

An Interview with Bernard R. Hodgson about High-Efficiency Mathematical Teaching Hypotheses

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Teaching efficiency is a common issue of concern in educational theoretical and practical areas. Now we are getting involved in one of the national education science "Eleventh Five-Year Plan" projects----Study on High-Efficiency Teaching Behavior of Elementary Education. This research attempts to discover characteristics and the attribution of high-efficiency mathematical teaching behaviors. Based on research of teaching videos of some acknowledged high-efficiency mathematics teachers, our research team formed initial hypotheses on characteristics and the attribution of high-efficiency mathematical teaching behaviors. We interviewed professor Bernard R.Hodgson, General Secretary of International Commission on Mathematical Instruction, about high-efficiency mathematical teaching hypotheses in June, 2010. And we recorded the interview process and made an analysis.

Key words: high-efficiency, mathematical teaching, interview, discussion.

Introduction

In 1632, J. A. Comenius, one of the world's most famous educators, wrote the preface of his book *Great Didactic of Comenius*. The purpose of writing this book was to advance school education, to make teachers teach less, to make students learn more, to make school full of joy instead of boredom and pressure. On the 20th century, Educationist Юрий Константинович Бабанский began to systematically study the optimal instructional problem. The famous Chinese Qingfu Mathematical Instruction Reform was successful

partly for its advanced educational ideas. They were: commonplace teachers narrate, good teachers explain, excellent teachers do, great teachers illuminate. Hitherto, many educators and scholars home and abroad have done further research into teaching efficiency and have gained some new insights about teaching efficiency. Our research, one on the national education science "Eleventh Five-Year Plan" projects——Study on High-Efficiency Teaching Behaviors of Elementary Education, will try to recover characteristics and the attribution of high-efficiency teaching behaviors. Based on our study of several acknowledged high-efficiency mathematics teachers utilizing video analyzing technology and our own observations, we formed our initial hypotheses. The definition of high-efficiency mathematics teaching composes our research's precondition. Here we define high-efficiency mathematics teaching as teaching for students' high-efficiency learning, that is, within a given time and energy the teaching activity can make students gain short-term and long-term development spending less time and energy.

We based our initial hypotheses mostly on domestic research achievements. We wonder whether these hypotheses can be established outside of China or if they can't the differences which occur. We also want to know the extent of relativity between each hypothesis and mathematics teaching efficiency. So we should heed the voices and opinions from large numbers of mathematics teaching experts at home and abroad. The 13th International Conference on Mathematics Education in China was held at Hangzhou Normal University from the 25th to the 28th of June, 2010. Bernard R. Hodgson who is Secretary-General of the International Commission on Mathematical Instruction, was Co-Chair of the conference. We asked Professor Hodgson and for an interview with him about high-efficiency mathematics teaching and, luckily he accepted it. It was held on the night of the 26th of June.

Interview

Time: 9p.m.-10p.m., 26th, July, 2010

Location: the meeting room of Hangzhou Huachen Ginza Hotel

Guest: Bernard R. Hodgson, Secretary-General of International Commission on Mathematical Instruction

Topic: High-efficiency mathematics teaching

In the following, "Q" will represent our questions and "H" will represent Professor Hodgson. The interview will include two aspects: the characteristics and the attribution of high-efficiency mathematics teaching behaviors, and the

characteristics will include five aspects: mathematics learning, mathematics education, teaching creativity, information feedback, and the art of teaching. The attribution will go from internal cause to external cause.

Characteristics of High-Efficiency Mathematics Teaching Behavior

From the perspective of mathematics learning

Q: Professor Hodgson, thank you for receiving our interview. It is really our great honor. We really appreciate that.

H: You are welcome! It's also my great pleasure.

Q: Ok, we begin now. The report "The Role of History of Mathematics in the Mathematical Preparation of Secondary School Teachers" which you presented this morning impressed us, for it was greatly relative to our research. Now we are getting involved in one of the national education science "Eleventh Five-Year Plan" projects—— Study on High-efficiency Teaching Behavior of Elementary Education, we formed our initial hypotheses on high-efficiency mathematics teaching behavior.

H: Really? Congratulations! Oh, yes, mathematics history really is very significant. Just like what I said, it can provide rich contexts for mathematics classroom discussion, and at the same time can make students understand mathematics knowledge from specific perspectives. Your research seems very meaningful. Can you give me some details about that?

Q: Ok, Professor. As we put forward our hypothesis, please give us your opinion. The first one, high-efficiency mathematics teachers focus on capturing students' attention and stimulating students' interest through examples of daily life or building upon students' existing mathematical knowledge.

H: I say yes. I quite agree with that.

Q: The next one, high-efficiency mathematics teachers focus on students' understanding of mathematical concepts and represent principal concepts in various ways to help them form good cognitive structures.

H: Yes, I quite agree with this. Multi-representation is very important. But, "various ways" to represent is not commonly seen in America. Most teachers would prefer one main form to teach instead of various ways. Maybe they are afraid of that multi-forms will lead students to thinking confusion.

Q: Ok, thank you for your explanation. Next, high-efficiency mathematics teachers focus on the standard use of mathematical language and symbols, training students to form good habits and abilities in using mathematical

language properly.

H: Yes. I agree with this. But be careful to not too much emphasized these, in case teachers teach just that.

Q: That's right. And next, high-efficiency mathematics teachers focus on explaining important and difficult points of the content for students to make the whole lesson flow rhythmically.

H: Yes. I agree.

Q: Next, high-efficiency mathematics teachers focus on the basis and levels of students' practice and exercise.

H: I am not very clear about the meaning of basis and level. Does the basis mean mathematical knowledge and experience students already have? And does the level mean doing exercises from easier questions then to harder ones?

Q: Yes, you grasp the meaning. And here basis means not just the knowledge students have known, but also means the basis of the whole mathematical entity.

H: Oh, I see. The basis here means that the exercise should not be too hard, right?

Q: That is right.

H: Ok, I agree with that.

From the perspective of mathematics education

Q: High-efficiency mathematics teachers always pay more attention to revealing the educational value of mathematics, such as the pursuit of the true, the good, and the beautiful, rationality, concise mathematics language, coherence of knowledge, connectivity of mathematics ideas, rigorous reasoning, beautiful mathematics thought, charms of application, and so forth.

H: Yes, just like the things you mentioned, they can make us better understand that mathematics is not just about numbers and graphs but culture, it has its own spirit.

Q: Next, high-efficiency mathematics teachers focus on showing students the origin and development of mathematical concepts and symbols to help the students better understand mathematical culture.

H: Aha, yes, that's my report! Symbol is just as a mathematics learning tool, but grasping the meaning it represents can help make the symbol easier to use. We should know that mathematics history is not for solving problems; it is to provide rich understanding perspectives for teachers and students.

Q: Thank you! And next, high-efficiency mathematics teachers focus on

summarizing mathematical law and spreading mathematical thought and method, letting students better understand that mathematics is a subject characteristic of thought and method.

H: Yes, summarizing of mathematical thought and method is necessary. Teachers always provide that summarization because it can save a lot of time, and you know that the class time is limited and there is not much time for students to rebuild. But understanding the formula is more important than the formula itself.

Q: Professor, we have a question. If students can't understand the formula very well, but know how to use it, when they meet a similar problem, they can solve it without thinking. From this angle, what do you think of the relevant relationship between grasping a formula proficiently and the teaching efficiency?

H: I don't know how you understand high-efficiency. If you take gaining high marks in the examination as one teaching indicator, grasping a formula proficiently indeed can be good for solving problems and be very important to examinations. But from my own opinion, mathematical beauty is more important.

Q: You mean, high-efficiency mathematics teachers not only can teach students how to solve problems but also can make students enjoy mathematical beauty and love mathematics. So teachers who can only teach students how to solve some kinds of problems to a certain extent are not high-efficiency teachers.

H: Yes, it is. For example, Japanese students always get high marks in international mathematics level tests. But when you ask them whether they like learning mathematics, most of them will shake their heads and say "no".

From the perspective of teaching creativity

Q: High-efficiency mathematics teachers focus on creating a relaxed and harmonious atmosphere in mathematics classroom.

H: Yes, I agree. Harmonious atmosphere is very important. It's good for students and teachers to imagine and create a lot of interesting ideas.

Q: Next, high-efficiency mathematics teachers focus on creating situations from the beginning of class to generate students' cognitive conflict and stimulate students' desire for knowledge.

H: We should know that generating students' cognitive conflict is a good method, but it is very hard to do it just right.

Q: High-efficiency mathematics teachers often reprocess the content,

teaching in one's own unique way.

H: Yes, "one's own unique way" seems very important. Mathematics teachers need to have this kind of ability to make their own decisions and make creation.

Q: High-efficiency mathematics teachers focus on visualizing knowledge through different ways to make it easier for students to understand.

H: Yes, visualizing mathematics knowledge is so important, and teachers need to have this ability.

Q: High-efficiency mathematics teachers focus on creating problems at different levels following students' cognitive order.

H: Yes, I agree.

Q: High-efficiency mathematics teachers focus on creating situations to let students experience the "re-creation" process through observation, experiment, induction, analogy and other activities.

H: Yes, I agree. The ability for teachers to create situations is very important, and students also need this "re-creation" experience.

Q: High-efficiency mathematics teachers focus on selecting examples with explorative value and doing proper variant teaching, to make students feel the beauty of mathematical method and mathematical thought.

H: Yes, I agree. In fact, how to select good examples just like you mentioned is very challenging for teachers, and the beauty of mathematical method and thought is what I have emphasized.

From the perspective of information feedback

Q: High-efficiency mathematics teachers make sure that every student has the same opportunity to answer questions.

H: Yes, this is right. Mathematics teaching is not for the few. Everyone should get the opportunity to develop from learning mathematics. But it is very difficult to do this when there are a lot of students.

Q: High-efficiency mathematics teachers always encourage students to ask questions boldly and freely in order to protect the enthusiasm of students asking questions.

H: Yes, of course it is very important. I don't understand why you said "encourage students to ask questions boldly"? What makes them not ask questions boldly? Chinese students seem to be shy. You won't see this in the USA.

Q: Chinese students sometimes are modest and cautious. They don't speak out when they are not very sure about their ideas.

H: Why do they act like this?

Q: Maybe this is relevant to Chinese traditional Confucian culture.

H: Yes, I think maybe so.

Q: When students do not answer questions precisely and completely, high-efficiency mathematics teachers will continue to make more detailed inquiry to help students find the answers.

H: Yeah, I can't agree with that more.

Q: High-efficiency mathematics teachers focus providing guidance for students in need in a timely way as they receive feedback from students.

H: Yes, this is very important but also is very challenging.

Q: When students get stuck in mathematics learning, high-efficiency teachers focus on their mathematical thinking and not just answers.

H: Yes, I agree with that.

From the perspective of the art of teaching

Q: High-efficiency mathematics teachers can integrate his or her own characteristics into teaching to vitalize the whole lesson and hold students' attention.

H: Yes, just like what we said, teacher's creativity is very important.

Q: High-efficiency mathematics teachers focus on teaching from a higher level of knowledge system to better establish vertical and horizontal linkages between knowledge points.

H: What does "vertical and horizontal linkages between knowledge points" mean?

Q: It means all kinds of relationships between knowledge points, and it can make students understand that knowledge is integral not singular.

H: Oh, I see. You mean, the mathematics teacher puts what he will teach into the whole mathematical structure, and when he teaches, he knows what to teach and how to teach, right?

Q: Yes, you are right, professor.

H: Ok, if so, I agree with that.

Q: In the process of instructing, high-efficiency mathematics teachers focus on revealing their thinking process while solving problems.

H: Revealing whose thinking process? Teacher's or student's?

Q: Here we refer to the teacher's problem-solving process. It can exemplify how to do it with the knowledge students have learned, and students can grasp the mathematics thought and method during it.

H: Yes, exemplification is necessary, but be careful not to let students just

stay on the imitative level. To myself, I think revealing the student's thought is more important than the teacher's.

Q: High-efficiency mathematics teachers are good at using a variety of instructional media and models to optimize the teaching process.

H: Yes, this is a technological requirement for mathematics teachers. Likewise, this is very important but is not emphasized enough.

Q: High-efficiency mathematics teachers focus on the effectiveness of rhetorical questions, setting questions from ability and methods, from the angle of the error-prone points, from the raising points, the growth points, and from the development potential points.

H: I am not very clear about these points. I am doing training for school teachers and have no experience in classroom teaching, so I think I'd better pass this one.

Q: OK. We understand thoroughly. Next, high-efficiency mathematics teachers focus on the design of a simple and decent blackboard layout with principal and difficult knowledge points made prominent.

H: Yes, blackboard layout is necessary, but not emphasized too much.

Q: High-efficiency mathematics teachers focus on making the final summarization refined and with the entire class as a whole.

H: Yes, I agree.

Q: High-efficiency mathematics teachers always have vivid and humorous teaching language, and rich teaching posture.

H: Yes, "vivid" seems very important in mathematics learning. Which one do you think is more important, rigour or intuition?

Q: Intuition?

H: Although rigour is the main feature of mathematics, I think intuition is more important, because it is source of creativity. Many great mathematicians create their ideas by intuition, and then prove them.

Q: High-efficiency mathematics teachers focus on reflecting on the teaching, and at the same time properly lead students to reflect on their learning.

H: Yes, the ability of self-reflection is very important, whether the teacher's or the student's.

The attribution of high-efficiency mathematics teaching behavior

Q: Now let's find causes of high-efficiency teaching behaviors. There are some internal causes such as: high-efficiency mathematics teachers have a

solid foundation and deep understanding of mathematics, have a foundation of mathematical methodology, mathematical philosophy and mathematical history, have reasonable views of mathematics teaching and education, have reasonable views of students, understand the content before walking into classroom, believe that mathematics teaching should build on the basis of mathematics learning.

H: Besides what you have mentioned, I think mathematics teachers should have epistemology.

Q: epistemology?

H: That is, what's knowledge, where they come from and how do we know.

Q: Thank you very much telling us about that. We summarize the external causes such as: the school or district of high-efficiency mathematics teachers gives high priority to teaching efficiency; High-efficiency mathematics teachers design the whole class for efficiency and have considered various factors that can affect effectiveness before class; students in high-efficiency classes are actively engaged in activities designed by the teacher; students in class have higher overall qualities than others.

H: The last item, "students in class have higher overall qualities than others", what does this mean?

Q: We mean students in high-efficiency class may have a better foundation than students in low-efficiency class.

H: You mean your students are divided into superior and inferior ones, and then put them in different classes?

Q: No, professor, we don't mean that. "Better foundation" may be affected by a lot of factors, such as teachers, families and so on.

H: Ok, what I will emphasize is that mathematical education is not for the few, let alone for getting high marks in examination. Mathematics is education for all. Different learners can understand mathematics differently and get benefits from it.

Q: Yes, you are absolutely right!

H: Ha, these hypotheses are very nice, but very difficult to do them just right. They are really very challenging. What I said just represents my own opinions, and I don't know whether it is useful for your research.

Q: As a matter of fact, your opinions are very precious. Sorry to disturb you so much. We really appreciate your sincere help! Thank you!

H: You are welcome, it's my pleasure. Hope your research goes well and makes new discoveries!

Discussion

Through analysis of interview, we found that professor Bernard R. Hodgson basically agreed with our hypotheses, and also advanced precious opinions to let us see the discrepancies between Western and Eastern mathematics instruction. This will make it better for us to understand mathematics teaching efficiency.

From the Respective of Mathematics Learning

In this part, professor Hodgson agreed with us that multi-representatives could help students understand mathematics concepts from multi-perspectives, but added that this phenomenon was not so common in the USA where mathematics teachers generally let students understand or grasp one main representation in order not to confuse students' minds.

This allows us to see discrepancy exists between Western and Eastern views of mathematical teaching. Generally speaking, Chinese mathematics instruction emphasizes student's deep understanding while Western instruction pays more attention to student's self-understanding of mathematics. Chinese students are required to understand multi-representations of concepts and learn to represent the key concepts using multi-forms, which can demonstrate whether students genuinely grasp the concept. What's more, the latest national mathematical curriculum standard added one more aim, which is to "cultivate students' sense of symbol". Western instruction allows students to use the way they like to represent concepts, after they construct new knowledge on their own cognitive structure. Then mathematics teachers begin to show them the formal way. This can make students easily understand that mathematical representations or symbols sometimes is a kind of "contract" between mathematicians. We need to reflect on our mathematics instruction, can multi-representations really optimize student's cognitive structure?

In addition, Professor Hodgson considered that formal mathematical language and symbols should not be over-emphasized in case that teachers set them as instructional aims and deviate from the natural—that is, students' understanding of mathematics. However, in our instruction we don't think so. On one hand mathematical language can embody the features of mathematics, that's the beauty of conciseness; on the other hand, concise language can help students clarify their thinking and find out the fallacious pivot.

From the Perspective of Mathematics Education

Professor Hodgson attached much importance to students' understanding

of mathematics knowledge, and indicated that mathematical symbols were just tools to help students understand the nature of mathematics. So professor gave his suggestion that mathematics teachers should look for and collect rich topics and materials from mathematical history. At the same time, he stressed that mathematical history was not used to help students solve problems but to offer rich understanding of their perspectives. In Western countries, getting students to understand and love mathematics is more important than getting students to grasp deep knowledge and skills.

In domestic mathematical instruction, teachers, especially in senior school, prefer to illustrate mathematical method or strategy to help students get high marks in entrance examinations for college. In their eyes, mathematical history or mathematical culture does not have relevant relationship with the entrance examination. By contrast, just as what Professor Hodgson said in the interview, Western instruction puts student's love and enthusiasm to mathematics in first place. Only students can really understand what they have learned and enjoy the nature of mathematics. They can accept it from their inner hearts. If they learn mathematics just due to some external motivation, they will drop it once the motivation doesn't exist. Mathematics learning like this kind is low-efficiency. Therefore, high-efficiency mathematics teachers not only can teach students how to solve problems, but can also make students enjoy mathematical beauty and love mathematics. If teachers only teach students how to solve problems to certain extent are not high-efficiency teachers.

From the Perspective of Teaching Creativity

Professor Hodgson pointed out that creating a harmonious and relaxed classroom atmosphere was very important for new ideas could easily be developed under these circumstances. But our point is trying to help students not feel simple, this may be to some extent relevant to Chinese traditional Confucian culture which confused Professor Hodgson. Generally speaking, Western instruction focuses on student's imagination and ideas while our instruction focuses on pursuing logical rigor and correct answers.

Professor Hodgson indicated that every mathematics teacher should have the abilities of self-judgment, self-decision and self-creation. The ability to self-creation is the most important. High-efficiency mathematical instruction requires that teachers should make continuous self-reflection, self-judgment, self-creation after probing teaching materials and students. On the other hand, if one teacher fails to recognize teaching materials and students correctly, his

teaching may be low-efficient.

From the Perspective of Information Feedback

Mathematician *George Polya* mentioned in one of his works, *How to Solve It* that there are two exercising forms in mathematical instruction. One is a practical problem which comes from real life; the other one is a written exercise which is compiled by writers, and the latter is the main form of exercises for students. Professor Hodgson emphasized that in mathematical instruction revealing student's thinking was more important than revealing the teacher's, in case that students remained on the imitative level and did not use their own thinking. For our instruction, the condition may be somewhat different. When a mathematics teacher shows his process of problem-solving with his students, students can benefit a lot, they can learn how to solve unknown from known, and they can learn how to apply their knowledge, they can gradually grasp its thought and method. What's more, it is good for students to solve similar questions and advance their learning efficiency. Sometimes training and exercises in mathematics are necessary, for they form the foundation for automatic mathematical skills. Our instruction advocates creation on a solid foundation. Generally speaking, domestic students usually have a more rigorous logical thinking but less imagination, while Western students have more fantastic ideas but less rigorous logical thinking.

In addition, Professor Hodgson pointed out that compared with mathematical rigour, mathematical intuition would be more significant, for many important hypotheses and theorems and even formulas come by way of the mathematician's intuition and not by deduction. This can lead us to consider whether some gorgeous ideas of our students are lost during over-training for "two basics" standard.

From the Perspective of the Art of Teaching

On this part, Professor Hodgson especially mentioned that high-efficiency mathematical teachers should have epistemology which means "what is mathematical knowledge, where it comes from and how do we know". This kind of knowledge can ensure whether a mathematics teacher has profound understanding about what he will teach, and whether he has the ability to deliver high-efficiency teaching.

When the interview came into the end, Professor Hodgson stressed that mathematical education was not for just a few, let alone for passing examinations. Mathematics is education for all, so different learners can

understand mathematics differently and get benefits from it. This is consistent with our pursuit of high-efficiency mathematics teaching.

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