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KOSINUS: An Approach to Long-Term Development of Mathematics Education

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The paper presents a program for in-service teacher training which aims at changing teachers' subjective theories of teaching mathematics, thereby achieving a sustainable impact on teachers' actions in the classroom. The concept of subjective theories will be briefly outlined distinguishing it from the concept of beliefs. The paper sketches the organizational framework taking into consideration the background of the German education system. A focus is on the contents of the training program. Their selection and arrangement is vital in altering teachers' subjective theories due to the chosen contents' effectiveness in initiating experiences of discrepancies. Examples of the selected contents are given to illustrate this point. Finally, the evaluation concept will be outlined and first findings will be presented from the different stages of evaluation. The program has been running for six years now and the experiences reported here can be a valuable resource for the development of future comprehensive in-service training programs.

Key words: *conceptual change, constructivist theories, in-service teacher training contents, subjective theories*

In this paper I am going to outline the KOSINUS[1]-program, an in-service teacher training program in the Saarland, a German federal state. The program has been planned, carried out, and evaluated by Saarland University in cooperation with the local teacher training institute. With this training program--now in its sixth and last year--we attempt to change teachers' subjective theories of mathematics in general as well as their teaching of mathematics with the particular aim of implementing a more constructivist view on math-teaching in secondary schools.
Our research aspires to answer the question whether a specifically designed training program can change teachers' subjective theories in a significant way into a constructivist direction. Another aim is to gain insights into the precise effects of specific course contents and design, with a view to making these available for other teacher training programs.

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[1] Kompetenzen stärken im Mathematikunterricht der Sekundarstufen im Saarland [Developing math teaching competencies for secondary schools in the Saarland]

I will first briefly explain the need for the program in the regional educational context. This is followed by a short clarification of the concept of subjective theories, differentiating it from other terms and concepts. Then I will sketch the organizational and the methodological framework of the training and give some examples of the training contents. After that I will formulate criteria for the in-service training course contents. Finally, I will give an overview of the program evaluation process.

The Need for KOSINUS

The implementation of nationwide Education Standards (Bildungsstandards) for secondary education in Germany since 2003 has triggered a massive transformation of math curricula with new contents, aims, and challenges. The concurrent introduction of centralized testing in most of the German federal states has added to the pressure on serving teachers so that there is a dramatic need in secondary schools for substantial support, in particular among older generations of math teachers. This need for support is best described by the German word 'Leidensdruck', the deep distress of teachers trying to cope with these new demands.

The task of university researchers in this context is to accompany these changes and provide the needed support in a critical and constructive way. This essentially means going into the schools to work collaboratively with teachers and school management. This connection of practical work in schools with scientific research on subject pedagogy and didactics allows for the establishment of a meta-view on the daily problems in the classroom--a meta-view necessary for a reflection of the teaching of mathematics. This effort is needed to work towards the ideal of the teacher as a reflective practitioner (Schoen, 1983; 1987) and with that towards the professionalization of teaching. Implementing KOSINUS we considered long term aims and long term effects to guarantee a process of change not only on the surface but in the special sets of cognitions of math teachers and consequently in the classroom (cf. Chroust, 2003; Lipowsky, 2004).

While empirical studies show that German math-teachers are, in principle, open to constructivist theories, the practice of German mathematics teaching is still largely shaped by a non-constructivist view on learning (cf. Dubberke, Kunter, McElvany, Brunner, & Baumert, 2008). Indeed, the constructivist view of learning mathematics is not anchored in teachers' subjective theories, and hence not put into practice in their daily work at school. Another goal of KOSINUS is to enable teachers to connect process-based competencies such as problem solving or argumentation with the mathematics curriculum content as well as to encourage them to accept that there is more than one way to solve a task and that there can be more than one possible solution for a math problem.

Finally, KOSINUS aims to encourage reflection and exchange of knowledge and experience within a professional context by establishing a climate of cooperation and communication in schools, which in the long run could lead to improved teaching practices. German teachers mostly consider themselves 'Einzelkämpfer', a common metaphor framing teaching as 'single combat', that is, they act alone and unobserved behind the closed doors of the classroom or their study. For most, cooperation is not part of their professional self-perception (Fastner & von Saldern 2010; Schmich & Burchert 2010).

Subjective Theories

Profound changes in teaching can only be accomplished by altering the above mentioned subjective theories (Dann, Humpert, Krause, & Tennstädt, 1982; Dann, 1983; 1989; 1994; Groeben & Scheele, 1982; Groeben, Wahl, Schlee, & Scheele, 1988; Hofer, 1981; Mutzeck, 1988; Mutzeck, Schlee, & Wahl, 2002). These are sets of cognitions that determine (math) teachers' practices in the classroom and everyday school life (Thompson, 2004).

Related terms employed in many publications on in-service teacher training are 'beliefs' and 'attitudes'. "Attitudes and beliefs are a subset of a group of constructs" (Richardson, 1996, p. 102), while subjective theories--through their similarity in name and structure to objective scholarly theories--show that there is a relationship to objective scholarly theories and their logical structure. Pajares (1992) states that the use of the concept of beliefs is justified when used in contrast with knowledge. "Belief is based on evaluation and judgment; knowledge is based on objective fact" (p. 316). However, he rightly criticizes: "The construct of educational beliefs is itself broad and encompassing. For purposes of research, it is diffuse and ungainly, too difficult to operationalize, too context free" (p. 316).

Therefore, focusing on subjective theories instead of the more widespread concept of 'beliefs' makes it possible to analyze very specific actions taken while teaching mathematics through examining teachers' explanations and justifications of these actions, in particular when distinguishing constructivist from transmission approaches (Dubberke, Kunter, McElvany, Brunner, & Baumert, 2008).

The organizational framework of close mentoring--on the basis of 'conceptual change' theories (Murphy & Mason, 2006; Posner, 1982)--together with the choice and arrangement of course contents, allow for an uncovering of teachers' subjective theories, which then in turn can be confronted with scholarly knowledge to initiate experiences of discrepancy between new ideas and their daily work and thus daily experience in school. In the long run, these experiences of discrepancy then can trigger changes in teachers' actions.

The Organizational and Methodological Framework of the Training

In order to achieve a confrontation of teachers' subjective theories with objective scholarly theories and trigger changes in their classroom activities, a long-term in-service training program is necessary. A significant issue is how to establish an organizational frame for such a long-term project, in particular how to guarantee a mentoring of all math teachers at one school for one year. This can only be achieved through an inter-institutional framework spanning the educational administration, the university, and the in-service training institute. This wide framework has allowed us, so far, to support nearly 60 secondary schools, overseeing the professional development of the participating school teachers, and the the professional development of the teacher trainers involved in the program. Inter-institutional cooperation allows for their high level qualification through special trainings and continually provides supervision as well as a regular exchange of experiences and ideas. School participation is voluntary; in fact it is only possible when the school's math division[2] reaches a consensus to join the program. The project runs for one year at each school. Because changes in the educational system are generally a very slow process, this is still only a short span and therefore follow-up programs will be required.

In order to accomplish the close mentoring of all math colleagues at participating schools for a year, KOSINUS provides the following organizational framework:

- One mentor (the original German term is 'Berater fuer Unterricht') who is responsible for four schools and counsels these schools with four full-day in-service training sessions as a base offer and more on a voluntary basis. Moreover, this mentor can be contacted and invited for further visits, if problems appear.
- Mentors give an intermediate and a final report to the head of school and to the head of the math department. They point out development options for the math division at each school.
- At the conclusion of the program, teachers are invited to participate in further in-service trainings tailored to their needs.
- In addition to the mentoring of the math teachers, there are also accompanying trainings for the school's administrative staff as well as for the head of the math division.

Because German schools traditionally have focused more on an individualistic than a cooperative work environment (Fussangel & Gräsel,

[2] All mathematics teaching stuff at one school.

2011; Helmke, Hosenfeld, Schrader, & Wagner, 2002; Rüegg, 2000; Schmich & Burchert, 2010), one crucial aim of the program is to develop a culture of cooperation and communication in the mathematics staffroom, providing a basis for profound changes in the classroom. Face-to-face exchanges between KOSINUS schools are made possible via an annual meeting of all participating math departments and electronically through an internet platform. The latter has not been used as much as planned, presumably by the fear of electronic teaching tools in general. Participating schools also receive a significant budget to buy classroom supplies for math teaching. This is restricted to hands-on materials to be used by students.

While conducting the program, we have continually been learning about the workings of such complex professional development, and were able to improve parts of the framework concerning the organization as well as the training contents. One of the difficulties we had to overcome was the adaption of the training program to the varying requirements of the different types of public secondary schools coexisting in the Saarland. After the fourth grade, the end of primary education in Germany, children are separated and attend either a comprehensive school (*Gemeinschaftsschule*) or a selective school (*Gymnasium*). Primary school teachers recommend their students to a particular school based on academic ability and learner autonomy. However, as in most German federal states, parents have the final say as to which school their child attends following the fourth grade. The *Gymnasium* is highly selective, it focuses on preparing students for university study and leads to a diploma called the *Abitur*, which is the university-entrance qualification. Pupils at the *Gemeinschaftsschule* graduate after 9 or 10 years leading to enrolment in vocational schools combined with apprenticeship training. Students with high academic achievement in the *Gemeinschaftsschule* can move on to a *Gymnasium* or a higher vocational school after graduation and acquire a university-entrance qualification. The comprehensive schools differ from school to school in the extent to which they teach students in mixed ability groups or stream them according to their academic abilities. Consequently, the training program had to be tailored to the needs of three groups of teachers ranging from highly selective and streaming systems to mixed ability systems. With parents having more of a say now and a general tendency towards allowing more heterogeneity, however, teachers in the more selective systems need most support in how to deal with mixed ability groups.

The development of the program has benefitted from SINUS[3] (Krebs, 2008; Ziegler, 2006;) and IMST[4] (Krainer, Hanfstingl, & Zehetmaier, 2009), in-service training programs well established in Germany and Austria,

[3] Steigerung der Effizienz des mathematisch-naturwissenschaftlichen Unterrichts [Improving science and mathematics instruction]

[4] Innovationen Machen Schulen Top [Better schools through innovation]

respectively. Hence the name KOSINUS, crediting the already established German program SINUS.

Selection of The Program Contents

One of the most important questions is which in-service training contents are necessary to reach changes in the math teachers' cognitions and which arrangement of these contents brings about the best opportunities for change. As we know, the knowledge base for teaching can be divided into, amongst others, 'subject-matter content knowledge' and 'pedagogical content knowledge' (Baumert & Kunter 2006; Schoenfeld 2006; Shulman, 1986). When selecting the contents for the in-service training sessions, these two need to be scrutinized as separate components as well as in their systemic and interdependent relations. Subject-matter content knowledge, in particular, is a necessary prerequisite for a good teaching experience. As the COACTIV[5]-study revealed in an alarming way, many secondary teachers in Germany only have a very basic subject-matter content knowledge. (Kunter et al., 2011) This is problematic because research has shown that without a solid base in subject-matter content knowledge pedagogical content knowledge cannot develop (Brunner et al., 2006; Ma, 1999).

Furthermore, contents that can change 'subjective theories' in teachers' doing must also establish a link between scholarly knowledge and everyday practical knowledge for the classroom. This means selecting new findings from current research on mathematics education and giving the teachers a chance to try those in their own teaching experience. Building a knowledge gained from experience is one of the aims of good in-service teacher training, that is, a knowledge which grows by integrating new contents into old structures while simultaneously adjusting these old structures with the help of this scholarly knowledge. We distinguish between 'demonstrative contents,' which are able to provoke discrepancies, and 'arranging contents,' which enable a cognitive re-arranging after perturbation.

Knowledge gained from experience grows horizontally. It has a narrative structure and is founded on somebody's own concrete teaching experiences (Herzog, 2004). The narrative structure renders this knowledge easy to spread throughout the staffroom. In order to create and spread new subjective theories, it is crucial to have teachers make these new experiences based on current scholarly theories.

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[5] Professionswissen von Lehrkräften, kognitiv aktivierender Mathematikunterricht und die Entwicklung mathematischer Kompetenz [Professional Competence of Teachers, Cognitively Activating Instruction, and Development of Students ´Mathematical Literacy]

On the basis of these premises, we established five criteria for the selection of course contents for KOSINUS:

1. Contents should not be selected solely on the basis of teachers' demands. This would cause a tautology because teachers tend to choose the contents they are already proficient in. (Brunner et al. 2006; Fastner & von Saldern 2010) One of the aims of in-service training is to bring new ideas into schools and into teaching experience--new ideas emerging from the current discourse in mathematics education research.
2. The contents must guarantee discrepancies with teachers' existing subjective theories, thereby provoking contradictions with their current routines and hence triggering a process of perturbation that causes the teachers to reflect on their daily work ('demonstrative contents').
3. The contents have to be directly relevant for the teaching practice. Teachers must be provided with a theoretical background of the content and they have to be able to try this content in one of their next lessons, and be able to arrange their old and their new experiences in a new way with the help of the 'arranging contents'.
4. Contents must be presented in a situated way, meaning that teachers must be familiarized with situations in which they can use the contents. This also means that teachers have to work on, for example, tasks connected to the contents by themselves before giving them to students.
5. The arrangement and order of the contents requires careful consideration because the construction of knowledge is cumulative. Therefore we progress from the smallest unit, the task, to the planning of a year's syllabus. With this kind of progression, the mentor is also able to gradually move into the background, granting participants more and more autonomy--in line with the concept of cognitive apprenticeship (Collins, Brown, & Newman, 1989).

Here are some examples of the contents meeting these criteria:

- having teachers classify tasks on the basis of criteria for the handling of knowledge-representation (Sjuts, 2001, 2002) to create a more effective variety of tasks used in lessons;
- raising teachers' awareness of different thinking styles in mathematics (Schwank, 2003) to improve their diagnostic skills;
- presenting teachers with innovative hands-on classroom materials that pupils can work with, exposing Bruner's (1974) three representations of enactive-iconic-symbolic particularly in mathematical fields in which this is not normally done (e.g. algebra);
- using student answers to practice the identification and analysis of typical mistakes in mathematics, especially in the lower grades, and to learn how to help students eliminate these mistakes;

- presenting examples of tasks for math lessons and exams and discussing their quality as well as working together on the improvement of these tasks for example with a view to mixed ability learning environments;
- promoting conceptualization as a key content of math teaching and giving advice to participants on how to implement this as an integral part of their own math teaching;
- working on the planning and evaluation of lessons and units focused on process based skills of mathematics, as e.g. problem solving, arguing, and communicating about mathematics. (Kultusministerkonferenz 2004, 2005);
- observing math lessons taught by colleagues and exchanging ideas about these as well as discussing and agreeing on common aims of math teaching in a school math divisions fostering cooperation and communication.

Knowing that the math divisions are very heterogeneous in their knowledge and experience--just like the pupils, we have adapted some of the contents depending on the individual schools' or math divisions' background. During the program, we learned that a rigid system of contents is not useful in promoting an atmosphere of trust and open communication. We encountered resistance in some math divisions and the support of the administration was not equally strong across all schools. This led us to change the terms of participation to the extent that there had to be a clear majority vote of all math teachers in a school to participate in the program. Only under this condition applications were considered.

Preliminary Evaluation of The Program

We are in the process of evaluating the program in three ways. First, we have asked the teachers to note down what they think they have learned during the program. Issues mentioned in these notes ranged from getting acquainted with different forms of tasks for learning mathematics, in particular in view of the requirements of the new educational standards to working with hands-on classroom materials such as geoboards.

Secondly, we conducted a survey amongst the participating teachers on their experiences with this program, also concluding on their preferences for future in-service trainings. Analyzing those surveys, the most striking result was that most teachers reported on better communication and cooperation in their math division through the program and that they highly appreciated this change. As desired further trainings they mostly specified trainings on computer software for math teaching (e.g. GeoGebra) as well as methods that allow pupils to work more autonomously in math lessons (e.g., cooperative learning).

Finally, we carried out interviews using the repertory-grid-method analyzing the personal constructs of participating teachers (Kelly, 1986; Neveling, 2007) to find out how their subjective theories changed during the

program. This method suggested itself due to the similarities between the concepts of subjective theories and personal constructs (cf. Mutzeck, Schlee, & Wahl, 2002). The benefit of using a repertory-grid interview rather than a standard interview is that the available data can be used for qualitative as well as quantitative analysis. For this project, however, the qualitative analysis provided most insight to the project.

For the interviews we met the teachers before they had started the program and we met with them a second time half a year after the conclusion of the program. In filling out the repertory grids the teachers had to justify their opinions. We transcribed those rationales and in the close analysis of these transcriptions we singled out descriptions of subjective theories on the basis of categories (Dann et al., 1982; Dann 1983; 1989; Laucken, 1973).
These descriptions were then classified in a matrix that ranges from object theories (if-then), to practical theories (in order to), and finally to practical work (concrete tasks and doing). Applying Bourdieu's theories (Bourdieu, 1979; 1982; 2001), they can then also be shown to range from practical sense as the lowest form of self-reflection to scholarly theories as the highest form of self-reflection.

	Object Theories	**Practical Theories**	**Implementation Aid**
Example (before)	**If** students are to master mathematics, they have to practise a lot.	**In order to** be able to practise a lot, students need to be provided with stereotypical exercises.	sets of stereotypical exercises
Example (after)	**If** students are to master mathematics, they have to practise intelligently.	**In order to** be able to practise intelligently, students need to be provided with rich mathematical tasks and exercises which allow varied access .	rich tasks
Practical sense			
Common sense			
Plausible generalization of experience			
Theories of science			

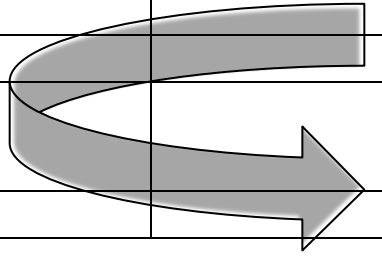

Figure 1. *Developmental path of teachers' subjective theories*.

Classifying the descriptions contained in teachers' first and second interviews in this matrix then allowed us to follow the development of teachers' subjective theories throughout the program by observing changes in the classifications. The ideal development of teachers' subjective theories as

prompted by the training program is sketched in the figure below: They move from practical theories to object theories, at which point they can be reflected on a meta-level matching them with scientific theories. In a final step, they then move back into the practical work via adapted practical theories and matching implementation aids (see

Figure **1**). The examples at the top of the table serve to illustrate the given categories and the developmental path.

The first analysis shows that those teachers who entered the program equipped with a willingness and competence to reflect on their actions have kept this high level of reflection throughout the program. Teachers who were less reflective at the outset have changed their actions less throughout the program, and accordingly, their subjective theories have not shifted significantly. Finally, those teachers already preferring a more constructivist approach of math teaching at the outset have successfully developed this approach further throughout the program.

Conclusion

It is self-evident that changes in subjective theories take years and that we are only able to see the beginnings of change. However, the program and its evaluation have at least given us some hints at the workings of long-term teacher training programs and their suitability for changing subjective theories. The short-term evaluation of the KOSINUS in-service training, after only six months, has shown that subjective theories have changed and have been partly exchanged by scholarly theories. However, an evaluation at this point in time does not yield sufficient evidence of the ensuing change of actions in the classroom towards a more constructive and less of a transmission approach. Knowing that subjective theories and teachers' actions in the classroom have a strong coherence let us presume that changes will follow. Hence, another evaluation at a later point in time is desirable.

Another finding is that open-minded and highly reflective teachers benefit more than others, presumably because it is easier for these teachers to (ex)change their subjective theories and improve their skills for math teaching. However, our efforts also have also revealed that it is impossible to appeal to all teachers in a math cluster. From our constructivist point of view this is not a surprising finding, but this still begs the question whether--when trying to implement new didactic or pedagogic ideas in a system--it is more efficient to work only with the motivated and open-minded teachers of a division or with the whole group.

KOSINUS also showed that a sustainable change in communication and cooperation patterns amongst teachers away from the typical single combat metaphor towards joint action can only be achieved through the establishment of long-term in-service trainings. It was the climate of confidence and the

eye-level discussions that gave us access to the most urgent challenges in the mathematics classroom and enabled us work on them with an academic input. This is confirmed by the ongoing interest of the first round of participating math divisions, who are still in touch with us and contact us about special trainings and academic advice whenever they have jointly come to realize a challenge in their classrooms. This program design therefore also forms the ideal basis for implementing the recently published national educational standards for the advanced secondary level (grades 11 to 13) (Kultusministerkonferenz, 2012).

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