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A Controversy between Mathematics and Mathematics Education

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The Math Reforms of the 1960s started a controversy between mathematicians and mathematics educators. This happened in China in the 1980s. In this article, we review briefly the history of geometry leading to the kind of geometry we teach in schools today. We record the views of mathematicians and mathematics educators on the new mathematics curriculum. Finally, we discuss a feasible path of moving forward in order to resolve the matter. This is a written version of a talk given at the Hangzhou conference in June 2010.

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A Controversy

The Math Reforms of the 1960s started a controversy between mathematicians and mathematics educators. This happened in China in the 1980s. It happened in the United States much earlier and it went on until recently. It was so serious that it was called a war. In this article, we refer mainly to what happened in China. In fact, there was no controversy as far as the teaching objective of mathematics is concerned between mathematicians and mathematics educators. One major change in the Math Reforms was the reduction of plane geometry in the syllabus (standards). The same happened in China as elsewhere in the world. The question of whether we should or should not reduce the geometry content created a deep divide between mathematicians and mathematics educators, particularly in China.

In this article, we review briefly the history of geometry leading to the kind of geometry we teach in schools today. We record the views of mathematicians and mathematics educators on the new mathematics curriculum. Finally, we discuss a feasible path of moving forward in order to resolve the matter. This is a written version of a talk given at the Hangzhou conference in June 2010.

History of Geometry

Euclidean geometry dominated the west for more than 2000 years. Historically it was a subject of academic pursuit. At one time mathematics was nothing but geometry. Professors of mathematics were called professors of geometry. The rise and fall of Euclidean geometry occupied a great part of the history of geometry. Even in the 1950s Euclidean geometry was taught as an axiomatic system. In schools, it meant proofs. For the past four hundred years, the world history was a history of dominance by the West. It began with the renaissance in Italy, then protestant reformation in Germany, and it culminated in the French revolution. In terms of the development of geometry, non-Euclidean geometries were discovered. Hence there were many geometries, and Euclidean geometry was only one of them. Furthermore, Euclidean geometry was not axiomatic as we were led to believe.

To make a long story short, it was David Hilbert (1971) who saved Euclidean geometry and then destroyed it. He saved it because he provided a sound list of axioms so that Euclidean geometry was once again valid as an axiomatic system. With such a long list of axioms, it was no longer suitable to teach it in schools. Therefore, we say that Hilbert destroyed it. In other words, Euclidean geometry as taught in schools in the past has to go. We still teach plane geometry in schools, but not the same as before. What we are teaching in schools now follows the approach of Birkhoff (1941). More precisely, we do not begin with points, lines, and parallel lines. Instead we assume without proof the congruence of triangles and prove the rest from there onward.

It is a myth that we teach less geometry because it was no longer an active area of research. In fact, the marriage of calculus and geometry gave birth to differential geometry and manifolds. Five mathematicians were awarded Fields Medals in the past fifty years for their work in manifolds. The fields medal is equivalent to the Nobel Prize in mathematics. So geometry has moved into a bright new home. For the study of convergence, we often use algebra. For the study of divergence, we may have to use geometry. It is true that many topics in school mathematics have gone algebraic. Geometry still plays a major role in the development of mathematics.

New Curriculum

A reduction of geometry was not the only change in the new curriculum. There were other changes. For example, new language was introduced such as set, function (always single-valued), and line segment. The presentation was more structural in the sense that we used the terms such as commutative law, associative law etc. Some other topics were removed, for example, elementary symmetric functions, and trigonometric identities. New topics were added, notably, statistics. But no other topics in the new curriculum generated such a heated debate as did geometry.

Other than content, another aspect of the new curriculum is the approach in teaching. It was no longer taught in the traditional sequence of examples, solutions and exercises. The discovery method was promoted. Later on, it was down graded to problem solving. Problem solving is nothing but guided discovery. Both mathematicians and mathematics educators had strong views about all the changes.

Here are some views of mathematicians. We may have expressed them in stronger words. As far as mathematicians are concerned, to teach geometry is to teach proof. Teaching proof is mainly done in plane geometry. A reduction of geometry would weaken the training of students in proving, that is, to be able to argue rigorously. It is very serious, and not acceptable.

Mathematicians find it difficult to endorse the kind of pedagogy promoted by mathematics educators. They have no objection to teaching being activity-based. In the new approach, everything is hands-on. They see hands-on and hands-on in the classroom, and never hands-off. In other words, time is wasted on low-level learning. One should advance further to a mathematics of substance. The quality of mathematics has been compromised.

An extreme view is that some mathematics taught is not mathematics at all. An example is tessellation. Another example is those so-called problems in context. The problems may be interesting by themselves but mathematical content is highly diluted.

For mathematics educators, a new element in teaching is pedagogy. In other words, we do not only look at the content but also the approaches. First of all, if we can, we should make it fun for students. We should teach in context and emphasize the process. This is nothing new. If we look at a mathematics treatise in ancient China, typically the book would contain a series of problems. Each problem is always posed in context. Then a process is given to lead to an answer. There is no formula. Such an approach is not alien to Chinese, except that they lost it after importing modern mathematics from the West. One other item is, whenever possible, engage in activities. Of course, this is a simplistic picture of the situation.

All the changes did not take place overnight. They were developed over a long period of time. It may be useful to trace the passage of mathematics reforms.

Passage of Mathematics Reforms

Let us look back for 50 years and give a summary. There were three waves of mathematics reforms. The first wave took place in the 1960s and 1970s, as mentioned in Section 3. The second wave refers to the years in the 1980s and 1990s when the back-to-basic movement came in and, eventually, problem solving dominated the scene. During the second wave, there were minor changes in content. The major changes occurred in the approaches in teaching. These were extensively documented in the publications of ICMI (International Commission on Mathematical Instruction). In the past 10 years, the changes were mainly in the classroom. We may call it the third wave. We shall not elaborate on it here. There are good reasons for the inclusion or exclusion of certain topics in school mathematics. See Lee (2009). Content is not a focus in the latest reforms.

One exciting outcome of the mathematics reforms is that mathematics education gradually became an academic discipline. See Kilpatrick (2008). Lots of money was poured into research in the so-called new mathematics during the reforms in the 1960s and 1970s. Research centers were set up all over the world, especially in America and Europe, though some centers closed down due to not linking to subjects outside mathematics. New journals sprang up catering to research articles in mathematics education. Currently, we seem to have too much research and not enough development. By development, we mean the kind of developments that would bring the fruit of research back to the classroom. Under the above-mentioned background, we look at the so-called controversy again.

Crossing Boundaries

In a way, to teach mathematics is to teach rigor and to teach applications.

For a long period of time, we taught rigor through proofs in plane geometry, and we taught applications in mechanics. The current issue seems to be the teaching of proofs. We have not come to the issue of applications yet. It will come. That is another totally different issue. So the key issue now is rigor, not proof, and proof is part of rigor. Thus the question is how to teach rigor and where to teach it.

Initially, we want to make mathematics concrete. Eventually, we have to go abstract. To go abstract, we need rigor. Hence the objective is to be able to reason rigorously and to be able to argue at an abstract level. In the past, the teaching of geometry served this purpose. The teaching of geometry may be the source of the controversy. But, it is not the real controversy. If we look at school mathematics carefully, we shall find that geometry is everywhere, not only in plane geometry, and that we can teach geometry anywhere, not necessarily in plane geometry. Ideally, we should teach algebra in geometry and teach geometry in algebra. Ideally we should teach proofs and indeed different kinds of proofs at different levels. We teach informal reasoning at the elementary school level, procedural argument at the middle school level, and formal proof at the high school level.

We had the Math Reforms in the 1960s and 1970s. Then we had back-to-basic movement in the 1980s. When we said back to basic, we did not really go back to where we were before the reforms. We moved forward. Like all other reforms, there is no return. We can only move forward. Again like all reforms, initially there will be setback. The initial setbacks should not be a reason for not moving forward.

Both mathematicians and mathematics educators are correct in their own views. What is missing is dialogue. If a problem was created over so many years, we cannot expect it to be solved in one day. We need to put in place a process. We need to create an environment that could lead to a consensus. To do that, the first step is to cross the boundary. I mean the boundary that divides mathematicians and mathematics educators. More precisely, we must be prepared to listen to each other and to understand the viewpoints of other people. We do not expect everyone to cross the boundary. Some of us must do so. In time to come, the matter will be resolved. If we want mathematics to be taught well in schools, we must resolve the matter in due course.

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