditions and values influencing the learning and the application of maths and estimation.

Limitations

Although present research design was unique in using pure fractional tasks and in finding more related factors and even predictors of estimation performance, there were limitations as well. For example, data collection mainly depended on students' self-report and teacher recommendations, which might cause problems in validity. Thus future research might find more sources for data collecting. Secondly, only addition and multiplication estimation tasks were included in the present estimation test. The age group was also limited. It would be more meaningful if more arithmetic operations and age groups were considered. However, all these limitations provide directions for future estimation research as well.

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