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Changes in Equality Problem Types Across Four Decades in Four Second and Sixth Grade Textbook Series

Robert M. Capraro, Mary Margaret Capraro, Rayya Younes,
Sun Young Han, & Kayla Garner
Texas A&M University & Aggie STEM Center

Textbooks can serve as artifacts that provide insights into how content was presented historically. In this study, we examined the equal sign and relational equality in four widely adopted textbooks (Grades 2 and 6) over a longitudinal period that spanned four decades from 1970-2010 (where possible). The textbooks (N = 29) were coded page by page using 11 categories. While the results of our study show that textbooks have made progress over the years towards including multiple contexts for the equal sign, there is still a need for inclusion of a greater variety of problem tasks to improve students' understanding of the equal sign. There were few differences between modern and longstanding textbooks examined in this study. The most dramatic difference in Grade 2 textbooks was the gradual reduction of treating expressions as equations in Holt, Scott Foresman (SF), and the University of Chicago School Mathematics Program. The SF textbook in both grades 2 and 6 showed the greatest improvement in the odds for students to see items conducive to understanding the equal sign over time. Across both grade levels, all the textbooks had a greater percentage of items conducive to understanding the equal sign as compared to Saxon.

Key Words: textbooks, equal sign, elementary, middle, longitudinal analysis

The purpose of this study was to examine the presentation of the equal sign and relational equality tasks in popularly adopted second and sixth grade textbooks from 1970 - 2010 in an attempt to contextualize findings about students' understanding of relational ideas. Early work in the U.S. identified broad and prolific deficits in elementary students' understanding of the relational symbol for equivalence (=) (Behr, Erlwanger, & Nichols, 1980). Later studies have shown that student misconceptions about the equal sign remain broad but are not nearly as prolific (Knuth, Stephens, McNeil, & Alibali, 2006; Li, Ding, Capraro, & Capraro, 2008; McNeil, 2007; Warren, 2003). In addition, subsequent studies have shown that textbooks (Baroody & Ginsburg, 1983), instruction (McNeil, 2007), and experience with different problem types (McNeil, 2008; McNeil & Alibali, 2005; McNeil, 2006) influence students' understanding of equivalence. What is lacking in the literature is a review of the equal sign presentation and problem types in textbooks over time. The potential impact of this study is that findings may show that textbooks have changed either in equal sign presentation or problem types presented across time, which could lead to greater accountability for textbook publishers and authors.

Review of the Literature

Textbooks

Textbooks serve as artifacts that can provide a glimpse backward into the archives of education at the time of publication. Examination of textbooks yields insight into what was taught and how concepts were viewed from an instructional perspective at a particular period in time. In addition, when examined across time, textbooks can provide a context for research findings (Capraro, Yetkiner, Ozel, Capraro, Ye, & Kim, 2009). As succinctly stated by Westbury (1990), "The textbook is, in fact, the heart of the school and without the ubiquitous text there would be no schools, at least as we know them" (p. 3). Thus, textbooks play a fundamental role in how students learn mathematics. "Teachers decide what to teach, how to teach it, and what sorts of exercises to assign to their students largely on the basis of what is contained in the textbook" (Reys, Reys, Tarr, & Chavez, 2006, p. 5). Therefore, textbook analyses can provide valuable data when trying to understand students' mathematics knowledge and achievement.

Textbook Analyses in Prior Studies

Earlier studies coded only a portion of textbooks. For example, one study coded a randomly selected sample of 50% of the pages in five middle-grades textbook series (McNeil, Crandau, Knuth, Alibali, Stephens, Hattijudur, & Krill, 2006). A coding scheme was used to examine the dual process for coding students' responses to definitions for the equal sign and for equation solving (Knuth et al., 2006). Tornroos (2005) examined mathematics textbooks to determine whether they contained examples that helped students answer high-stakes items correctly across a range of seventh grade mathematics topics.

Sequencing Instruction

Research showed that students had a better understanding of equivalence when they were first exposed to concrete tasks, which were then followed by reflexive examples, before lastly experiencing abstract problems. Students who were exposed to all three methods performed better than those who were exposed to only one method (Fyfe & McNeil, 2009). Providing students with four different presentations of equivalence (operations on the left and right hand sides, reflexive, filling in missing numbers, and operations on both sides-the most abstract) enhanced their understanding of the equal sign (McNeil et al., 2006). When students were faced with solving problems like 4 + 3 +2 = 1 + ____, which contained an operation on both sides of the equal sign and filling in missing numbers, students responded predominately in one of two ways (McNeil, 2007). Students either summed the numbers on the left hand side of the equation and considered their answer to be the missing number, or the students added all the numbers together (ignoring the equal sign) and considered their answer to be the missing number. Both responses showed a lack of relational understanding of the equal sign. In this study, students were taught how to solve operations on the left hand side and were shown reflexive examples. However, these students still did not perform well when faced with questions that had operations on both sides (McNeil, 2007). Thus it is important for students to first be exposed to concrete examples of the equal sign and then to a variety of problems involving non-standard contexts that focus on the relational meaning of the equal sign.

Equal Sign

Algebra has been characterized as the gatekeeper mathematics course with the potential for advancing students into higher-level mathematics and career opportunities (Ladson-Billings, 1997), or conversely, deterring them from participation in higher level courses (Herscovics & Linchevski, 1994). Without competency in algebra, students are often excluded from participation in more advanced mathematics courses. The concept of the equal sign is fundamental to understanding algebra. Research showed that teachers often overestimated the number of students who understood the equal sign as a relational symbol, which may indicate an underlying problem for learning algebra (Asquith, Stephens, Knuth, & Alibali, 2007).

Student misconceptions about the equal sign have been studied for over thirty years, which indicates that students have long-standing difficulties with relational thinking (Bernstein, 1974; Ginsburg, 1989; Hiebert, 1984; Kieran, 1981; Li et al., 2008). The ability to define the meaning of the equal sign symbol is important, as it has been linked to later success in algebra (Knuth et al., 2006) and further success in more advanced mathematics courses (Usiskin, 1995). It has been commonly suggested that the equal sign should be carefully taught to prevent student misconceptions and to ensure that the relational meaning of the equal sign is emphasized (Baroody & Ginsburg, 1983). Importantly, there has been limited research examining how representation of different problem types in textbooks impact students' relational understanding of the equal sign. In this regard, no studies have been able to identify a "textbook effect," as most students have not experienced one textbook consistently. Studies conducted in the advanced elementary and middle grades (Knuth et al.; Li et al.; McNeil et al., 2006; Rittle-Johnson & Alibali, 1999; Sáenz-Ludlow & Walgamuth, 1998; Seo & Ginsburg, 2003) were confounded by students' potential exposure to many different textbooks. Thus, there is a need to examine a cross section of textbooks to determine the representation of problem types aligned with building relational understanding.

Several studies have attempted to identify mathematical sentences that could contribute to students' misconceptions about the equal sign. "Most often, sentences do ask children to perform a calculation; if so, why should they interpret them otherwise?" (Ginsberg, 1989, p. 113). Only 31% of fourth- and fifth- grade students correctly solved problems such as $3 + 4 + 5 = 3 +$ ___ (Rittle-Johnson & Alibali, 1999) and only 32% of sixth grade students were able to provide a correct definition of the equal sign (Knuth et al., 2006). Overall, these studies suggested that children from upper elementary to middle grades often misunderstood the equal sign as an operational symbol, (i.e. a signal for "doing something)" rather than as a relational symbol indicating quantity sameness (Sáenz-Ludlow & Walgamuth, 1998).

More recently, researchers (Li et al., 2008; McNeil et al., 2006; Seo & Ginsburg, 2003) have used mathematical sentences that revealed equal sign misconceptions to examine the relationship between the presented contexts and student understanding. This work has shown that students' understanding of the equal sign depends on how it is presented during instruction. For example, second grade students were shown to have a context-dependent understanding of the equal sign based on their textbook (Seo & Ginsburg). Similarly, in an examination of four middle-grades textbooks, students' interpretations of the equal sign were found to be shaped by the context of the textbook (McNeil et al., 2006).

None of the previous studies on equivalence have examined patterns in textbooks over time. Therefore, the driving question of this study was to determine how second and sixth grade textbooks have changed over time to reflect the findings in recent studies about students' understanding of the equal sign. More specifically, the following questions were addressed: How have the types of representations in second and sixth grade textbooks changed over time, and to what extent do the most recent textbook editions reflect the most current research findings about teaching and learning the equal sign and relational concepts?

Methodology

Mathematics textbooks from second (n = 15) and sixth (n = 14) grades were coded page-by-page to determine how the equal sign and relational equality tasks were presented. Textbook series were selected for coding based on either availability from 1970 through 2010 or because they reflected innovation in mathematics education (although lacking a long publication history). We identified two textbooks that could be traced across time (Scott Foresman-Addison Wesley [SF], and Holt/Houghton Mifflin [Holt]) and were adopted in the major markets. Two textbook publishers were also included: Saxon and University of Chicago School (UCSMP) Mathematics Program to represent contemporary thought about mathematics education. Both textbooks were adopted in the same major markets as the other textbooks and arose from an immediate need in mathematics education. In addition, these two textbooks had not yet been systematically examined.

The coding was divided into two main categories – standard and non-standard contexts. The standard context presents the problem in the form of 3+5= (operation on the left side only with the answer on the right) or vertical $\begin{array}{r} 11 \\ +3 \\ \hline 14 \end{array}$, where there is an equivalency bar between the computation above and the answer below. The non-standard context consisted of all other problem presentations. The standard context is described in previous research as leading students to view the equal sign as an operator (place the answer in the blank or box) in contrast to those in non-standard context that conveyed a relational meaning of the equal sign that encouraged students to balance both sides of the equal sign (McNeil et al., 2006). The nine non-standard contexts (Li et al., 2008) used in previous work were incorporated in this study to facilitate comparison and included: name part of the operation (e.g. 4__4 = 8; place a + sign on the line), filling in missing numbers (e.g. 5 + ____ = 9), no explicit operation on either side (1 foot = 12 inches), operation on the right side only (__ = 7 + 9), operations on both sides (6 + __ = 7 + __), use/insert relational symbols (< [is less than], > [is greater than], = [equals], ≠ [is not equal to], i.e., $6\,\underline{\ <\ }\,9$, and verbal representation (with words, i.e. *are equal to*, *is the same as*). For comparison purposes, we retained the relational symbols of greater than and less than because they were included in prior work (McNeil et al., 2006). The following three categories (cf. Capraro et al., 2009) added to the Li et al. (2008) coding were also included in this study: (a) without an equal sign (e.g. 3 + 2); (b) match to an equivalent quantity or statement, using an arrow to connect two quantities (e.g. 7 --→ 3 + 4), and (c) the equivalency bar $\begin{array}{r} 11 \\ +3 \\ \hline 14 \end{array}$. Table 1 outlines each of the categories and provides examples of each.

Table 1
Coding Descriptions and Examples

	Code Number	Code Description	Example
Standard Representation	Code 1	Operation on Left Side Only	9+5=14
	Code 2	Equivalency Bar	$\begin{array}{r} 11 \\ +3 \\ \hline 14 \end{array}$
Non-Standard Representation	Code 3	Without Equal Sign	7 +3 or match to an equivalent quantity
	Code 4	Name Part of Operation	4__4 = 8
	Code 5	Using Arrow to Connect	7 → 3 + 4
	Code 6	Filling in Missing Numbers	5 + ____ = 9
	Code 7	Reflexive: No Explicit Operations on Either Side	12 inches = 1 foot
	Code 8	Operation on Right Side Only	___ = 7 + 9
	Code 9	Operations on Both Sides	6 + __ = 7 + __
	Code 10	Use/Insert Relational Symbols	6 ___9; insert <, >, or =
	Code 11	Verbal Representation	three plus four equals

Coding Reliability

Textbook coding took place over a one-year period. Because of the extensive coding scheme and scope of the study it was important to examine interrater reliability and intrarater reliability. More specifically, it was important that each rater be able to accurately classify the problem type and to prevent rater drift over time. To assess reliability, a second rater recoded a random 10% sample of the data. Agreement between coders was initially 92% and 97%, but reached 96% and 100% agreement after discussion of discrepancies. The intrarater reliability was 100% where each coder randomly recoded 10% of their original codings monthly.

Logistic Regression

The logistic regression model was used to generate odds to facilitate the discussion on the likelihood that students would encounter specific instantiations of the equal sign or relational equality. Logistic regression is used to predict dichotomous outcomes based on how individual textbooks changed over time or to compare the most current textbook editions. The odds represent the ratio of the number of occurrences to the number of non-occurrences. If the odds equal 1 this means that both outcomes have an equal probability of occurring. If the odds are less than 1 then the likelihood of occurring favors the outcome that was coded as 0 and if the odds are greater than 1 then the likelihood of occurring favors the outcome that was coded as 1 (Thompson, 2006). The 95% confidence interval (CI) was then computed from each odds, providing a graphic for easy comparison. For group comparisons, CIs that lie to right or left favor the group to that side of the comparison. For individual book comparisons, CIs that lie to right of 1 indicate a greater likelihood students will encounter that specific code in the latest edition of the book, while CIs that lie to the left of 1 indicate a greater likelihood that the students will encounter the code in the earlier versions. A natural grouping also emerged because SF and Holt have a long history of publication, while UCSMP and Saxon have publication inception dates in the 1990's. Therefore, we contrasted the likelihood of encountering the problem types by group followed by disaggregated findings for each book. For individual books, the 2010 edition was compared to its earlier editions.

Results

The second and sixth grade textbooks were coded for 11 contexts involving the representations of equivalence over four decades. In general, there was limited presentation of equivalence symbols coded as name part of the operation, using arrow to connect, operation on right side only, and operations on both sides across the years and across the textbooks in grade 2. Additionally, there was a limited number of instantiations for name part of the operation and filling in missing numbers in grade 6. The categories that were hypothesized to be aligned best with understanding the equal sign as a relational symbol were not overtly prominent, but were more evident in recent textbooks. For students to develop a relational understanding of the equal sign concept, it has been posited that students need experience with a greater variety of problem types including operations on both sides of the equal sign (McNeil & Alibali, 2005; McNeil, 2008) as well as scaffolded practice with operations on the left hand side, reflexive, filling in missing numbers, and operations on both sides (McNeil et al., 2006). The percentages were based on the instances of a context divided by the sum of all the other instances of all the other contexts multiplied by 100. This analysis allows for direct comparison of each textbook's change overtime as well as in comparison to other textbooks over time.

Grade 2 Results

The standard context was predominant across the textbooks and across the years, however the proportion of specific categories within both standard and non-standard contexts varied across textbooks and years. In 1990, Holt used the standard context the least (31%) while Saxon used the standard context more heavily (94%) (see Table 2). USCMP contained the greatest percentage of the problem types aligned with building relational understanding in its initial publication. However, the percentage of problem types aligned with building relational understanding decreased overtime and was slightly lower than Holt and significantly lower than SF. Thus, students who used either Holt or SF in 2010 were more likely to see the equal sign used in contexts conducive to building understanding of the equal sign, compared to students using either UCSMP or Saxon.

Table 2

Percentages of Equal Sign Contexts in Second-Grade Mathematics Textbooks

		Scott Foresman					*UCSMP*		*Saxon*			*Holt*				
Coding		*1970*	*1980*	*1990*	*2000*	*2010*	*2000*	*2010*	*1990*	*2000*	*2010*	*1970*	*1980*	*1990*	*2000*	*2010*
Standard Representation	Code 1	*36*	*13*	*16*	*29*	*28*	*31*	*26*	*11*	*14*	*17*	*25*	*11*	*27*	*25*	*23*
	Code 2	*35*	*71*	*71*	*52*	*27*	*10*	*47*	*83*	*79*	*75*	*42*	*43*	*4*	*58*	*42*
	Total standard	*71*	*84*	*87*	*81*	*55*	*41*	*73*	*94*	*93*	*92*	*67*	*54*	*31*	*83*	*65*
Non-Standard Representation	Code 3	*21*	*13*	*8*	*8*	*3*	*29*	*9*	*1*	*0*	*1*	*10*	*3*	*61*	*5*	*10*
	Code 4	*1*	*0.6*	*0.1*	*2*	*1*	*0*	*0*	*0*	*0*	*0.1*	*0.3*	*1*	*1*	*0*	*0*
	Code 5	*7*	*0*	*0*	*0*	*2*	*0*	*2*	*0*	*0*	*0.6*	*0.2*	*3*	*0*	*0*	*0*
	Code 6	*1*	*0*	*0.86*	*4*	*18*	*3*	*4*	*4*	*7*	*4*	*11*	*9*	*2*	*2*	*11*

Code 7	*0*	*0*	*0*	*0*	*3*	*14*	*5*	*0*	*0*	*0.3*	*1*	*0*	*0.5*	*2*	*1*
Code 8	*0*	*0*	*0*	*0*	*3*	*4*	*3*	*0*	*0*	*0*	*2*	*5*	*0*	*2*	*2*
Code 9	*0*	*2*	*0.2*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0.2*	*2*	*1*
Code 10	*1*	*0.3*	*4*	*2*	*8*	*9*	*0.6*	*0.5*	*0*	*1*	*5*	*17*	*4*	*4*	*5*
Code 11	*0*	*0*	*1*	*2*	*7*	*0*	*3*	*0*	*0.2*	*0.6*	*2*	*8*	*0.5*	*0.5*	*2*
Sum of codes 6-7-8-9	*1*	*2*	*1*	*5*	*26*	*21*	*12*	*4*	*7*	*4*	*15*	*14*	*3*	*8*	*15*

The results show that over time, the SF text reduced its overall use of the standard context, but doubled its use of operations on the left side only, from a low of 13% in 1980 to 28% in 2010. Concurrently, the SF text reduced presentations of the equivalency bar from a high of 71% in 1980 to 27% in 2010 and removed nearly all instances of without an equal sign from 21% to only 3%. The key point here is that the problem types which would most likely to lead to misconceptions about the equal sign were diminishing, while there was a greater representation of diverse problem types that promote better understanding of equal signs. Most notably, was an increase in the use of filling in missing numbers and relational symbols (both of which were seldom used in earlier decades) to a greater inclusion of 18 and 8 percent, respectively in 2010. While operations on both sides of the equal sign increased from zero to 2% over the decades, which is positive as it is aligned with research on relational understanding, the percentage is still very low.

In the UCSMP textbook the use of the standard context overall increased (41% to 73%, respectively), use of the equivalency bar dramatically increased from 10% to 47%, and the use of operations on the left side only slightly decreased. These net changes were not supported by research or aligned with recent research. However, without an equal sign decreased from 29% to only 9%, which was recommended by recent research and may help to limit the confusion of expressions and equations in later mathematics courses. Even though research has shown that operations on both sides are beneficial for improving student understanding of equivalence (cf. McNeil & Alibali, 2005), UCSMP did not include this category.

The Saxon textbook maintained its use of the standard context (94%, 93%, & 92%, respectively) and slightly increased the use of operations on the left side only (11% to 17%, respectively) but decreased presentations of the equivalency bar from 83% to 75%. The changes were not aligned with current research recommendations and the high percentages of standard context may foster equal sign misconceptions. Finally, the Saxon textbook had no tasks that included use of explicit operations on the right side only or operations on both sides, which are problem types supported by current research.

The Holt text fluctuated in its use of the standard context over the years of textbook publication and increased presentations of the equivalency bar from 42% to 56%. While this textbook had the second lowest percentage of standard context problems, it also had the greatest percentage of problems without an equal sign. According to recent research, these two changes are contradictory and not aligned with research suggestions. The use of problems without an equal sign may contribute to students' inaccurate interpretation. Scott Foresman and Holt have generally remained constant over time with the most abstract form, operations on both sides being represented approximately 2% across the years; SF and Holt had the greatest representation of items that are suggested by current research. Thus students using the most current versions of these books are more likely to encounter better problem types than those using either UCSMP or Saxon. SF also decreased the percentage of problems of without an equal sign, which is aligned with current research recommendations.

Grade 6 Results

The without an equal sign context was predominant across SF, Saxon, and Holt in grade 6 textbooks, which may cause students to believe that expressions and equations are the same. In UCSMP, the without an equal sign context remained consistently low overtime. The three textbooks that had closest alignment to recent research recommendations for high-quality problems (codes 6-9) were SF, UCSMP, and Holt with little difference between UCSMP and Holt. However, SF and Holt reduced the percentage of standard context instances to 20% or less, which indicates greater attention to recent research recommendations. In general, more non-standard contexts were used in sixth-grade textbooks over the years than in second-grade textbooks. The use of name part of the operation context was nearly nonexistent across all books for all years in grade 6 (see Table 3).

Table 3
Percentages of Equal Sign Contexts in Sixth-Grade Mathematics Textbooks

		Scott Foresman					*UCSMP*		*Saxon*			*Holt*				
	Coding	*'70*	*'80*	*'90*	*'00*	*'10*	*'00*	*'10*	*'90*	*'00*	*'10*	*1970*	*1980*	*1990*	*2000*	*2010*
Standard Representation	Code 1	*9*	*6*	*7*	*11*	*17*	*29*	*28*	*55*	*7*	*14*	*9*	*9*	*12*	*Unable to obtain this book either though library loan or through the publisher*	*15*
	Code 2	*6*	*26*	*13*	*1*	*3*	*9*	*9*	*26*	*30*	*7*	*30*	*33*	*19*		*3*
	Total Standard	*15*	*32*	*20*	*12*	*20*	*38*	*37*	*81*	*37*	*21*	*39*	*42*	*31*		*18*
Non-Standard Representation	Code 3	*55*	*39*	*54*	*56*	*34*	*14*	*13*	*4*	*50*	*39*	*31*	*27*	*35*		*35*
	Code 4	*0*	*0*	*0*	*0*	*0.2*	*1*	*0.2*	*0*	*0*	*0*	*0.5*	*0*	*0.2*		*0*
	Code 5	*1*	*1*	*0*	*0.2*	*1*	*0.4*	*3*	*0.4*	*1*	*3*	*0*	*0.2*	*0.2*		*5*
	Code 6	*14*	*10*	*5*	*19*	*23*	*1*	*3*	*3*	*4*	*14*	*11*	*11*	*12*		*1*
	Code 7	*8*	*11*	*4*	*4*	*7*	*12*	*16*	*3*	*2*	*6*	*6*	*4*	*6*		*22*
	Code 8	*1*	*2*	*7*	*3*	*7*	*11*	*9*	*0.2*	*0*	*2*	*3*	*9*	*3*		*8*
	Code 9	*1*	*2*	*7*	*1*	*3*	*11*	*8*	*1*	*0.4*	*2*	*3*	*1*	*2*		*2*
	Code 10	*2*	*2*	*2*	*5*	*4*	*3*	*4*	*4*	*3*	*4*	*5*	*5*	*9*		*5*
	Code 11	*3*	*2*	*0*	*0.4*	*0.4*	*8*	*7*	*3*	*3*	*8*	*2*	*1*	*1*	*X*	*5*
	Sum of Codes 6-7-8-9	*24*	*25*	*23*	*27*	*40*	*35*	*36*	*7*	*6*	*24*	*23*	*25*	*23*		*33*

Over time, the use of the standard context was reduced in the SF textbook but the use of operations on the left side only, nearly doubled, from 9% to 17% and presentations of the equivalency bar were reduced from 6% to 3%, which was consistent with recent research recommendations. SF also contained the greatest percent (19 and 23 percent) of filling in missing numbers context across textbooks.

The use of the standard context remained consistent over time in the UCSMP textbook and the use of operations on the left side only slightly decreased, but the inclusion of the equivalency bar remained unchanged at 9%, which is not consistent with recent research recommendations and can lead to student misconceptions about the equal sign. By sixth grade, UCSMP had the greatest percentage of operations on both sides (11% and 8%), however, the trend decreased over time with a greater variety of non-standard problem types as compared to the other textbooks, which is aligned with recent research but does not indicate dramatic improvement.

The overall use of the standard context significantly decreased in the Saxon textbook; presentations of the equivalency bar decreased from 26 to 7 percent, and the use of operations on the left side decreased from 55 to 14 percent. These changes are all clearly aligned with recent research recommendations. However, the Saxon textbook also had the greatest percentage of without an equal sign problem types of all the textbooks and the least percentages of operations on both sides, which is contrary to recent research recommendations. The overall use of the standard context decreased in the Holt textbook and presentations of the equivalency bar decreased from 30% to 3%, which is consistent with current research. However, the Holt textbook had an increase from 9% to 15% for operations on the left side only, which is not conducive to building a relational understanding of the equal sign. The items without an equal sign remained consistent across time (approximately 35%), which serves as another indicator that research was attended to for this code. The use of reflexive items increased across time becoming the most prevalent as compared to the other textbooks at 22% and the greatest decrease was filling in missing numbers items (12% in the early years to 1% in 2010). Both of these findings are aligned with efforts to improve students' understanding of the equal sign.

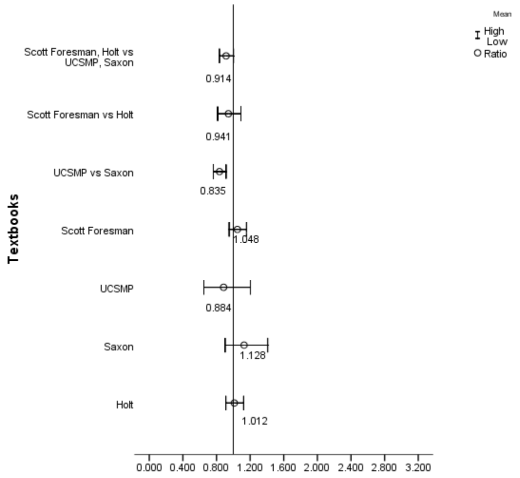
How did the two textbooks with the greatest longevity to date change presentations across time? In grade 2, SF authors increased their use of non-standard contexts, in some cases dramatically while decreasing the instances of without an equal sign. It is possible that not using an equal sign helps to perpetuate the interpretation of the equal sign as an operator. When students are asked to find the product, sum, difference, or to solve and are given a list problems to solve (i.e., 4X4, 5+14, and 13-6 etc.), this may perpetuate students belief that the equal sign is simply a command to compute and not equivalence.

At the same time, authors of the SF textbook increased the use of filling in missing numbers and operations on both sides of the equal sign. Both changes are aligned with current thinking about developing suitable understandings of the equal sign. The Holt textbook remained the most heavily invested in the standard context and had the lowest percentage of items in each of the categories suggested to improve students' conceptualization of the equal sign as compared to all the other textbooks (e.g., Fyfe & McNeil, 2009). In grade 6, both the SF and Holt textbooks showed a decreased use of the standard context, while increasing filling in missing numbers and no explicit operation on either side of the equal sign, respectively. However, neither textbook showed an appreciable increase in a majority of the non-standard context items.

In examining the contexts in which the equal sign and relational equality tasks were found, there was no clear distinction between UCSMP and Saxon. At grade 2, UCSMP (73%) and Saxon (92%) contain a majority of standard context presentations of the equal sign. However, the two textbooks differ in important ways. The authors of Saxon make little use of five of the coded categories: using arrow to connect, no explicit operations on either side (reflexive), operations on right side only, operations on both sides, and verbal representations as compared to UCSMP. At grade 6, the differences shift: UCSMP contains 37% standard context presentations of the equal sign as compared to 21% for Saxon. The two books are nearly equivalent on representation of four of the categories, but UCSMP authors emphasize use of operations on both sides of the equal sign. Neither UCSMP nor Saxon showed meaningful increases in the variety of problem types or in problem types associated with better conceptualization of the equal as suggested by research.

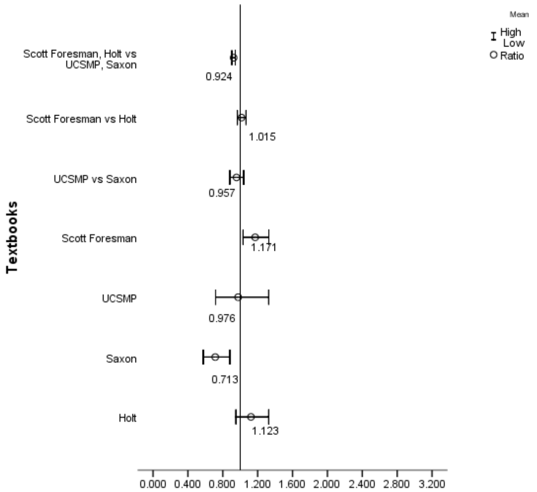
Logistic Regression Results

The logistic regression provides an odds ratio for interpreting the odds students would see any one code or group of codes. It also provides an estimate of the number's importance. The number for the odds ratio is bracketed by an interval, the wider the interval the less precise the estimate, the narrower the interval the more precise the point estimate or odds ratio. If the interval for the odds ratio covers 1, it is best interpreted as not being overly important. Therefore, the odds ratio allows the reader to determine if the odds of seeing a particular code or group of codes are an unimportant or important difference. Odds ratios are interpreted based on what is being compared, thus an odds ratio less than one indicates the first entry is more likely than the second entry.



Grade 2. Operation on left side only

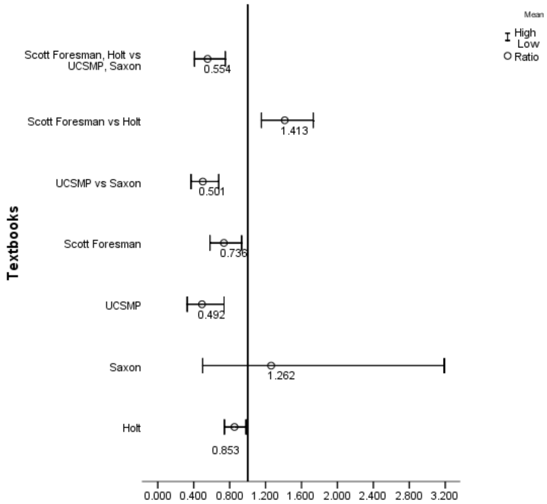
For grade 2 (see Figure 1) the paired textbook comparison showed students using the SF and Holt textbooks were more likely to encounter the operation on the left side only than those using UCSMP and Saxon textbooks. When comparing the latest editions of SF to Holt, and UCSMP to Saxon there was no difference between SF and Holt textbooks, but students were more likely to see operation on the left side only in UCSMP than in Saxon textbooks. When comparing each 2010 textbook to its cumulative ratings from prior years none of the second grade textbooks changed significantly overtime.



Grade 6. Operation on left side only

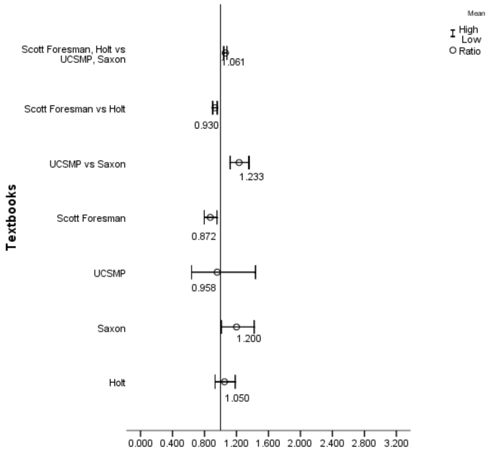
In the paired textbook comparison for grade 6 of SF and Holt to UCSMP and Saxon, there was a slightly greater chance of encountering an operation on the left side only in SF and Holt than in UCSMP and Saxon (see Figure 2). When comparing the latest editions of SF to Holt, and UCSMP to Saxon, there was no difference between SF and Holt or between UCSMP and Saxon.

However, meaningful changes included increased odds for encountering operation on the left side only in SF and decreased odds for encountering it in Saxon. However, these changes were only in comparison to each textbook's own prior editions.



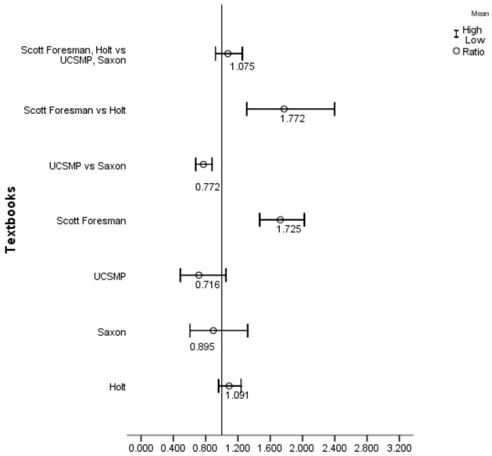
Grade 2. Without equal sign

The without an equal sign (or expressions) is arguably the single most confounding problem facing students in the U.S. as it creates ambiguity surrounding the role and function of the equal sign. In the grade 2 paired textbook comparison of SF and Holt to UCSMP and Saxon (see Figure 3), students using SF and Holt would be more likely to see expressions and asked to treat them as equations than students using UCSMP and Saxon. To examine the prevalence of expressions treated as equations in individual textbooks, we compared each pair of textbooks. Our findings showed that it was much more likely for grade 2 students to encounter expressions treated as equations in Holt than in SF and more likely in UCSMP than in Saxon textbooks. When comparing each 2010 textbook to its cumulative ratings from prior years, students using any of the textbooks except for Saxon were less likely to see expressions treated as equations as compared to their earlier editions.



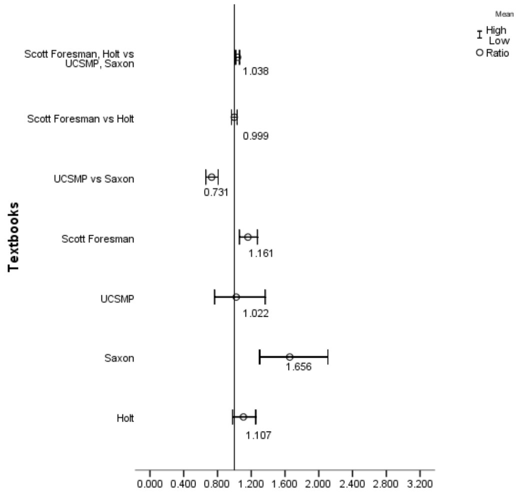
Grade 6. Without equal sign

For grade 6, there was no meaningful difference among the grouped textbooks because their value was not different from 1, however, when comparing SF to Holt and UCSMP to Saxon there was no difference between the former and the odds for students encountering expressions as equations in UCSMP was much less than in the Saxon textbook (see Figure 4). Most notably, when comparing each 2010 textbook to its cumulative ratings from prior years, the odds of encountering expressions as equations decreased slightly in SF, slightly increased in Saxon, and relatively no change in the odds for the other two textbooks.



Grade 2. Most conductive presentations for equal sign understanding

There were four problem contexts that were suggested to foster a relational meaning of the equal sign, (filling in missing numbers, reflexive, operation on the right side, and operations on both sides), thus these problem contexts were the basis for analysis (see Figure 5). In the grade 2 paired textbook comparison of SF and Holt to UCSMP and Saxon, there was no difference in the likelihood of second grade students encountering one of the four problem types in the most current edition of the textbooks. However, when comparing SF to Holt, students would be much more likely to see one of the four problem types when using Holt. When comparing UCSMP to Saxon, students were more likely to see the four problem types in UCSMP. Most notably, when comparing each 2010 textbook to its cumulative ratings from prior editions, students using SF had the greatest odds of seeing the four problem types, Holt and Saxon saw no changes, and students using UCSMP and Saxon were less likely to see the four problem types.



Grade 6. Most conductive representations for equal sign understanding

In Grade 6 textbooks, there were no meaningful differences between paired textbooks (see Figure 6). There was no difference when comparing SF to Holt, but when comparing UCSMP to Saxon, the odds of encountering the four problem types was much greater in UCSMP. When comparing each 2010 textbook to its cumulative ratings from prior editions, there were greater odds of encountering the four problem types in SF and a much greater likelihood in the current Saxon book than in previous years, with no change in Holt and UCSMP.

Discussion

One might expect that research identifying problem types aligned with better mathematics understanding would be adopted across time in student textbooks. This study partially supports this hypothesis. There is some association between textbook use of problem context and the historical findings accounting for student understanding of the equal sign. For example, early reports indicate that few students understood the equal sign as a relational symbol and indeed the textbooks used fewer problems contexts and there was little use of problem types suggested as the most effective for developing understanding of the equal sign. However, research studies since 2000 show that students now exhibit a greater facility with the equal sign (cf. Capraro, Capraro, Yetkiner, Corlu, Özel, Ye, & Kim, 2011; Knuth et al., 2006; Li et al., 2008; McNeil, 2008; McNeil et al., 2006) than did earlier studies (cf. Baroody & Ginsburg, 1983; Behr et al., 1980; Bernstein, 1974; Falkner et al., 1999; Kieran, 1981) and the identified textbooks also show an increased usage in the percentage and variety of problem contexts, which is conducive to a better understanding of the equal sign. While these textbooks were not necessarily the textbooks used in the reported studies, this research shows that textbooks during specific eras are typically similiar. Thus changes in textbooks over time are consistent with the changes observed in the student population. Most notably, except for Saxon, the recent edition of each textbook compared to its prior years was less likely to treat expressions as equations, a major change that is aligned with recent research. Recent studies report that about 30% of students have facility with the equal sign (Capraro et al., 2011), while SF, UCSMP, and Holt include at least 12% of the problem types most conducive to understanding the equal sign. While this study does not permit causal conclusions, this method provides insights for examination if the theoretical framework designed around problem variety or the use of specific problem types have been included to any greater degree since 2000 when improvements in students' relational understanding have been shown. One compelling issue is the use of expressions for students who are developing understandings of relational symbols. In early research, the "equals" button on the calculator was identified as a contributing factor in students' view of it as an operator. Therefore, do expressions without an equal sign, when accompanied by directions such as "compute" "simplify" or "solve" foster this same interpretation of the equal sign as the button on the calculator? This problem type continues to predominate across textbooks necessitating a better understanding about its role in developing relational understanding. In addition, do symbols such as the "equivalency bar" function to cloud relational understandings? When students are introduced to addition in the vertical format, directions in the teacher's manual often tell them use the word equals. However, this meaning can become convoluted in multiplication of 2-digit by 2-digit numbers when two equivalency bars are used. While the idea holds for multiplication, it does not fit long division. If the student, in his or her head. translates those "bars" as equals, then the mathematical sentence is not true. For example in

$$\begin{array}{r} 66 \\ 5\overline{)331} \\ \underline{30} \\ 31 \\ \underline{30} \\ 1 \end{array}$$

, this would be translated as 33 minus 30 equals 31 minus 30 equals 1, which are not true statements. This work provides several new ideas worthy of examination, for example, how students use both representations and text in developing their understandings. To date the operational framework has focused on item types that have changed little from the original studies in 70s and 80s. There is a need to examine the influence of items using the equivalency bar and without an equal sign.

Two compelling questions for future research are 1) How does textbook inclusion of key problem types assist in students' learning of relational symbols?; and 2) Would students with greater conceptualization of the equal sign at 2nd and 6th grades show improvement over their peers when they enroll in algebra in either 7th or 8th grade? While this study does not answer these questions, it provides a framework for considering an iteration in the theoretical framework that includes two new item types that may account for greater variance in understanding.

What we hope to accomplish from this study is that publishers and textbook authors find a middle ground for incorporating research findings into textbook development. In conclusion, if we expect teachers to infuse instruction with a greater variety of problem types that support learning about the equal sign and relational symbols, then textbooks need to provide greater representation of different problem types that support the understanding of equal signs. The research findings of this study and others (e.g. Capraro et al., 2011; McNeil, 2008; McNeil & Alibali, 2005) call for a greater variety in the presentations of the equal sign. However, as was shown in the textbook analyses, the variety of equal sign presentations has been very slow to emerge (cf. Li et al., 2008; McNeil et al., 2006). Thus it is incumbent upon teachers to provide students with supplemental material and examples of a greater variety of problem types when introducing, practicing and learning relational symbols because most textbooks are insufficient in this domain. Teacher guides for textbooks should include caveats that the equivalency bar should not be substituted for "equals" during the introduction of vertical addition, subtraction, or multiplication but introduced as a command to compute absent relational meaning. Finally, because of the preponderance of its representation in some textbooks, further research needs to be conducted to determine whether treating expressions as equations is detrimental to operational and/or relational understandings of the equal sign.

References

**Indicates references for textbooks examined longitudinally*

Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: Equal sign and variable. *Mathematical Thinking and Learning 9*, 249–272.

Baroody, A. J., & Ginsburg, H. P. (1983). The effects of instruction on children's understanding of the "equals" sign. *The Elementary School Journal, 84*, 198-212.

Behr, M., Erlwanger, S., & Nichols, E. (1980). How children view the equals sign. *Mathematics Teaching*, *92*, 13-15.

*Bell, M., Bell, J., Bretslauf, J., Dillard, A., Hartfield, R., Issacs, A., McBride, J., Pitvorec, K., & Saecker, P. (2000). *Everyday mathematics: The university of Chicago school mathematics project - Grade 2.* Chicago: Wright Group/McGraw Hill.

*Bell, M., Bell, J., Bretslauf, J., Dillard, A., Hartfield, R., Issacs, A., McBride, J., Moran, C., Pitvorec, K., & Saecker, P. (2010). *Everyday mathematics: The university of Chicago school mathematics project - Grade 2.* Chicago: Wright Group/McGraw Hill.

*Bell, M., Bretslauf, J., Dillard, A., Hartfield, R., Issacs, A., McBride, J., Pitvorec, K., Saecker, P., Balfanz, R., & Carroll, W. (2000). *Everyday mathematics: The university of Chicago school mathematics project - Grade 6.* Chicago: Wright Group/McGraw Hill.

*Bell, M., Bell, J., Bretslauf, J., Dillard, A., Hartfield, R., Issacs, A., McBride, J., McCarty, A.Pitvorec, K., & Saecker, P. (2010). *Everyday mathematics: The university of Chicago school mathematics project - Grade 6.* Chicago: Wright Group/McGraw Hill.

*Bennet, J. M., Burger, E. B., Chard, D. J., Jackson, A. L., Kennedy, P. A., Renfro, F. L., et al. (2007). *Holt mathematics course 1* (Texas ed.). Austin, TX: Holt.

Bernstein, B. E. (1974). "Equation" means "equal." *The Arithmetic Teacher, 21*, 697-698.

*Bolster, L.C., Cox, G. F., Gibb, E.G., Hansen,V. P., Kirkpatrick, J. E., Robitaille, D. F., Trimble, H. C., Vance, I., Walch, R., & Wisner, R. (1978). *Mathematics around us: Skills and applications (Grade 6).* Glenview, IL: Scott, Foresman and Company.

*Bolster, L. C., Gibb, E. G., Hansen,V. P., Kirkpatrick, J. E., McNerney, C. R., Robitaille, D. F., Trimble, H. C., et al. (1983). *Scott Foresman mathematics (Grade 6).* Palo Alto, CA: Scott, Foresman and Company.

*Bolster, L. C., Gibb, E. G., Hansen,V. P., Kirkpatrick, J. E., McNerney, C. R., Robitaille, D. F., Trimble, H. C., Vance, I., Walch, R., & Wisner, R. (1983). *Scott Foresman mathematics (Grade 2).* Palo Alto, CA: Scott, Foresman and Company.

*Bolster, L. C., Boyer, C., Butts, T., Cavanagh, M., Channel, M., Crown, W., Gibb, Fair, J., . . .Wisner, R. (1991). *Scott Foresman: Exploring mathematics (Grade 6).* Glenview, IL: Scott, Foresman and Company.

*Bolster, L. C., Boyer, C., Butts, T., Gibb, E. G., Cavanagh, M., Channel, M., Crown, W., Fair, J., . . .Wisner, R. (1991). *Scott Foresman: Exploring mathematics (Grade 2).* Glenview, IL: Scott, Foresman and Company.

Capraro, R. M., Capraro, M. M., Yetkiner, Z. E., Corlu, M. S., Özel, S., Ye, S., & Kim, H. G. (2011). An international perspective on students' understanding of the equal sign. *Mediterranean Journal for Research in Mathematics Education: An International Journal, 10,* 187-213.

Capraro, M. M., Yetkiner, E., Ozel, S. Capraro, R., Ye, S., & Kim, H. (2009, April). *An international perspective on sixth graders' interpretation of the equal sign*. Presented at the annual meeting of the American Educational Research Association, San Diego, CA.

*Charles, R., Chancellor, D. Harcourt, L., Moore, D. Schielack, J., Van de Walle, J. & Wortztman, R. (1999) *Math: Grade 2.*Menlo, CA: Scott Foresman-Addison Wesley.

*Charles, R., Dossey, J. Leinwand, S. Seeley, C. & Vonder Embse, C. (1999). *Middle school math: Course 1.* Menlo, CA: Scott Foresman-Addison Wesley.

*Charles, R., Crown, W., & Fennell, F. (2007). *Scott Foresman - Addison Wesley mathematics grade 2.* Upper Saddle River, NJ: Pearson Education.

*Duncan, E., Quast, W. G., Haubner, M. A., Cole, W. L., Gemmill, L. M., Alen, C. E., Cooper, A. M., & Capps, L. R. (1985). *Mathematics Grade 2.* Boston: Houghton Mifflin.

Erlandson, D., Harris, E., Skipper, B., & Allen, S. (1993). Gathering data. In *Doing naturalistic inquiry : A guide to methods* (pp. 80 – 110) . Newbury Park, CA: Sage**.**

Falkner, K. P., Levi, L., & Carpenter, T. P. (1999). Children's understanding of equality: A foundation for algebra. *Teaching Children Mathematics, 6,* 232-236.

*Fennell, F., Ferrini-Mundy, J., Ginsburg, H. P., Greenes, C., & Murphy, S. J. (1998). *Silver Burdett Ginn mathematics (Grade 2).* Parsippany, NJ: Silver Burdett Ginn.

*Fennell, F., Reys, B., Reys, R., & Webb, A. (1991). *Mathematics unlimited: Grade 6.* Orlando, FL: Holt, Rinehart & Winston.

Fyfe, E. R., & McNeil, N. M. (October, 2009). *Benefits of "concreteness fading" for children with low knowledge of mathematical equivalence*. Poster presented at the biennial meeting of the Cognitive Development Society (CDS), San Antonio, TX.

Ginsburg, H. (1989). *Children's arithmetic: How they learn it and how you teach it*. New York: Van Nostrand.

*Greenes, C., Leiva, M., & Vogeli, B. (2002). *Mathematics (Grade2).* Morris Plains, NJ: Houghton Mifflin.

*Greenes, C., Larson, M., Leiva, M. & Shaw, J. M., Stiff, L., Vogeli, B., & Yeatts, K. (2007). *Houghton Mifflin Math (Grade2).* Boston: Houghton Mifflin.

*Hake, S. (2008). *Saxon math: Course 1*. Orlando, FL: Saxon.

*Hake, S. & Saxon, J. (1987). *Saxon math 65: An incremental development*. Norman, OK: Saxon.

*Hake, S. & Saxon, J. (1999). *Saxon math 65: An incremental development* (2nd ed). Norman, OK: Saxon.

Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics, 27*(1), 59-78.

Hiebert, J. (1984). Children's mathematical learning: The struggle to link form and understanding. *The Elementary School Journal, 84*, 496-513.

*Keedy, M., Dwight, L. A., Nelson, C. W., Schluep, J., & Anderson, P. A. (1970). *Exploring elementary mathematics (Grade2).* New York: Holt, Rinehart & Winston.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12,* 317-326.

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37,* 297-312.

Ladson-Billings, G. (1997). It doesn't add up: African American students' mathematics achievement. *Journal for Research in Mathematics Education, 28,* 697-708.

*Larson, N. (2004). *Saxon math 2*. (Part 1 & 2). Norman, OK: Saxon.

*Larson, N. (1991). *Saxon math 2*. (Part 1 & 2). Norman, OK: Saxon.

Li, X., Ding M., Capraro, M. M., & Capraro, R. M. (2008). Sources of differences in children's understandings of mathematical equality: Comparative analysis of teacher guides and student texts in China and in the United States. *Cognition and Instruction, 26,* 195-217.

McNeil, N. M. (2007). U-shaped development in math: 7-year olds outperform 9-year olds on mathematical equivalence problems. *Developmental Psychology, 43*, 687-695.

McNeil, N. M. (2008). Limitations to teaching children 2+2 = 4: Typical arithmetic problems can hinder learning of mathematical equivalence. *Child Development*, *79,* 1524 – 1537.

McNeil, N. M., & Alibali, M. W. (2005). Why won't you change your mind? Knowledge of operational patterns hinders learning and performance on equations. *Child Development, 76,* 883 – 899.

McNeil, N. M., Grandau, L., Knuth, E. J., Alibali, M. W., Stephens, A. C., Hattikudur, S., & Krill, D. E. (2006). Middle-school students' understanding of the equal sign: The books they read can't help. *Cognition and Instruction, 24,* 367-385.

*Nichols, E. D., Anderson, P. A., Hoffman, S. A., Dwight, L. A., Kalin, R., Flournoy, F., Schluep, J., & Simon, L. (1981). *Holt mathematics (Grade 6)*. New York: Holt, Rinehart and Winston.

Reys, R., Reys, B., Tarr, J., & Chavez, O. (2006). *Assessing the impact of standards-based middle school mathematics curricula on student achievement and the classroom learning environment.* Washington, DC: National Center for Education Research.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural understanding: Does one lead to the other? *Journal of Educational Psychology, 91*, 175-189.

Sáenz-Ludlow, A., & Walgamuth, C. (1998). Third graders' interpretations of equality and the equal symbol. *Educational Studies in Mathematics, 35*, 153-187.

Seo, K-H., & Ginsburg, H. P. (2003). "You've got to carefully read the math sentence…": Classroom context and children's interpretations of the equal sign. In A. J. Baroody & A. Dowwker (Eds.), *The development of arithmetic concepts and skills.* Mahwah, NJ: Erlbaum.

Thompson, B. (2006). *Foundations of behavioral statistics: An insight-based approach*. New York: Gilford.

Tornroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation, 31*, 315-327.

Usiskin, Z. (1995). Why is algebra important to learn? *American Educator, Spring*, 31-37.

Warren, E. (2003). *Young children's understanding of equals: A longitudinal study*. In N. Pateman, G. Dougherty, & J. Zillox (Eds.). Proceedings of the 27th Conference of the International Group for the psychology of Mathematics Education and the 25th Conference of Psychology of mathematics Education North America (Vol 4, pp. 379-387). Honolulu, HI: University of Hawaii.

Westbury, I. (1990). Textbooks, textbook publishers, and the quality of schooling. In D. L. Elliott & A. Woodword (Eds.*), Textbooks and schooling in the United States.* Chicago: University Press.

What Works Clearinghouse. (2007). *Intervention report: Saxon mathematics.* Washington, DC: US Department of Education.

Authors:

Robert M. Capraro
Texas A&M University
rcapraro@tamu.edu

Mary Margaret Capraro
Texas A&M University
mmcapraro@tamu.edu

Rayya Younes
Texas A&M University
rayya@neo.tamu.edu

SunYoung Han
Texas A&M University
sunyounghan@tamu.edu

Kayla Gardner
Texas A&M University & Bryan ISD